FISCAL AND MONETARY RULES FOR A CURRENCY UNION

by Andrea Ferrero

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CONTENTS

Abstract

This paper addresses the question of the joint conduct of fiscal and monetary policy in a currency union. The problem is studied using a two-country DSGE framework with staggered price setting, monopolistic competition in the goods market, distortionary taxation and nominal debt. The two countries form a currency union but retain fiscal policy independence. The policy problem can be cast in terms of a tractable linear-quadratic setup. The stabilization properties and the welfare implications of the optimal commitment plan are compared with the outcome obtained under simple implementable rules. The central result is that fiscal policy plays a key role to smooth appropriately the impact of idiosyncratic exogenous shocks. Fiscal rules that respond to a measure of real activity have the potential to approximate accurately the optimal plan and lead to large welfare gains as compared to balanced budget rules. Monetary policy shall focus on maintaining price stability.

JEL Classification: E63, F33, F42

Key Words: Currency Union, Optimal Policy, Flexibility, Welfare

NON-TECHNICAL SUMMARY

I study the problem of the joint conduct of fiscal and monetary policy in a currency union. The economy features staggered price setting and imperfect competition in the goods market. The government can levy distorting taxes and issue nominal riskless debt to finance a given process of spending. These elements generate non-trivial interactions between the two sources of policy for stabilization purposes. Monetary policy decisions have fiscal implications given the effects on the real value of government debt. Alternative fiscal policies influence production plans, which are determined not only by supply but also by demand considerations, given the presence of sticky prices.

In the context of this framework, I first show the existence of a symmetric steady state which entails zero inflation and constant positive debt. Then, I derive a linear-quadratic approximation of the non-linear optimal policy problem. A purely quadratic objective allows to evaluate welfare correctly by using a first order approximation of the equilibrium conditions. The loss function consists of quadratic deviations of total output, the terms of trade and each country's GDP inflation rate from their targets compatible with full stabilization. The presence of tax distortions does not lead to a direct presence of tax rates in the loss function. Nonetheless, the effects of such distortions show up in the weights attached to the different elements of the objective function.

I characterize the optimal plan that a benevolent central planner would implement under a timeless perspective commitment. The planner chooses monetary policy for the whole area and fiscal policy for the two countries to maximize the present discounted value of the welfare objective defined for the monetary union. The optimal plan then serves as a benchmark to evaluate the performance of alternative simple but implementable rules for monetary and fiscal policy. After calibrating the model to European data, I compare the response of the endogenous variables to a number of exogenous shocks. Finally, I evaluate quantitatively the welfare properties of al-

ternative regimes, searching for the value of the parameters in the simple rules that minimize the departures from the optimal plan.

The main finding is that a regime characterized by flexible debt rules for fiscal policy and strict inflation targeting for monetary policy approximates accurately the optimal plan. Flexibility is granted to fiscal policy by allowing real debt in each country to respond to departures of output from its target level. Monetary policy shall focus on maintaining price stability. The welfare gains of flexible fiscal rules, as compared to balanced budget rules, are quantitatively substantial.

1   Introduction and Related Literature

In the 1960's, in a series of influential papers, Mundell [1961], McKinnon [1963] and Kenen [1969] posed the foundations of the theory of Optimum Currency Area. In recent years, the macroeconomic aspects of a currency union have gained renewed attention in the academic research agenda. The creation of the European Monetary Union (EMU) in 1999 and the introduction of the Euro in 2002 among the participant members has stimulated a notable debate on several issues concerning the appropriate functioning of economic relations among sovereign countries that share a common central bank.

The new monetary regime has also been accompanied by the institution of the Stability and Growth Pact. According to the Pact, national fiscal policies are bound to respect an upper threshold for the deficit-GDP and the debt-GDP ratios of 3% and 60% respectively. Since the constitution of the EMU, several countries have violated these fiscal rules without incurring in the sanctions prescribed by the Pact itself. As of today, the fiscal rules that support the existence of the Euro are under revision. These episodes bear two fundamental questions. What are the appropriate rules to include in the Pact? What is the appropriate mechanism to enforce the rules written in the Pact? This paper focuses on the first question. It aims at analyzing, from a theoretical and quantitative perspective, the stabilization role of fiscal and monetary policies within a monetary union.

