EUROPEAN CENTRAL BANK

THE NATURAL REAL INTEREST RATE AND THE OUTPUT GAP IN THE EURO AREA

A JOINT ESTIMATION

by Julien Garnier
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CONTENTS

Abstract

The notion of a natural real rate of interest, due to Wicksell (1936), is widely used in current central bank research. The idea is that there exists a level at which the real interest rate would be compatible with output being at its potential and stationary inflation. This paper applies the method recently suggested by Laubach and Williams to jointly estimate the natural real interest rate and the output gap in the euro area over the past 40 years. Our results suggest that the natural rate of interest has declined gradually over the past 40 years. They also indicate that monetary policy in the euro area was on average stimulative during the 1960s and the 1970s, while it contributed to dampen the output gap and inflation in the 1980s and the 1990s.

Key words: Real interest rate gap, output gap, Kalman filter, euro area

JEL Classification Numbers: C32, E43, E52, O40

Non-technical summary

The notion of a *natural rate of interest* knows a revival of interest in current monetary policy research. From a theoretical point of view, the natural real rate of interest is a central concept in the literature because it provides policymakers with a benchmark for monetary policy: Interest rates above the natural rate are expected to lower inflation, whereas rates below the natural rate are expected to raise inflation. In practice, however, the natural real rate is unobservable and has to be estimated.

According to recent contributions to the literature, the concept of the natural real interest rate and the concept of potential output should be consistent, so that given the definition of potential output it will be clear how to define the natural real interest rate. The two equilibrium-variables are expected to change together over time with shocks and should therefore not be estimated independently. In line with this argument we estimate the natural real rate of interest and potential output simultaneously using data for the euro area, Germany and the US.

The modelling framework consists of a small macroeconomic model encompassing an IS curve and a Phillips curve that connects inflation, the output gap and the real interest rate gap, defined as the difference between the real short term interest rate and estimates of the natural real interest rate. In addition, the natural real interest rate is linked to the potential growth rate of the economy. This produces an estimate of the natural real interest rate that varies over time in harmony with lower-frequency evolutions in the real economy. We use relatively long datasets, starting in the early 1960s.

Our results suggest that the natural real rate of interest in the euro area and Germany has declined gradually over the past 40 years, while the natural real interest rate in the US has been more stable around its long term average. The natural rate of interest in the euro area was, on average, higher than the actual rate in the 1960s and 1970s, while it was lower compared to the actual rate most of the time in the 1980s and 1990s. In other words, monetary policy was stimulative during the 1960s and 1970s, while it was on average tight in the 1980s and 1990s.

Regarding the output gap, the length of the business cycle's booms and busts are in line with the consensus view in the business cycle literature. However, the estimated output gap is influenced by the real interest rate gap, which implies that its average level is positive in periods of loose monetary policy (1960s and the 1970s) and negative in periods of tight monetary policy (1980s and 1990s). Indeed, the coefficient of correlation between the real interest rate gap and the output gap is strongly negative for the euro area. Moreover, we argue that the real interest rate gap may contain valuable information about future inflation in the euro area. However, it should be borne in mind that estimates of the natural real interest rate are imprecise.

1 Introduction

In the long run, economists assume that nominal interest rates will tend toward some equilibrium, or "natural", real rate of interest plus an adjustment for expected long-run inflation. The natural rate of interest is a central concept in the monetary policy literature since it provides policymakers with a benchmark for monetary policy. In theory, it is important for evaluating the policy stance since rates above (below) the natural rate are expected to lower (raise) inflation.

From an empirical point of view, the "natural" real rate of interest is unobservable. The estimation of the natural real interest rate is not straightforward and is associated with a very high degree of uncertainty. In practice, therefore, policymakers cannot rely exclusively on the real interest rate gap, defined as the difference between the real short term interest rate and estimates of the natural real interest, as an indicator of the monetary policy stance. Rather, a comprehensive approach using a wide set of information is required. This notwithstanding, central bank economists have increasingly devoted attention to developing estimation strategies for the natural real interest rate. The methods used range from as simple as calculating the average actual real interest rate over a long period to building dynamic stochastic general equilibrium (DSGE) models with nominal rigidities.

A recent contributions to the literature on how to empirically approach the concept of the natural rate is a paper by Laubach and Williams (2003; henceforth LW)[1] with an application to data for the United States. They suggest to estimate the natural real interest rate and potential output growth simultaneously, using a small-scale macroeconomic model and Kalman filtering techniques. In this model, the natural real interest rate is related to the potential growth rate of the economy. Thus, the estimate of the natural real interest rate is time-varying and related to long-term developments in the real characteristics of the economy, consistent with economic theory. This method has become popular since it strikes a compromise between the theoretically coherent DSGE approach and ad-hoc statistical approaches, as emphasized by Larsen and McKeown (2002).

In this paper we employ the technique of LW for the euro area. The present work differs from others in that we use a relatively long (synthetic) dataset starting in the early 1960s. Besides, we estimate the natural real interest rate in Germany and the US for comparison. Such comparison is interesting because, prior to 1999, monetary policies in Europe were considerably influenced by the Bundesbank policy. At the same time, the US is often considered as a useful proxy for global influences that affect monetary policy worldwide. Third, we apply simple statistical tests to investigate the leading indicator properties of the baseline estimate of the euro area real interest rate gap on inflation and economic activity.

The paper is structured as follows. In section two we take a look at the data for the real interest rate from 1960. Sections three reviews the recent empirical literature and discusses the natural rate concept in the context of different horizons. Section four presents the

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[1]Some papers have already used this framework on European data, including Sevillano and Simon (2004) for Germany, Larsen and McKeown (2002) for the United Kingdom and Crespo-Cuaresma et al (2003) and Mésonnier and Renne (2004) for the euro area.

modeling approach and displays the estimation results. Section five briefly discusses the leading indicator properties of the estimated real rate gap. Section six concludes.

2 The data

In this section we take a closer look at the data used in this study. The data covers a total of 161 quarterly observations from 1963q1 to 2004q1 for the euro area and Germany and 162 quarterly observations from 1961q1 to 2002q4 for the US. The dataset consists of short-term interest rates, inflation and Gross Domestic Products for the three economies[2]. The computation of real interest rates is subject to several practical and conceptual difficulties. Ideally, an estimate of the real interest rate should be obtained by substracting ex ante inflation expectations from nominal interest rates. However, the lack of good data for inflation expectations forces us to take a more straightforward approach. In this paper, the real interest rates are calculated from the three-month money market rates and annual consumer price inflation rates[3]. While we believe that the problem with using current inflation as a proxy for inflation expectations is less severe when assessing developments over longer horizons, the deviations may be stronger in periods with unanticipated inflation, notably in the 1970s.

It should be borne in mind that monetary policy regimes differed significantly over time and across countries. Moreover, in many euro area countries, specifically in the 1960s and early 1970s, other instruments than interest rates were important in the conduct of monetary policy. In particular, capital controls prevailed in many euro area countries. Furthermore, inflation, economic growth and interest rates were very volatile in some euro area countries. Finally, the euro area real interest rate has been significantly influenced by other factors than monetary policy, such as tensions within the Exchange Rate Mechanism (ERM) at the turn of the 1990s (Cour-Thimann et al, 2004).

The euro area real interest rate fell dramatically in the 1970s, when overheated economies and rising oil prices pushed up inflation at a level that could not be offset by the nominal real interest rates (See Figure 1). Following the trough in the mid-1970s, as European monetary authorities gradually put more emphasis on disinflationary policies, the real interest rate increased slowly over a period of more than 15 years. After peaking in the early 1990s, the real rate declined gradually again, influenced by the monetary authorities' achievement of more favourable inflation developements.

Similar to the euro area aggregate, the real interest rate in the United States was also lower than average in the 1970s and higher than its average in the 1980s. However, the persistence in the data seems less pronounced, as indicated by the quick rise in the real rate at the turn of the 1980s. In Germany the real interest rate has been more stable around its long-term average, reflecting the achievment of lower and more stable inflation over the whole sample.