In the model economy, two countries form a currency union but retain fiscal policy independence. Given the focus on stabilization, optimal policy will be characterized from a fully centralized perspective[1]. The optimal policy outcome will provide a well-defined benchmark for the evaluation of alternative operational policy rules and for the assessment of their welfare costs and benefits. Overall, the results will prove helpful to indicate the appropriate mandate of the fiscal and monetary authorities in the specific context of a currency union.

The model builds upon the recent work by Benigno and Woodford [2003] (henceforth BW). The structure of the economy allows for non-trivial interactions between fiscal and monetary decisions. The typical public finance approach to dynamic optimal taxation is conjugated with the also typical, but somewhat distinct, monetary approach to optimal stabilization policies. Nominal rigidities, in the form of staggered prices, lead to real effects of monetary policy whereas distortionary taxation introduces non-Ricardian effects of fiscal policy. These two key ingredients generate non-trivial policy spillovers. Variations of the nominal interest rate and the equilibrium inflation rate affect fiscal decisions through the real burden of debt. The tax rate influences the pricing decisions of the firms. Since output is partly demand-determined, fiscal policy interacts with the real effects of monetary policy. Should the governments be allowed to use lump-sum instruments, the first channel would be shut down. Similarly, if prices were fully flexible, monetary policy would be neutral and aggregate demand would play no role in output determination. The present

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[1]See, for instance, Chari and Kehoe [1998] for an insightful view on the elements of strategic interaction among policymakers.

framework provides enough richness to study important sources of feedback from one policy to the other and to characterize a normative benchmark for the evaluation of alternative rules.

In line with the seminal contribution of Lucas and Stokey [1983], a number of papers have studied non-linear optimal fiscal and monetary policy problems, under various assumptions on the economic environment[2]. This so-called Ramsey approach to optimal policy generally characterizes the set of allocations that can be implemented as an equilibrium for a given set of instruments assigned to the policymakers. More recently, Kim, Kim, Schaumburg and Sims [2003], Juillard [2003] and Schmitt-Grohé and Uribe [2004a] have put forward fully numerical methods, based on an accurate second order approximation of the model, to solve optimal policy problems with pre-specified policy rules[3]. In this paper, I will follow the proposal of BW, which blends elements of the two aforementioned methodologies and allows to compare the stabilization properties and evaluate the welfare outcomes of simple and implementable rules against the benchmark constituted by the optimal commitment plan.

In the open economy macroeconomics literature, the emphasis has most often been on studying the nature of the optimal exchange rate regime (i.e., cooperation versus independence) for a given stabilization problem[4]. Benigno and Benigno [2003] reconsider and generalize some of those results in a framework closely related to this paper but where taxes are lump-sum, so that fiscal considerations are not part of their analysis. My work takes as given the existence of a monetary union, as in Benigno [2004], but it also considers explicit fiscal decisions by adding a stabilization role for fiscal policy. Very recently, Beetsma and Jensen [2004] and Galí and Monacelli [2004] discussed the optimal determination of fiscal and monetary policy in a currency union in a model similar to the present one[5]. Although in both these models fiscal decisions are made explicit by the possibility for the government to choose the amount of "useful" public spending, the presence of lump-sum taxation precludes any analysis of taxation smoothing motives[6]. Finally, Canzoneri, Cumby and Diba [2005] assess quantitatively the performance of operational rules in a model of a currency union which features capital accumulation and sticky wages. Once again, the presence of lump-sum instruments potentially allows fiscal authorities to be over-active without affecting any relevant margin of the decision problem of agents in the private sector.

From a methodological perspective, the absence of lump-sum taxes and the pres-

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[2]Recent examples, in models with nominal rigidities and distortionary taxation, are Correia, Nicolini and Teles [2002] and Siu [2004].

[3]See Canzoneri, Cumby and Diba [2005], Kollmann [2004] and Schmitt-Grohé and Uribe [2004b] and [2004c] for applications to models with fiscal and monetary policy.

[4]Examples of this type are Clarida, Galí and Gertler [2002], Devereux and Engel [2003] and Obstfeld and Rogoff [2002].

[5]Lombardo and Sutherland [2004] propose a two-period version of the model by Beetsma and Jensen [2004] with prices set one period in advance and investigate costs and benefits from fiscal cooperation.

[6]Duarte and Wolman [2003] study the implications of national fiscal policy decisions for inflation differentials in a two-country general equilibrium model of a currency union with flexible prices, debt and distortionary taxation.

ence of positive steady state debt and public spending make unfeasible using the tax rate as a subsidy to eliminate steady state monopolistic distortions. Without further corrections, a linear-quadratic approximation of the optimal policy problem would lead to an incorrect welfare ranking of alternative policies. Following the analytical method proposed by BW, it is possible to preserve the tractability of LQ problems and correctly evaluate welfare up to the second order by using a first order approximation of the equilibrium conditions. The characterization of optimal policy in terms of dynamic response to exogenous shocks is investigated in relation to the performance of alternative policy rules. These rules are 'simple', in the sense that can be considered the natural formalization, in the context of a theoretical model, of the rules that policymakers follow in practice. However, such rules are generally 'suboptimal', in the sense that imply welfare losses with respect to the optimal policy benchmark. The numerical experiments in the last section evaluate the stabilization properties and assess quantitatively the welfare losses associated to simple rules.

The central result of the paper is that flexibility in either fiscal or monetary policy is always a desirable feature of the simple rules but the welfare gains from a flexible monetary rule are marginal as compared to the gains from flexible fiscal policy rules.

The rest of the paper is organized as follows. In the next section, I outline the building blocks of the model. Then, I define the optimal policy problem and show that it can be approximated by a linear-quadratic framework where the objective function derives from a second order expansion of individual utility and the constraints are a linear approximation of the set of equilibrium conditions. In line with the existing literature, the resulting per-period loss function can be expressed in terms of the deviations of average output, the terms of trade and the GDP inflation rates from their respective desired levels, appropriately defined as the targets for complete stabilization. Next, I characterize the joint optimal fiscal and monetary policy for the model in the cooperative solution. Finally, I compare the dynamic responses to exogenous disturbances under these two regimes and evaluate the associated welfare implications. The technical details are derived in the appendix[7].

2 A Model of a Currency Union

The world is composed of two countries, Home and Foreign (also denoted by H and F hereafter). The total population is ordered on a continuum of measure one. The size of country H is $n \in (0,1)$ and that of country F is $1-n$. In each country, there are two sectors (households and firms) and one fiscal authority. The two countries are part of a currency union so that monetary policy is chosen for the whole area. Financial markets are assumed to be complete both at the national and international level. Goods markets are characterized by monopolistic competition and nominal price rigidities. Labor markets are segmented. Households offer a specialized labor input for the production of a specific final good. The implied monopolistic power in labor supply is captured by the introduction of an exogenous and time-varying wage

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[7]An additional mathematical supplement with more complete analytical derivations and further details is available on the web at http://homepages.nyu.edu/~apf210/research/research.htm.

markup. In what follows, I discuss in detail each sector of the economy and the role of the policy authorities.

2.1 Households

All households, indexed by $j \in [0, 1]$, have the same preferences defined over consumption and leisure and discount future utility at rate $\beta \in (0, 1)$. Individual lifetime utility is

$$u_0^j \equiv E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_t^j) - \tilde{V}(\ell_t^j) \right] \right\},\tag{1}$$

where $U' > 0$, $U'' < 0$, $\tilde{V}' > 0$ and $\tilde{V}'' > 0$. The operator $E_0 \{\cdot\}$ refers to the expectation conditional upon the information set available at time 0.

Household j sequentially trades in a complete set of one period state-contingent securities that span all possible states of nature. The random variable D_{t+1}^j denotes the payoff of a portfolio of such state-contingent securities, purchased by household j at time t. The random variable $Q_{t,t+1}$ represents the price of D_{t+1}^j. I assume that households offer fully specialized labor inputs so that the wage rate is household-specific and indicated by w_t^j. The flow budget constraint for household j is

$$P_t C_t^j + E_t \left\{ Q_{t,t+1} D_{t+1}^j \right\} = w_t^j \ell_t^j + \Gamma_t^j + D_t^j,\tag{2}$$

where Γ_t^j stands for profits (net of taxation) from ownership of the firms.