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[2]We are grateful to the ECB for providing the data for Germany and the euro area, and Thomas Laubach and John C. Williams for providing the US data.

[3]For the euro area, national levels for interest rates and consumer prices have been aggregated prior to 1999 using GDP and consumer spending weights respectively at PPP exchange rates, see ECB (2003).
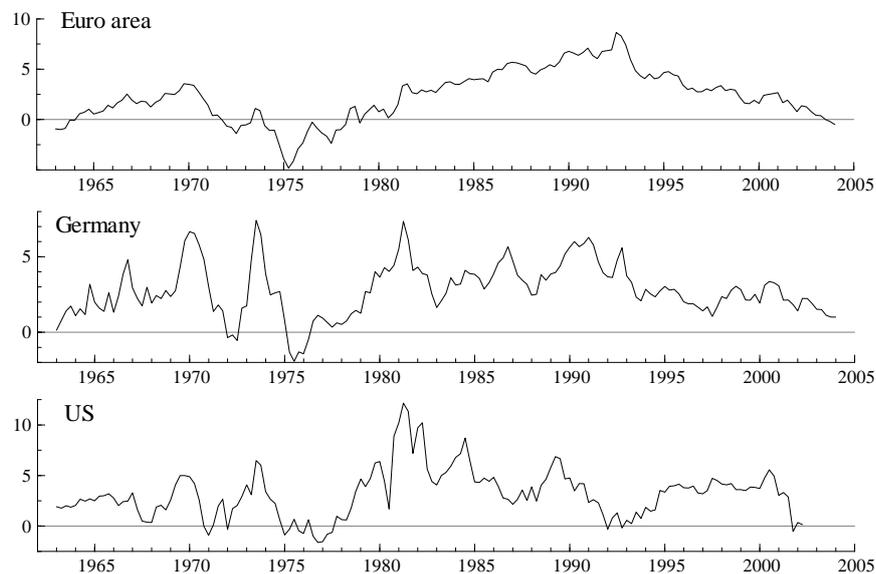
Figure 1: Real interest rates

3 A review of the recent empirical literature: short vs long-run perspectives

The concept of a natural rate of interest was first introduced by Knut Wicksell in the late 19th century (1898, with 1936 translation). Today the concept knows a revival of interest following Woodford's seminal book, Interest and Prices. According to the recent literature, fluctuations in the real interest rate may be decomposed into two different components: a natural real rate and a real rate gap (Woodford, 2003; Neiss and Nelson, 2003; Cour-Thimann et al, 2004). The natural real rate is related to structural factors and is the real interest rate that in theory would prevail under perfectly flexible prices. This is commonly referred to as the "Wicksellian" definition of the natural rate of interest. The real interest rate gap is related to the business cycle and reflects the existence of nominal rigidities in the economy.

The available estimates of historical developments in the euro area natural real interest rate differ considerably from one author to the other. We briefly address these differences below and classify estimates of the natural real rate, taking as cirterium the time horizon at which they should be interpreted.

Some papers find that most of the fluctuations in the real interest rate should be attributed to fluctuations in the real interest rate gap rather than the natural real interest rate. This group of papers, which includes Giammarioli and Valla (2003), Mésonnier and Renne (2004), Neiss and Nelson (2003), Sevilliano and Simon (2004) and LW, associate the fluctuations in the natural real interest rate with the evolution of real fundamentals such as determinants of trend GDP growth and preferences. These variables are typically stable in the short to medium term, but may display some variation in the longer run. Consequently,

the natural real interest rate is also relatively stable in the short run, and the natural rate in these papers should be considered in a "long-run" perspective. It refers indeed to the level expected to prevail in, say, the next five to ten years, after any business cycle "booms" and "busts" underway have played out. Note however that the estimated natural real rate of Mésonnier and Renne (2004) is much more volatile that that of LW or Sevilliano and Simon (2004).

On the contrary, other papers conclude that fluctuations in the natural real interest rate explain most of the variation in the real interest rate (Basdevant et al, 2004; Cuaresma et al, 2004; Cour-Thimann et al, 2004; Larsen and McKeown, 2002). The papers consistent with this view typically make use of the Kalman filter or other filtering techniques to split the actual real rate into a trend (the natural real rate) and a cyclical component (the real rate gap). However, the models they use do not necessarily contain judgements about the determinants of the natural rate. Rather, the approach they take is closer to a pure statistical measure. Consequently, variations in the natural rate are more pronounced, because the natural rate tends to follow more closely the medium term fluctuations in the actual real rate. The interpretation of the natural real interest rate in this context is therefore likely to be more relevant in a "shorter" time perspective in that it refers to a neutral monetary policy stance in a situation where the economy has not necessarily settled at its long-run levels.

4   Estimating the natural rate of interest

This paper takes a "long-run" time-perspective and uses economic theory as a benchmark for determining the developments in the natural real interest rate. As recalled by LW, standard growth models imply that the natural real interest rate varies over time in response to shifts in preferences and the trend growth rate of output, themselves unobservable variables.

4.1   The model

The empirical framework suggested by LW is to run the Kalman filter on a system of equations to jointly estimate the natural real interest rate, potential output growth and the output gap. They propose a model of a neo-keynesian inspiration, that jointly characterises the behaviour of inflation and the output gap through modified IS and Phillips curves. Neo-keynesian models are not so much interested in the levels of variables composing these curves, but rather the deviations from equilibrium values. The main equations of the model are given by:

$$a_y(L)\tilde{y}_t = a_r(L)\tilde{r}_t + \varepsilon_{1,t} \tag{1}$$

$$b_\pi(L)\pi_t = b_y(L)\tilde{y}_t + \varepsilon_{2,t} \tag{2}$$

where $\tilde{r}_t$ is the real interest rate gap, $\tilde{y}_t$ represents the output gap defined as the difference between the (log) GDP y_t and (log) potential output y_t^* such that

$$\tilde{y}_t = 100(y_t - y_t^*) \tag{3}$$

π_t is consumer price inflation, $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are white noise errors, and a_y, a_r, b_π and b_y are polynomials in the lag operator L such that $a_y(L) = -\sum_{i=0}^{n} a_{y,i} L^i$, with $a_{y,0} = -1$.

The laws of motion of unobservable potential output and its trend growth rate are specified as the following:

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{4,t} \qquad (4)$$
$$g_t = g_{t-1} + \varepsilon_{5,t} \qquad (5)$$

where $\varepsilon_{4,t}$ and $\varepsilon_{5,t}$ are white noise errors.

The economic theory imposed by LW is represented by the following relationship for the natural real interest rate:

$$r_t^* = cg_t + z_t \qquad (6)$$

where g_t is the unobservable trend growth rate of the economy, which is linked to the natural real interest rate r_t^* with the parameter c, capturing the relative risk aversion. z_t represent other possible determinants of the natural rate of interest, such as households time preferences, variation in public saving and uncertatinty about interest rates. z_t is assumed to follow a stochastic process determined by:

$$z_t = \alpha z_{t-1} + \varepsilon_{3,t} \qquad (7)$$

In LW, z_t, is either a stationary AR process or a random walk. The measure of the random determinants of the natural real interest rate, z_t,is obviously associated with a considerable degree of uncertainty. We face here a technical problem in that r_t^* is an unobserved variable, itself composed of two unobserved components. This difficulty has already been pointed out by Mésonnier & Renne (2004). In most of the specifications that we have tried, the results were highly sensitive to initial conditions and were often not reasonable. This is especially the case when z_t follows a random walk. Indeed, while the first element g_t is explicitly linked to the output through (3) and (4), z_t is only defined through (7), which makes it more sensitive to small variations in the initial specifications of the model. To overcome this problem, we only consider the case where z_t follows a stationary AR process, and we restrict its variance σ_3^2 through a signal-to-noise ratio, λ_z. We elaborate further on this point in the next section. In addtion, we claim that the possible elements composing z_t should be stationary, thereby making a random walk specification useless.