The consumption index C_t^j is defined as the Dixit-Stiglitz aggregator over the bundles of goods produced in country H and F respectively

$$C_t^j \equiv \left[n^{\frac{1}{\theta}} (C_{H,t}^j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} (C_{F,t}^j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where $\theta \geq 1$ is the elasticity of substitution between the Home and Foreign bundles. The indexes $C_{H,t}^j$ and $C_{F,t}^j$ are Dixit-Stiglitz aggregators defined over the continuum of differentiated goods produced in each country

$$C_{H,t}^j \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_{F,t}^j \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c_t^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution across goods produced within one country.

All the prices are set in the common numeraire. The law of one price is assumed to hold, so that the price of each variety of good is the same across countries. The implied overall consumption-based price index is

$$P_t = \left[n P_{H,t}^{1-\theta} + (1-n) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}},\tag{3}$$

while the implied country-specific price indexes are given by

$$P_{H,t} = \left[\frac{1}{n} \int_0^n p_t(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \qquad P_{F,t} = \left[\frac{1}{1-n} \int_n^1 p_t(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}, \tag{4}$$

where $p_t(h)$ and $p_t(f)$ are the prices of the Home-produced and Foreign-produced goods respectively.

The expenditure minimization problem consists of two steps. First, households minimize total consumption expenditure subject to a minimum level of C_t^j. Then, the optimal allocation of spending among different varieties is chosen by minimizing expenditure on the two consumption bundles, given a minimum level of $C_{H,t}^j$ and $C_{F,t}^j$. In the spirit of the public finance literature, I will assume that the total amount of public spending in country i is given exogenously in each period by $G_{i,t}$. Each government chooses optimally the composition of a Dixit-Stiglitz aggregator over all goods produced in its own country[8] to minimize expenditure. Given the appropriate CES aggregators for Home and Foreign production, one can express national GDP as a function of total consumption, relative prices and public spending as

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_{W,t} + G_{H,t}, \qquad Y_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_{W,t} + G_{F,t}. \qquad (5)$$

where $C_{W,t}$ is defined as $C_{W,t} \equiv \int_0^1 C_t^j dj$. Total demand for good h and f can be written as a function of relative prices and national GDP only as

$$y_t^d(h) = \left[\frac{p_t(h)}{P_{H,t}}\right]^{-\sigma} Y_{H,t}, \qquad y_t^d(f) = \left[\frac{p_t(f)}{P_{F,t}}\right]^{-\sigma} Y_{F,t}. \qquad (6)$$

It is useful to define the terms of trade as the relative price of the Foreign bundle of goods in terms of the Home bundle ($T_t \equiv P_{F,t}/P_{H,t}$). From the expression for the price index [3], it can be seen that there exists a simple relation between the terms of trade and each country's relative price, given by

$$\left(\frac{P_{H,t}}{P_t}\right)^{\theta-1} = n + (1-n) T_t^{1-\theta}, \qquad \left(\frac{P_{F,t}}{P_t}\right)^{\theta-1} = n T_t^{\theta-1} + (1-n). \qquad (7)$$

Movements in the terms of trade reflect movements in relative prices, and, hence, imply demand shifts across countries.

For each household j, the optimality condition for the allocation of wealth among state-contingent securities characterizes the stochastic discount factor as

$$Q_{t,t+1} = \beta \left(\frac{P_t}{P_{t+1}}\right) \frac{U_C(C_{t+1}^j)}{U_C(C_t^j)}. \qquad (8)$$

Under complete markets, idiosyncratic risk is completely shared across households, both within and across countries. The risk-sharing conditions result from equating the expression [8] for each couple of households j and k in the population. Hence, for every $\{j,k\}$, marginal utility of consumption is equalized up to the same constant, which is further set to one. This latter assumption means that the initial state-contingent distribution of wealth is such that the life-time budget constraints of all

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[8]The functional form is identical to the consumption bundles $C_{i,t}^j$.

households are identical[9]. It follows that one obtains $C_t^j = C_t$, $\forall j \in [0, n]$, $C_t^j = C_t^*$, $\forall j \in [n, 1]$ and $C_t = C_t^* = C_{W,t}$. Finally, given the expression of the stochastic discount factor [8], no arbitrage implies that the gross return on a one-period risk-free bond[10] R_t satisfies

$$R_t^{-1} = E_t \left\{ Q_{t,t+1} \right\}. \tag{9}$$

Labor markets are characterized by an exogenous country-specific wage markup ($\mu_{i,t}^w > 1$, $i = \{H, F\}$) that can be interpreted as capturing monopolistic distortions in input supply[11] or a wedge introduced by inefficient contracting. The optimality condition for labor supply is

$$\frac{w_t^j}{P_t} = \mu_{i,t}^w \frac{\tilde{V}_\ell(\ell_t^j)}{U_C\left(C_{W,t}\right)}. \tag{10}$$

The last first order necessary condition for household's optimization is the intertemporal budget constraint which corresponds to the flow budget constraint [2] coupled with the appropriate transversality condition

$$\lim_{T \to \infty} E_t \left\{ Q_{t,T} D_T^j \right\} = 0. \tag{11}$$

2.2 Firms

I assume the existence of a continuum of firms of measure n in country H and of measure $1 - n$ in country F. Let $h \in [0, n]$ and $f \in [n, 1]$ be the indexes for generic Home and Foreign firms respectively. For simplicity, I focus on the optimization problem of firm h in country H. Such a firm produces the differentiated consumption good $y_t(h)$, which is traded internationally without frictions, using a linear technology of the form

$$y_t(h) = a_{H,t} \ell_t(h), \tag{12}$$

where $a_{H,t}$ is a technology shock.

Labor is immobile across countries and labor markets are assumed to be segmented[12]. Each firm acts as a wage-taker and hires workers from an imperfectly competitive labor market (due to the presence of the wage markup $\mu_{H,t}^w$) in which a continuum of individuals supply a firm-specific input according to the optimality condition [10].

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[9]A further implication is that the current account position is irrelevant for the determination of the equilibrium value of the other endogenous variables and the direction of the flow of state-contingent securities across countries can be neglected insofar it ensures equality of consumption at each point in time and in every state of the world.

[10]Under the assumed structure for financial markets, a one-period risk free bond is obtained as a portfolio of state-contingent securities that pays off one unit of currency in each state of the world with certainty.

[11]Erceg, Henderson and Levin [2000] endogenize this wage markup by assuming time-varying elasticity of substitution of labor supply in a model where also wages are sticky.

[12]See Woodford [2003] for the equivalency between this specific formulation and alternative assumptions on the labor market structure under complete asset markets.

Production plans may differ across firms because prices are assumed to be set on a staggered basis. Following Calvo [1983], I impose that in each period, independently of previous adjustments, each firm faces a probability $(1 - \alpha_H)$ of adjusting its price. The problem of the firm consists of choosing the price[13] $p_t(h)$ at time t as to solve

$$\max_{p_t(h)} E_t \left\{ \sum_{T=t}^{\infty} \alpha_H^{T-t} Q_{t,T} \left[(1 - \tau_{H,T}) \, p_t(h) \, y_{t,T}(h) - w_T(h) \, \ell_T(h) \right] \right\},$$

$$s.t. \ y_{t,T}(h) = \left[\frac{p_t(h)}{P_{H,T}} \right]^{-\sigma} Y_{H,T}, \qquad \forall T \geq t,$$

given the technology constraint [12]. The notation $y_{t,T}(h)$ stands for the demand at time T (from expression [6]) conditional on the fact that the price of good h has not changed since period t. The first term represents sales revenues net of taxes[14] $\tau_{H,t}$. The second term denotes the total nominal cost of producing $y_t(h)$ units of output. Finally, profits are discounted by the appropriate stochastic discount factor for nominal assets as individuals are the ultimate shareholders of the firm.

In the presence of sticky prices, monetary policy affects real activity because output is demand-determined for those firms that are not allowed to readjust their price in any given period. Fiscal policy decisions are relevant because lower taxes[15], ceteris paribus, imply a lower price for those firms that do reset.