Equation (4), (5), (6) and (7) constitute the state (transitory) equations of our state-space model, and the IS curve (1) and the Phillips curve (2) constitute the observation equations (see Harvey (1989)). On this system, the Kalman filter is run twice: first in order to identify parameters by maximum likelihood, and second in order to estimate the unobserved components r_t^*, y_t^*, g_t and z_t. The model can be written under its state-space form (see appendix A):

$$\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \mathbf{B}\mathbf{x}_t + \mathbf{G}\boldsymbol{\varepsilon}_t \qquad (8)$$
$$\boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \mathbf{H}\boldsymbol{\varepsilon}_t \qquad (9)$$

4.2 Model estimation

The procedure follows different steps, in line with the recommendations of LW. The first one is to get a prior estimation of the output gap. For this purpose, we use a segmented linear trend with breaks in 1973 and 1993, as a proxy for potential output. The initial output gap is then used to estimate the coefficients of the simplified system by OLS: $a_y(L)\tilde{y}_t = a_r(r_{t-1}+r_{t-2})+\varepsilon_{1,t}$ (for the IS equation) and $b_\pi(L)\pi_t = b_y(L)\tilde{y}_t+\varepsilon_{2,t}$ (for the Phillips curve). This provides us with adequate starting values for the maximum likelihood estimation of the coefficients.

In a second step, we consider a simplified system similar to the previous one, except that we estimate the coefficients by maximum likelihood and we use the Kalman filter. Potential output y_t^* is treated as an unobserved component:

$$a_y(L)\tilde{y}_t = a_r(r_{t-1} + r_{t-2}) + \varepsilon_{1,t}$$

$$b_\pi(L)\pi_t = b_y(L)\tilde{y}_t + \varepsilon_{2,t}$$

$$y_t^* = y_{t-1}^* + \bar{g} + \varepsilon_{4,t}$$

where $\tilde{y}_t$ is the output gap, r_t is the real interest rate, and $\bar{g}$ is a constant.

The third step is dedicated to finding a median unbiased estimate of the variance of potential output growth, σ_5^2.[4] For this purpose, we use the estimate of y_t^* from the previous step in order to run the median unbiased technique of Stock & Watson (1998). The procedure works as follows: 1) Regress for every date t the potential output growth[5] on a constant and a dummy with a break at time t. 2) Compute the t-ratios corresponding to the coefficients of the dummies. 3) Compute the Exponential Wald (EW) statistic[6]. It adds the t-ratios obtained at every date. 4) Compare the values obtained with that of Stock & Watson's table that maps these statistics to the value of median unbiased signal-to-noise ratios λ. Once we have found the adequate ratio λ_g,[7] it suffices to plug it into $\sigma_5 = \lambda_g \sigma_4$ in order to get the adequate measure of potential output. We provide below (appendix B) a sensitivity analysis of the model by taking different percentiles of the distribution of λ_g's, computed from 10.000 draws of the monte carlo simulation procedure used by LW.

As exposed above, the variance of z_t is set according to the signal-to-noise ratio $\lambda_z = \frac{a_r}{\sqrt{2}}\frac{\sigma_3}{\sigma_1}$.[8] We use once again the median unbiased estimator. Although this technique is only needed in theory when z is non-stationary, it should provide an adequate way to estimate

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[4]The reason for using this approach is that the bulk of the distribution of the parameters that control for the variance is often very close to zero. Consequently, the maximum likelihood estimates of these parameters are often statistically insignificant, and are far below the median of the distribution. This would imply for instance that g_t would be constant.

[5]i.e. Δy_t^*, where y_t^* is computed as in step 2 above.

[6]such that $EW = \ln(\frac{1}{T}\sum_{i=1}^{T}\exp(s_i^2/2))$ where s_i is the t-ratio corresponding to a break at time i.

[7]We keep here the same notation as LW.

[8]$\sqrt{2}$ comes from the assumption that the output gap in equation (1) is determined by a moving average of the real interest gap of order 2. That is, $\tilde{y}$ is influenced by z_{t-1} and z_{t-2} through a single coefficient, a_r. See the following section.

λ_z, even with stationary processes. For this purpose, we compute the monte carlo procedure of LW to get a distribution for λ_z. We use the median of this distribution as our baseline value. That is, we take $\lambda_z = 0.064$. The 5th percentile is 0.046 and the 95th 0.076.

The final step estimates the whole system (1) to (7) by maximum likelihood[9], with the two ratios λ_g and λ_z imposed. We proceed in two steps: First, we estimate the whole system and store the IS curve output lag coefficients. Second, we re-estimate the system with these coefficients fixed. For some reason, the estimated output gap with this procedure is much more in line with the existing literature than in the case of a simple, one-step estimation.

In order to identify the model, we have to restrict some parameters. For instance, the variance parameters were restricted to be strictly positive. This is common practice in the literature. We also use some constraints that are specific to the model. For example, we impose $a_g = c.a_r \leq 0$, since there should be a mechanical negative relation between the growth rate g_t of potential output y_t^* and the output gap $\tilde{y}_t = y_t - y_t^*$. This in turn implies that the coefficient c is positive, which is also intuitive because this coefficient is supposed to capture consumers' relative risk aversion. Following LW, we also take a simple moving average of the first and second lags of $\tilde{r}_t$. This comes down to imposing the same coefficient on these two elements. We impose in addition that the resulting coefficient a_r is less than or equal to zero, since the real rate gap should be countercyclical.

In practice, we follow LW and assume that the polynomials $a_y(L)$ and $a_r(L)$ in equation (1) are of order 2, while $b_y(L)$ in (2) is simply of order 1. $b_\pi(L)$ is of order 3, but instead of taking the second and third lags of π_t, i.e. π_{t-2} and π_{t-3}, we take moving averages of the last three quarters of the first year and the whole previous year, such that : $\pi'_{t-2} = \sum_{i=2}^{4} \pi_{t-i}$ and $\pi'_{t-3} = \sum_{i=5}^{8} \pi_{t-i}$.

4.3   Results

This section reports and discusses the estimation results. Table 1 shows parameter estimates for the euro area, Germany and the US. The natural rate estimates, and the uncertainty surrounding the estimates, are fairly similar to those reported by LW on US data.

As regards the IS curve, the sum of the coefficients of the autoregressive components of the output gap, $a_y(L)$, lies between 0.83 and 0.93 for all countries. The effect of a change in the real interest rate gap on the output gap, a_r, seems to be somewhat weeker in the euro area than in the US and Germany. The effect of a change in the output gap on inflation, on the other hand, seems to be slightly stronger in the euro area compared to the US and Germany. The null hypothesis that the coefficients of the inflation terms, $b_\pi(L)$, sum to one in the Phillips curve is not rejected by the data.

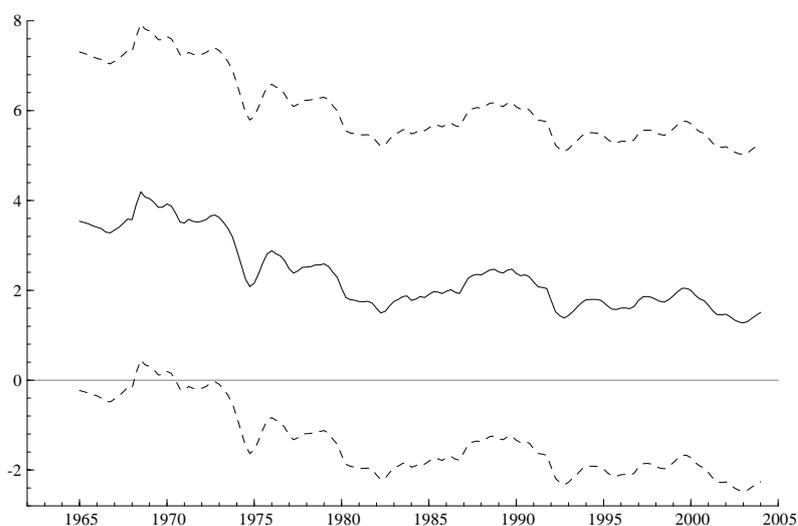[9]The BFGS procedure for numerical optimization is used for this purpose.