BW show that, under the isoelastic functional forms assumed here for $U(\cdot)$ and $\tilde{V}(\cdot)$, the first order condition of the firm's problem can be arranged so that the optimal relative price is a function of aggregate variables only as

$$\frac{\tilde{p}_t(h)}{P_{H,t}} = \left(\frac{K_{H,t}}{F_{H,t}} \right)^{\frac{1}{1+\sigma\eta}}, \tag{13}$$

where $\tilde{p}_t(h)$ is the optimal price chosen by firm h and η is the inverse of the Frisch elasticity of labor supply. The determinants of the previous expression are defined as

$$K_{H,t} \equiv E_t \left\{ \sum_{T=t}^{\infty} (\alpha_H \beta)^{T-t} \, k_{H,T} \left(\frac{P_{H,T}}{P_{H,t}} \right)^{\sigma(1+\eta)} \right\}, \tag{14}$$

$$k_{H,T} \equiv \left(\frac{\sigma}{\sigma - 1} \right) \mu_{H,T}^w V_y \left(Y_{H,T}, a_{H,T} \right) Y_{H,T}, \tag{15}$$

$$F_{H,t} \equiv E_t \left\{ \sum_{T=t}^{\infty} (\alpha_H \beta)^{T-t} (1 - \tau_{H,T}) \frac{P_{H,T}}{P_T} f_{H,T} \left(\frac{P_{H,T}}{P_{H,t}} \right)^{\sigma-1} \right\}, \tag{16}$$

$$f_{H,T} \equiv U_C \left(C_{W,T} \right) Y_{H,T}. \tag{17}$$

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[13]If a firm is not allowed to choose the price, it adjusts its quantity to meet demand. Given the assumption of monopolistic competition, this does not necessarily imply negative profits. In fact, I shall assume that profits are always non-negative in what follows.

[14]This tax rate takes the form of a Value-Added Tax (VAT) rate.

[15]See Eggertsson and Woodford [2004] for a discussion of the effects of VAT rates as compared to alternative formulations.

The numerator $K_{H,t}$ of equation [13] is the present discounted value of a time-varying gross markup over all current and future (total) marginal costs. The gross markup can be decomposed in a combination of the price markup (the constant term) and of the distortions in the labor market due to the exogenous wage rigidities. The denominator $F_{H,t}$ is the present discounted value of all current and future (total) revenues net of taxation. These expressions mimic the closed economy result in BW with the difference of the adjustment brought about by the terms of trade effect from sales abroad.

All firms that reset the price will choose the same optimal figure. By the law of large numbers, the relevant price for all the firms that do not adjust is the country price index of the previous period. From the definition of the price index [4], the resulting Phillips curve for country H is

$$
\left(\frac{1 - \alpha_H \Pi_{H,t}^{\sigma-1}}{1 - \alpha_H} \right)^{\frac{1+\sigma\eta}{\sigma-1}} = \frac{F_{H,t}}{K_{H,t}} \tag{18}
$$

which only depends on aggregate variables and where GDP inflation is defined as $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$.

2.3 Policy Authorities

The monetary policy instrument is the nominal interest rate factor R_t. Following the recent literature, I will abstract from monetary frictions and consider the limit of a "cashless economy" (see Woodford [2003] for an extensive treatment). This implies that seigniorage is not a source of revenues for national governments. Nonetheless, monetary policy has important implications for fiscal decisions, as the level of the interest rate determines the debt burden and the inflation rate affects the real value of debt. Moreover, the presence of nominal rigidities ensures non-trivial effects of monetary policy on real activity.

Fiscal policy consists of choosing the mix between taxes and one period nominal risk-free debt to finance an exogenous process of public spending. The flow government budget constraints are

$$
nB_{H,t} = R_{t-1}nB_{H,t-1} - \int_0^n p_t(h) \left[\tau_{H,t}y_t(h) - g_t(h) \right] dh,
$$

$$
(1-n) B_{F,t} = R_{t-1} (1-n) B_{F,t-1} - \int_n^1 p_t(f) \left[\tau_{F,t}y_t(f) - g_t(f) \right] df. \tag{19}
$$

The variables $B_{H,t}$ and $B_{F,t}$ represent the per-capita issues in nominal terms of the Home and Foreign risk-free bonds respectively[16], which are in zero international net

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[16]Per-capita issues refer to the population size of the country of issuance. Households in each country can purchase public debt issued by the other country's government while governments are prohibited from holding each other's debt. Under complete markets, the latter assumption involves no loss of generality as long as no non-negativity restrictions on $B_{H,t}$ and $B_{F,t}$ are imposed. In principle, the government of country H can finance country's F deficit (and viceversa) by lending to the households who have purchased debt issued by country F.

supply. Using the expression for the demand of good h and f [6] and the definition of the price indexes [4], the previous equations can be readjusted to give

$$B_{H,t} = R_{t-1} B_{H,t-1} - P_t s_{H,t}, \qquad B_{F,t} = R_{t-1} B_{F,t-1} - P_t s_{F,t}, \qquad (20)$$

where real surpluses in per-capita terms are defined as

$$s_{H,t} \equiv \frac{P_{H,t}}{P_t} \left(\tau_{H,t} Y_{H,t} - G_{H,t} \right), \qquad s_{F,t} \equiv \frac{P_{F,t}}{P_t} \left(\tau_{F,t} Y_{F,t} - G_{F,t} \right). \qquad (21)$$

In an open economy, the household's transversality condition does not necessarily imply a correspondent restriction on the value of debt issued by each national government. Even if governments are limited by assumption not to purchase each other's debt, in principle, there might exist equilibria where the government of one country indefinitely borrows from the public, the other government lends to the public and asset trading in state-contingent securities is such that the households' transversality conditions are never violated[17]. Indeed, the only constraint brought about by the transversality conditions of the private sector is a correspondent transversality condition on the sum of the asset positions of the two governments (i.e., on consolidated debt). The scenario just depicted above may be plausible in the case of a benevolent planner that seeks to maximize the welfare of the whole union subject to the flow constraints for fiscal policy in the two countries, as the leverage of total debt can be managed arbitrarily at the centralized level. I will argue below that such paths for government debt are not relevant for the scope of this paper[18].

Given the definition of debt in per-capita terms, the consolidated government budget constraint[19] is the average of the two expressions in [20]

$$nB_{H,t} + (1-n) B_{F,t} = R_{t-1} \left[nB_{H,t-1} + (1-n) B_{F,t-1} \right] - P_t \left[ns_{H,t} + (1-n) s_{F,t} \right]. \qquad (22)$$

The appropriate transversality condition for government assets is

$$\lim_{T \to \infty} E_t \left\{ Q_{t,T} \left[nB_{H,T} + (1-n) B_{F,T} \right] \right\} = 0. \qquad (23)$$

Starting from [22], the resulting consolidated intertemporal budget constraint can be

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[17]A requirement for government debt to be strictly positive, as in Canzoneri, Cumby and Diba [2001], would rule out such schemes and would be sufficient for the private sectors' transversality condition to imply an analogous constraint on each national government's budget constraint.

[18]Bergin [2000] points out that an equilibrium with one government indefinitely purchasing bonds issued by the other country has unpleasant politico-economic features but cannot be ruled out a priori. A similar argument is proposed also by Woodford [1998]. Bergin [2000] adds that considering optimizing fiscal authorities would introduce a transversality condition on each government's value of debt, just in the same spirit of the one for the households' problem.

[19]The consolidated budget constraint presented here differs from the case of a fiscal federation, where fiscal policy is managed at the supranational level, insofar only one riskless bond is issued by the fiscal authority in the latter. In such an institutional framework, the household transversality condition does imply an analogous transversality condition on government assets, as in the closed economy model of BW.

written as

$$\frac{U_C\left(C_{W,t}\right)}{\Pi_t}\left[nb_{H,t-1} + (1-n)\,b_{F,t-1}\right] = E_t\left\{\sum_{T=t}^{\infty}\beta^{T-t}U_C\left(C_{W,T}\right)\left[ns_{H,T} + (1-n)\,s_{F,T}\right]\right\},\tag{24}$$

where I have defined the real value of debt at maturity in per-capita terms as $b_{i,t} \equiv R_t B_{i,t}/P_t$ and CPI inflation as $\Pi_t \equiv P_t/P_{t-1}$.

In the cooperative solution considered here, the benevolent central planner is required to respect the two flow budget constraints [20] for each period but is only subject to the single transversality condition [23] on the value of the sum of debt issued in the two countries. If the optimal plan satisfies the two constraints in [20] and the transversality condition [23], then, the solution always satisfies [24] as well. But the contrary is not necessarily true[20] and similarly it is not necessarily true in general that the two intertemporal government budget constraints are satisfied either. I will elaborate on these issues in the next sections.