Figure 2: Natural real interest rate r_t^* estimate and ± 2 s.d.

Table 1 : Parameter estimates, baseline model

	Euro Area	*Germany*	*US*
	63q1-04q1	*63q1-04q1*	*61q1-02q2*
Variances			
λ_g	0.081	0.081	0
λ_z	0.064*	0.064*	0.064*
σ_{IS}	0.005	0.008	0.006
$\sigma_{Phillips}$	0.396	0.473	0.776
σ_{y*}	0.003	0.004	0.004
$\sigma_g = \lambda_g.\sigma_{y*}$	2.43×10^{-4}	3.24×10^{-4}	0
IS curve			
a_{y1}	0.70 (1.88)	0.47 (1.23)	1.63 (6.63)
a_{y2}	0.14 (1.81)	0.36 (1.23)	-0.70 (6.75)
a_r	-0.056 (2.42)	-0.172 (1.47)	-0.18 (1.96)
c	0.880	0.653	1.179
Phillips curve			
$b_{\pi1}$	1.18 (6.25)	1.07 (4.65)	0.77 (3.04)
$b_{\pi2}$	-0.28 (5.34)	-0.14 (4.00)	0.13 (2.60)
$b_{\pi3} = 1 - (b_{\pi1} + b_{\pi2})$	0.1*	0.07*	0.09*
b_y	0.051 (9.31)	0.041 (7.73)	0.103 (18.01)

t statistics in parentheses

*: imposed coefficient

Regarding the estimate of the natural real interest rate in the euro area, Figure 2 reveals a very high uncertainty around the estimate of the *level* of r_t^*. However, as indicated by the sensitivity analysis in the appendices, we argue that this is essentially due to the uncertainty surrounding the coefficient c. If we impose a constraint on the possible range of values this coefficient can take, the range of estimates of the level of the natural real interest rate diminishes. For almost all signal-to-noise ratios in the model (having fixed the c coefficient to its baseline estimate), the estimated natural real interest rate seems to have been higher in the 1960s and early 1970s than in the 1990s and 2000s. Moreover, the natural real interest rate seems to have been higher in 1990, when the reunification of East and West Germany took place, compared to the period after the Stage Three of Monetary and Economic Union (EMU). Furthermore, the estimated natural real interest rate was also lower in 2004 compared to the start of Stage Three of the EMU in 1999. Finally, our baseline estimate (the bold line in Figure 2) suggests that the natural real interest rate has declined from around 4% in the 1960s to less than 2% in 2004.

In the model, the decline in the natural real interest rate in the euro area is largely due to a fall in the estimated trend growth rate of the economy (see Figure 3)[10]. Given the imprecision of the Kalman filter estimate of trend GDP growth, we compare our estimate with estimates based on the Hodrick-Prescott filter for which we use two different values of the smoothing parameter lambda (1600 and 50 000). The larger value of lambda makes the resulting trend smoother (less high-frequency noise), while the smaller lambda means the trend follows the data more closely. Figure 3 shows that the volatility of the Kalman filtered trend growth is, on average, fairly similar to the Hodrick-Prescott filter estimate when using a relatively large value of the lambda. For all estimates, the trend growth of the economy has declined over the sample.

Figure 4 shows our baseline estimate of the euro area output gap (compared with estimates from HP filter and Baxter and King's bandpass filter), whereas figure 5 and 6 show data for consumer price inflation and our baseline estimate of the real interest rate gap respectively. The estimated output gap is consistent with the commonly held view that monetary policy was loose in the 1970s, contributing to a positive output gap and a persistently high level of inflation for most of the decade. Moreover, in the early 1980s, as monetary policy authorities in many countries pursued tight monetary policy oriented towards disinflation[11], the output gap turned negative. Except for a small positive output gap in the beginning of the 1990s, the output gap remained negative until the start of Stage Three of EMU.

Interestingly, Figure 5 shows that inflation began to decline almost immediately after the estimated real interest rate gap turned positive in the 1980s. As noted by LW, univariate filtering would, by their nature of two-sided weighted averages, lead to estimates of the natural real interest rates and potential output that are, on average, relatively close to

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[10] Cour-Thimann et al (2004) provide very plausible arguments for the view that increases in government debt in the 1980s and higher exchange rate risk premia in the early 1990s might have put upward pressure on the natural real rate in the euro area. These arguments imply that our estimate of the natural real interest rate in this period is somewhat low.

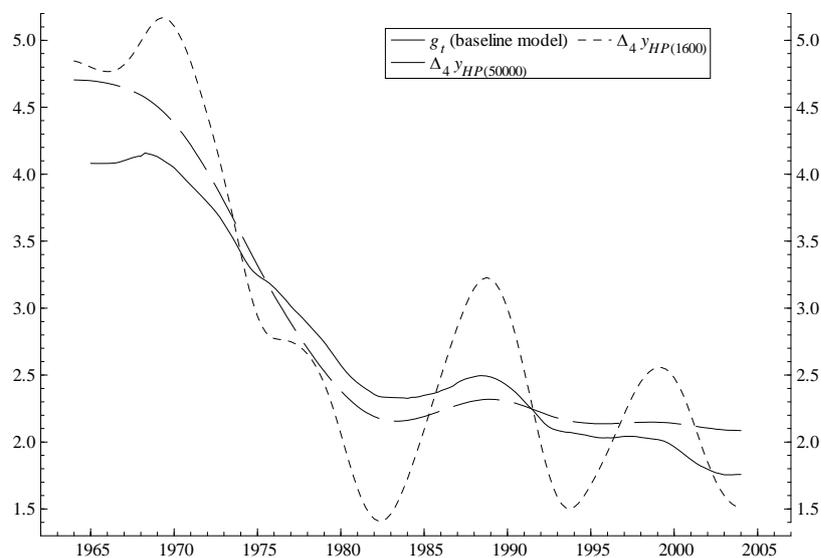[11] See for instance Taylor (1992) for a description of the disinflation policy in the US.

Figure 3: Estimated potential output growth g_t and fourth differences of HP-filtered output.

the actual real interest rate and actual output during the 1970s, even though inflation was increasing. For this reason, the Kalman filter generally provides more reasonable results of the real interest rate gap and the output gap than univariate time-series methods.

Figure 7 compares the baseline estimates of the natural real interest rates for the euro area, Germany and the US. Evidently, while the estimated natural real interest rate in the euro area and Germany has declined over the sample, the natural real interest rate in the US has been more stable around its long term average. The estimates also indicate that the level of the natural real interest rate is lower in the euro area, and in particular in Germany, than in the US.

It is important to stress that all estimates of the natural real interest rate are very imprecise and that caveats are associated with all estimation methods. Regarding the pitfalls with the approach taken in this paper, the estimation results are very sensitive to initial specifications of the model and the selection of starting values for the parameters. A second aspect concerns the measurement of time-variation in preferences (see equation 3). Third, non-textbook factors that may contribute to time variation in the natural rate are treated arbitrarily. Within the empirical framework of this paper, variable z_t is supposed to represent all other factors than trend output growth to explaining the developments in the natural real interest rate. Arguably, the preciseness of this measure is very doubtful, which could make the estimates difficult to interpret.
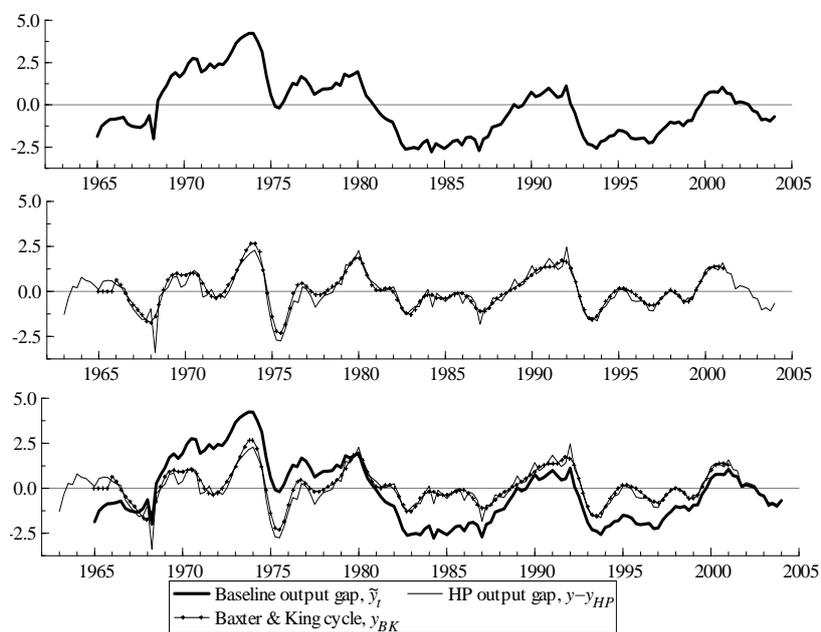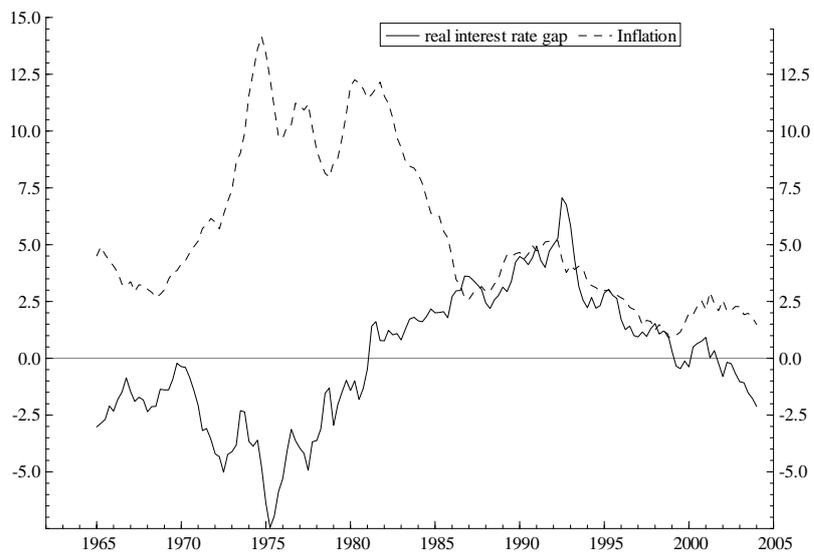
Figure 4: Baseline estimate of output gap

Figure 5: Natural real interest rate gap $\tilde{r}_t$ $(= r_t - r_t^*)$ and inflation rate, euro area.
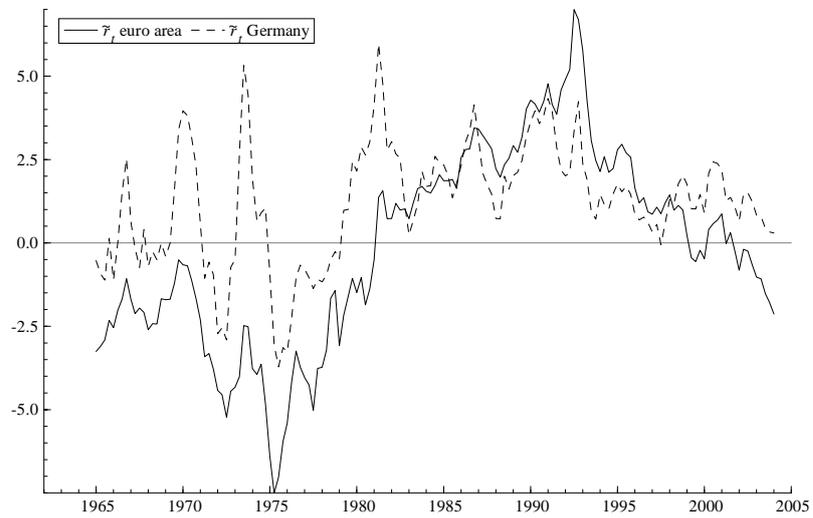
Figure 6: Natural real interest rate gap $\tilde{r}_t$ $(= r_t - r_t^*)$, euro area and Germany.
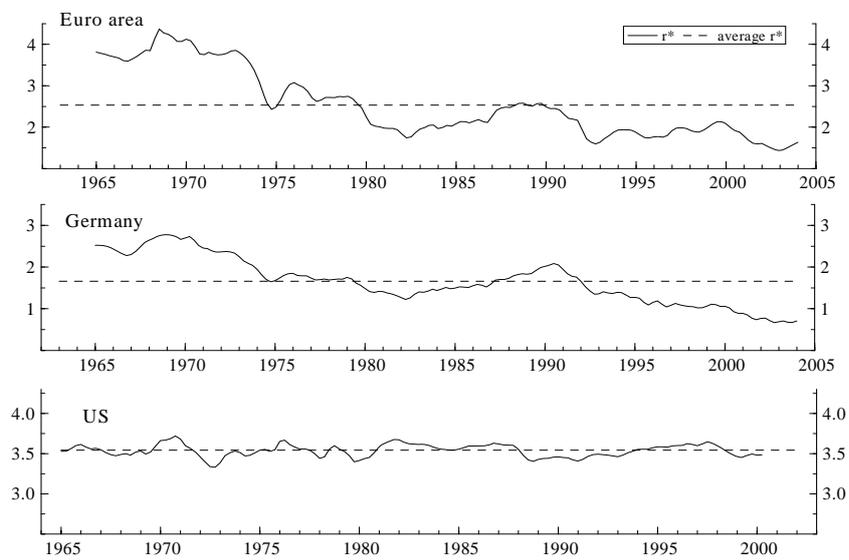

Figure 7: Baseline estimates of r_t^* for the euro area, Germany and the US

5   Statistical properties of the real interest gap in the euro area

We now examine some statistical properties of the euro area model, focusing on simple statistics that describe the relationship between the real interest rate gap and inflation. Table 2 and 3 report standard deviations and correlations of selected variables used in this analysis, namely log output y_t, log potential output y_t^*, the output gap $\tilde{y}_t$, the actual and the natural real rate and the real rate gap (r_t, r_t^* and $\tilde{r}_t$) and inflation π_t. A notable feature of the reported statistics is that the correlation between the actual real interest rate and the real interest rate gap are high and their standard deviations are roughly identical. In other words, the variation in the real interest rate is not primarily related to variation in the natural real interest rate. This is consistent with the results in Giammarioli and Valla (2003) for the euro area, Neiss and Nelson (2003) for the UK and LW for the US, but stands against the results of Cour-Thimann (2004) for the euro area.

Table 2:
Standard deviations, euro area

y_t	0.35
y_t^*	0.30
$\tilde{y}_t$	1.72
π_t	3.50
r_t	2.55
r_t^*	0.80
$\tilde{r}_t$	2.93

Table 3 : Correlation coefficients, euro area

	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
$Corr(r_t, \tilde{r}_{t-k})$	0.98				
$Corr(\pi_t, \tilde{r}_{t-k})$	-0.47	-0.48	-0.50	-0.53	-0.55
$Corr(\tilde{y}_{tt}, \tilde{r}_{t-k})$	-0.55	-0.57	-0.59	-0.60	-0.62

Interestingly, the correlation between inflation and the real rate gap is strongly negative at all lags. This indicates that the developments in euro area inflation since 1960 are, in part, related to the evolution of the real interest rate gap. In line with this hypothesis we find that the real rate gap is also strongly and negatively correlated with the output gap. Next, we investigate the leading indicator properties of the real interest rate gap for inflation. Following Neiss and Nelson (2003), we estimate:

$$\pi_t = a + b_1 \pi_{t-1} + b_2 (r_{t-k} - r_{t-k}^*) + \varepsilon_t$$

where annual inflation π_t is regressed on past inflation and lagged values of the real interest rate gap. The regression on the 1962:1 - 2003:1 sample is summarised in table 4 (numbers in parantheses are standard errors). The results indicate that lagging the real interest rate gap 3 to 5 quarters yields statistically significant parameter estimates when added to an autoregression for inflation. The long lags seem consistent with the common view that monetary policy affects inflation after a significant delay. This simple exercise suggests that the estimated real interest rate gap may contain valuable information about future inflation.

Table 4 : Parameter estimates, euro area

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
a	0.08	0.10	0.13	0.15	0.18
b_1	0.98	0.98	0.97	0.97	0.97
b_2	-0.02	-0.03	-0.04	-0.05	-0.06
	(-1.53)	(-1.97)	(-2.85)	(-3.07)	(-3.70)
R_2	0.98	0.98	0.98	0.98	0.98

6 Conclusion

This paper estimates the natural real interest rate for the euro area, considering the currency union as a single entity over the period 1963 to 2004, and for Germany and the US. Following closely the methodology suggested by Laubach and Williams (2003), we apply the Kalman filter to a small scale macroeconomic model, which encompasses a Phillips curve and an IS curve. This allows us to estimate the natural real interest rate, potential output and trend growth rate of the three economies simultaneously. Overall, the results are quite comparable with the original results of Laubach and Williams (2003) using US data.

According to our baseline estimate for the euro area, the fluctuations in the natural real interest rate have been relatively low since 1963. The natural rate has declined gradually over the past 40 years, from an estimate of around 4% in the 1960s to slightly less than 2% in 2004. The real interest rate gap is relatively persistent over longer periods, with low short-term fluctuations. Moreover, the real interest rate gap indicated that monetary policy was on average stimulative in the 1960s and 1970s, while it contributed to dampen economic activity and inflation in the 1980s and 1990s.

Regarding the output gap, the length of the business cycle's booms and busts are in line with the consensus view in the business cycle literature. However, this model produces an output gap that is influenced by the real interest rate gap. Its average level is negative in periods of tight monetary policy and positive in periods of monetary laxism. This is interesting since it might be taken as an indicator of the degree at which the central bank policy influences the real economy. In the 1970s when inflation become high. the real interest rate gap was negative and the output gap was positive, on average. Likewise, in the 1980s and 1990s, when inflation fell to lower levels, the real interest rate was positive and the output gap was on negative, on average.

Simple empirical tests also suggest that the estimated real interest rate gap is negatively

correlated with the output gap and inflation. Furthermore, the tests show that the real rate gap may contain valuable information about future inflation in the euro area. The general caveats associated with interpreting estimates of the natural real interest rate, which are highly uncertain, also applies for this paper.

References

[1] Barro, R.J. and Sala-i-Martin, X. (1995): "Economic Growth.": McGraw-Hill.

[2] Basdevant, O.N. Björksten and O. Karagedikli (2004): "Estimating a time varying neutral real interest rate for New Zealand ", Reserve Bank of New Zealand Discussion Paper, DP2004/01

[3] Blanchard, O. and S. Fischer (1989): "Lectures on Macroeconomics.": Cambridge, MA: MIT-press.

[4] Cour-Thimann, Philippine, Rasmus Pilegaard and Livio Stracca (2004): "The output gap and the real interest rate gap in the euro area, 1960-2003", paper presented at the Bank of Canada workshop on Neutral Interest Rates, 9-10 September 2004 (available on Bank of Canada's website).

[5] Cuaresma, J.C., Gnan, E . and D. Ritzberger-Grunenwald (2004): "Searching for a natural rate of interest: a euro area perspective", Empirica, vol 31, p. 185 - 204

[6] De Long and J. Bradford (1997): "America's Peacetime Inflation: the 1970s ", in C.D. Romer and D.H. Romer (eds.), Reducing Inflation: Motivation and Strategy. Chicago: University of Chicago Press. 247-276

[7] Ford, Robert and Douglas Laxton (1999), "World Public Debt and Real Interest Rates", Oxford Review of Economic Policy, Vol. 15 No.2.

[8] Griammarioli, Nicola and Natacha Valla (2003), "The Natural Rate of Interest in the Euro area", ECB Working Paper No. 233

[9] Harvey, Andrew C. (1989): "Forecasting, structural time series model and the Kalman filter", Cambridge University Press, Cambridge.

[10] Koopman S.J., Shephard N. and Doornik J.A. (1998), "Statistical algorithms for models in state space using SsfPack 2.2", Econometrics Journal, 1, 1-55 and http://www.ssfpack.com/

[11] Larsen, Jens D. J. and Jack McKeown (2002), "The information content of empirical measures of real interest rate and output gaps for the United Kingdom", BIS papers No 19

[12] Laubach, Thomas and John C. Williams (2003), "Measuring the Natural Rate of Interest", *Review of Economics and Statistics*, vol 85(4), p. 1063-1070

[13] Levin, A., V. Wieland and J.C.Williams (2003). "Robustness of Simple Monetary Policy Rules under Model Uncertainty " in John B. Taylor, ed. Monetary Policy Rules, Chicago. University of Chicago Press. pp, 263-299

[14] Maravall, A. (1995) "Unobserved Components in Economic Time Series", in M.H.Pesaran and M.R.Wickens (eds.), Handbook of applied econometrics. Volume 1. Macroeconomics. Handbooks in Economics, Blackwell, Oxford and Malden, 12-72.

[15] Mésonnier, J.-S, and J.-P. Renne (2004) "A Time-Varying Natural Rate of Interest for the euro area", Working paper No 115, Banque de France.

[16] Neiss, K. and Nelson, E (2003): "The real rate gap as an Inflation Indicator ", Macroeconomic Dynamics, 7, pp. 239 -262.

[17] Nelson, E. (2004): "The Great Inflation of the Seventies: What really Happened?", Fed working paper, Januar 2004

[18] Nelson, E. and K. Nikolov (2002): "Monetary Policy and Stagflation in the U.K. " Bank of England Working paper No. 155, May

[19] Nelson, E. and K. Nikolov (2003): "U.K. Inflation in the 1970s and 1980s: the Role of Output Gap Mismeasurement", Journal of Economics ans Business, Vol. 55 (4), 353-370.

[20] Proietti, T. (2001) "Unobserved Components Models for Measuring Output Gaps, Capacity Utilisation and Core Inflation", mimeo, notes prepared for the course 'Unobserved Components Models and their Applications in Macroeconomics', European Central Bank, 11 and 12 June 2001.

[21] Sevillano, J. M. M. and M. M. Simon (2004): "An empirical approximation of the natural rate of interest and potential growth", Working paper No 0416, Banco de Espana.

[22] Stock J. and M.Watson (1998), "Median unbiased estimation of coefficient variance in a time-varying parameter model", *Journal of the American Statistical Association*, March, 93, p.349-358.

[23] Stokey, N and S. Rebelo (1995): "Growth Effects of Flat-Rate Taxes. " *Journal of Political Economy.* Vol. 103, No. 3.

[24] Taylor, John B. (1992): "The great inflation, the great disinflation and policies for future price stability. " In A. Blundell-Wignall (ed), Inflation, Disinflation and Monetary Policy. Sydney. Ambassador press. 9-34.

[25] Wicksell, K. (1898) "Interest and prices", London, Macmillan, 1936. Translation of 1898 edition

[26] Williams, J.C. (2003) "The natural rate of interest", FRBSF Economic letter 2003-32

[27] Woodford, M. (2003): "Interest and prices: foundations of a theory of monetary policy", Princeton University Press.

Appendices

A State-space form

The system to be estimated, eq (1), (2), (4), (5) and (7), has to be put into the general state-space form:

$$\boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \mathbf{H}\boldsymbol{\varepsilon}_t \tag{10}$$

$$\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \mathbf{B}\mathbf{x}_t + \mathbf{G}\boldsymbol{\varepsilon}_t \tag{11}$$

where $\mathbf{Z}, \mathbf{B}, \mathbf{G}, \mathbf{T}$ and $\mathbf{H}$ are matrices of coefficients[12]. (10) is the so-called measurement equation and (11) the state equation. $\mathbf{y}_t$ is a vector of dependent variables, corresponding to y_t and π_t in our model, $\boldsymbol{\alpha}_t$ is a vector corresponding to the unobserved components, here y_t^*, g_t and z_t. r_t^* is suppressed from the model because it can be fully recovered from g_t and z_t, provided we obtain c. $\mathbf{x}_t$ is a vector of deterministic variables. $\boldsymbol{\varepsilon}_t \sim NID(0, \mathbf{I})$ and $\boldsymbol{\alpha}_0 \sim N(a, \mathbf{P})$. Note that the representation of (12) is particular in that the vector of parameters $\mathbf{B}$ is treated as a vector of unobserved variables. This is a feature of the library *Ssfpack* for *Ox* that we use in this paper.

The system can be rewritten

$$\begin{pmatrix} \boldsymbol{\alpha}_{t+1} \\ \mathbf{B} \\ \mathbf{y}_t \end{pmatrix} = \begin{pmatrix} \mathbf{T} & 0 \\ 0 & \mathbf{I} \\ \mathbf{Z} & \mathbf{x}_t \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_t \\ \mathbf{B} \end{pmatrix} + \begin{pmatrix} u_t \\ 0 \\ v_t \end{pmatrix} \tag{12}$$

$$= \boldsymbol{\Phi} \begin{pmatrix} \boldsymbol{\alpha}_t \\ \mathbf{B} \end{pmatrix} + \mathbf{u}_t, \qquad \mathbf{u}_t \sim NID(0, \mathbf{u}_t \mathbf{u}_t')$$

where $\alpha_t' = \begin{pmatrix} y_t^* \, y_{t-1}^* \cdots y_{t-n}^* \, g_t \, g_{t-1} \, z_t \, z_{t-1} \end{pmatrix}$, $\mathbf{B}' = \begin{pmatrix} C \, a_x \cdots a_{rx} \, b_x \cdots b_{sx} \end{pmatrix}$, where C equals 0 or ρ, depending on the specificiation of z_t (see eq. 7).

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 \\ 1 & \cdots & & 0 & 0 & 0 & & \\ \vdots & \ddots & & \vdots & \vdots & & \ddots & \vdots \\ 0 & \cdots & & 1 & 0 & & & 0 \\ & & & & 1 & 0 & & \\ \vdots & & & & 1 & 0 & & \\ & & & & & & 1 & 0 \\ 0 & & \cdots & & & 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{Z} = \begin{pmatrix} 1 & -a_{y1} & \cdots & -a_{yn} & 0 & -c.a_r & 0 & -a_r \\ 0 & -b_{y1} & \cdots & 0 & 0 & \cdots & \cdots & 0 \end{pmatrix}$$

$$\mathbf{x}_t = \begin{pmatrix} 0 & x_{1t} & \cdots & x_{rt} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & x_{1t}' & \cdots & x_{st}' \end{pmatrix}$$

---

[12] We assume here that they are constant since this is the specification used in our model, but they could just as well be defined as time-varying parameters.

and $\mathbf{u}'_t = \begin{pmatrix} \varepsilon_{4,t} \, 0 \cdots 0 \, \varepsilon_{5,t} \, 0 \, \varepsilon_{3,t} \, 0 \cdots \cdots 0 \, \varepsilon_{1,t} \, \varepsilon_{2,t} \end{pmatrix}$.

$$\mathbf{Y}_t = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} y_t - a_{y1}y_{t-1} - ... - a_{yn}y_{t-n} - a_r r_{t-1} \\ \pi_t - b_{\pi1}\pi_{t-1} - ... - b_{\pi p}\pi_{t-p} - b_y y_{t-1} \end{pmatrix}$$

The specification of the vector of dependent variables allows us to impose constraints on the coefficients on some of the variables (e.g. y_t and y_t^*). The other exogenous variables, namely the dummies x_t, are estimated as in a classical regression model, i.e. are treated as unobserved variables, upon which no constraint can be imposed.

B Sensitivity analysis

B.1 Shorter sample

Figure 8: Shorter sample. r_t^* estimates 1973Q2-2004Q2

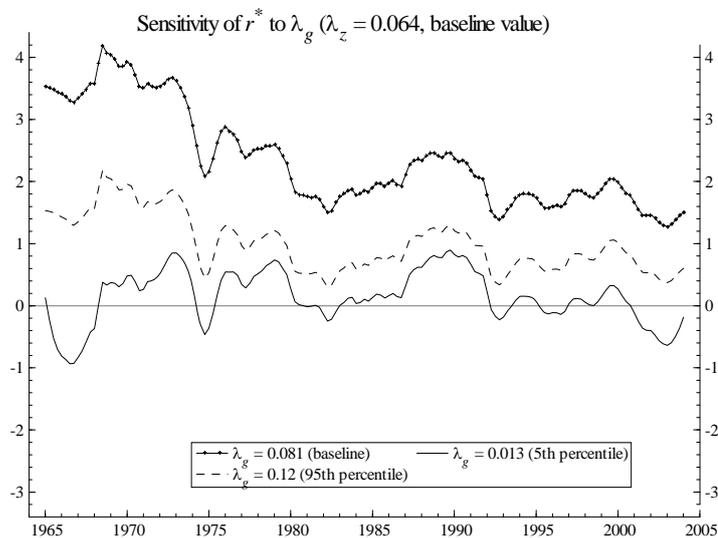## B.2 Sensitivity to signal-to-noise ratios: unrestricted c

Figure 9: The natural real interest rate. Sensivity to signal to noise ratios with unrestricted c.
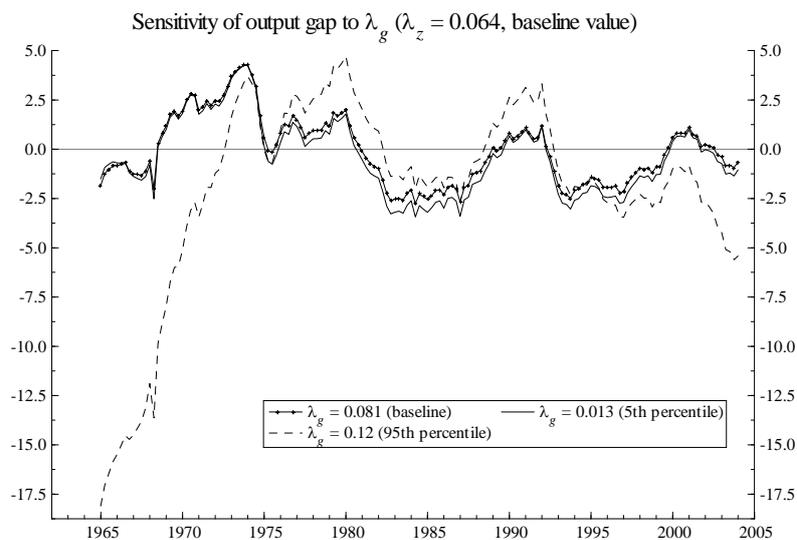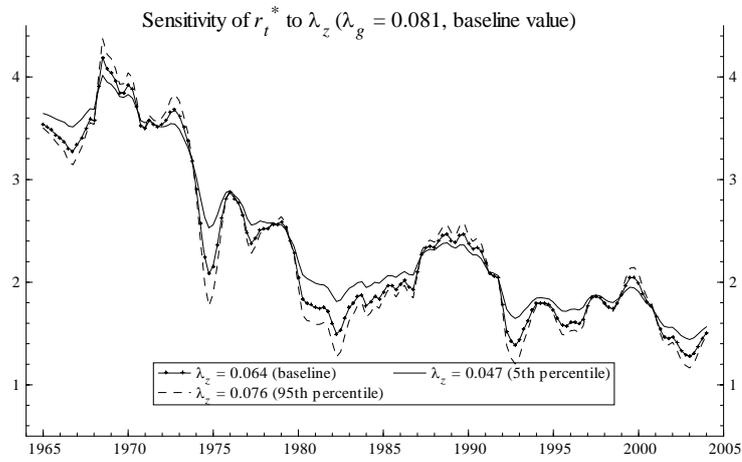

Figure 10: The output gap. Sensivity to signal to noise ratios with unrestricted c.

Figure 11: The natural real interest rate. Sensivity to signal to noise ratios with unrestricted c.
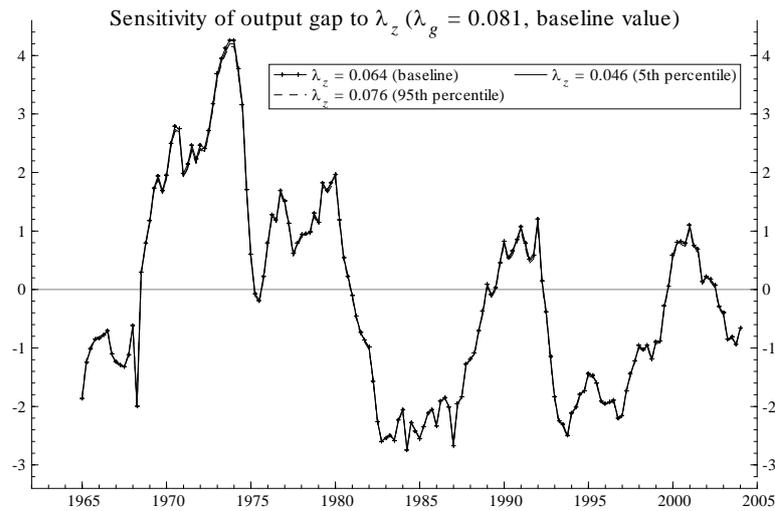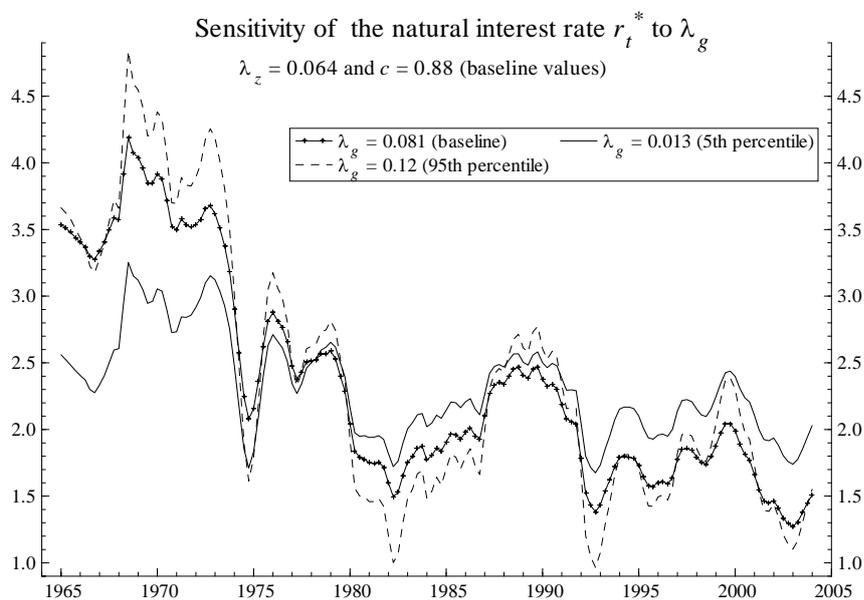

Figure 12: The output gap. Sensivity to signal to noise ratios with unrestricted c.

B.3 Sensitivity to signal-to-noise ratios: c restricted to baseline value

Figure 13: The natural real interest rate. Sensivity to signal to noise ratios with restricted c.
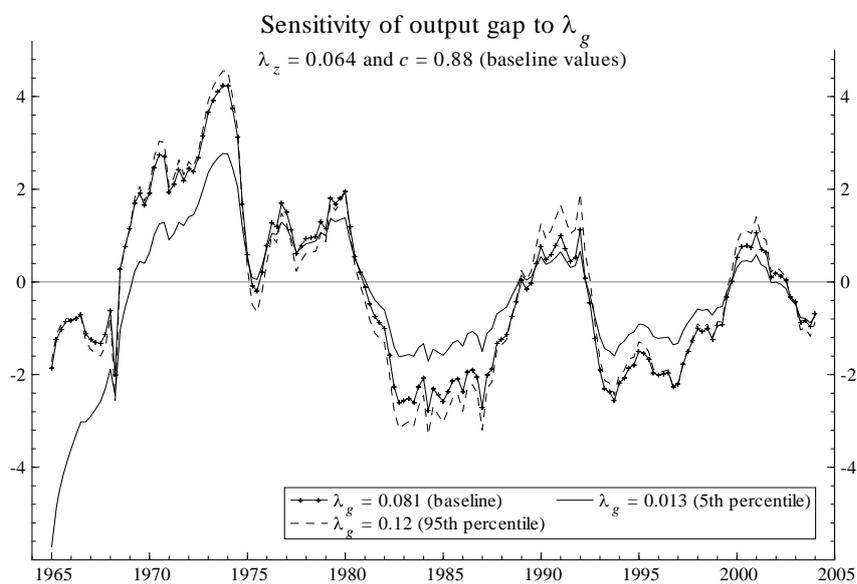

Figure 14: The output gap. Sensivity to signal to noise ratios with restricted c.
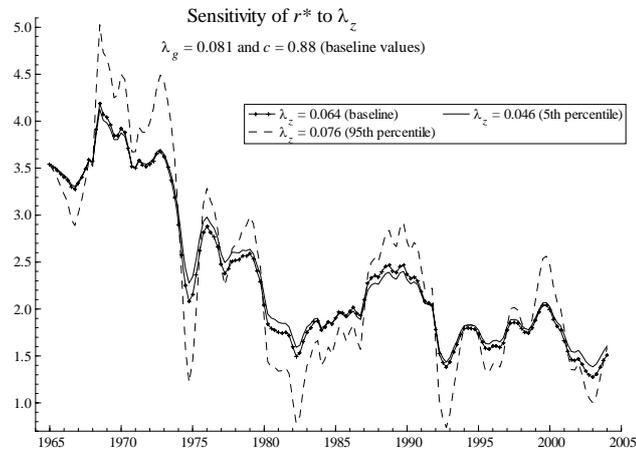
Figure 15: The natural real interest rate. Sensivity to signal to noise ratios with restricted c.
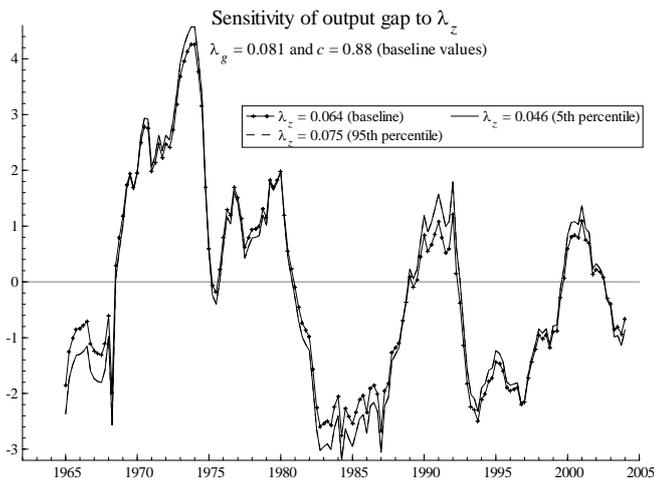

Figure 16: The output gap. Sensitivity to signal to noise ratios with restricted c.

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