2.4 Equilibrium

I restrict attention to rational expectation equilibria where the nominal interest rate is strictly positive $(R_t > 1)$. It is useful to define the relative price in country $i = \{H, F\}$ as $p_{i,t} \equiv P_{i,t}/P_t$ and the index $\Delta_{i,t}$, which measures the level of price dispersion for country i, as

$$\Delta_{H,t} \equiv \frac{1}{n}\int_0^n\left[\frac{p_t\left(h\right)}{P_{H,t}}\right]^{-\sigma(1+\eta)}dh, \qquad \Delta_{F,t} \equiv \frac{1}{1-n}\int_n^1\left[\frac{p_t\left(f\right)}{P_{F,t}}\right]^{-\sigma(1+\eta)}df.\tag{25}$$

The consumption Euler equation derives from substituting the expression for the stochastic discount factor [8] into the no-arbitrage condition [9]

$$1 = \beta R_t E_t\left\{\frac{1}{\Pi_{t+1}}\frac{U_C\left(C_{W,t+1}\right)}{U_C\left(C_{W,t}\right)}\right\}\tag{26}$$

I can then rewrite the demand equation for goods produced in country i [5] as

$$Y_{i,t} = p_{i,t}^{-\theta}C_{W,t} + G_{i,t}.\tag{27}$$

The relation between the relative price in country i and the terms of trade [7] becomes

$$p_{H,t}^{\theta-1} = n + (1-n)\,T_t^{1-\theta}, \qquad p_{F,t}^{\theta-1} = nT_t^{\theta-1} + (1-n)\,.\tag{28}$$

Expressions [26]-[28] describe the aggregate demand block of the economy.

Given the definition of the price index and the assumption of Calvo price setting, the measures of price dispersion in [25] evolve according to

$$\Delta_{i,t} = \alpha_i\Delta_{i,t-1}\Pi_{i,t}^{\sigma(1+\eta)} + (1-\alpha_i)\left(\frac{1-\alpha_i\Pi_{i,t}^{\sigma-1}}{1-\alpha_i}\right)^{\frac{\sigma(1+\eta)}{\sigma-1}}.\tag{29}$$

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[20]The result is an "if and only if" statement only when the intertemporal consolidated budget constraint is accompanied by at least one flow budget constraint at the country level.

The characterization of the supply side is completed by the Phillips curve of country i

$$\left(\frac{1 - \alpha_i \Pi_{i,t}^{\sigma-1}}{1 - \alpha_i}\right)^{\frac{1+\sigma\eta}{\sigma-1}} = \frac{F_{i,t}}{K_{i,t}}, \tag{30}$$

where expressions [14]-[17] can be rewritten in order to be consistent with the rest of the model as

$$K_{i,t} = k_{i,t} + \alpha_i \beta E_t \left\{\Pi_{i,t+1}^{\sigma(1+\eta)} K_{i,t+1}\right\}, \tag{31}$$

where

$$k_{i,T} \equiv \left(\frac{\sigma}{\sigma-1}\right) \mu_{i,T}^w V_y\left(Y_{i,T}, a_{i,T}\right) Y_{i,T}, \qquad T \geq t, \tag{32}$$

and

$$F_{i,t} = f_{i,t} + \alpha_i \beta E_t \left\{\Pi_{i,t+1}^{\sigma-1} F_{i,t+1}\right\}, \tag{33}$$

where

$$f_{i,T} = (1 - \tau_{i,T}) U_C\left(C_{W,T}\right) Y_{i,T}^{\sigma-1} p_{i,T}, \qquad T \geq t. \tag{34}$$

The flow budget constraints of the two fiscal authorities can be cast in terms of the real value of debt at maturity as follows

$$\frac{U_C\left(C_{W,t}\right) b_{i,t-1}}{\Pi_t} = U_C\left(C_{W,t}\right) p_{i,t}\left(\tau_{i,t} Y_{i,t} - G_{i,t}\right) + \beta E_t \left\{\frac{U_C\left(C_{W,t}\right) b_{i,t}}{\Pi_{t+1}}\right\}, \tag{35}$$

together with the definition $b_{i,t} = R_t B_{i,t}/P_t$. As noted before, the long run fiscal sustainability is subject to a transversality condition on the value of consolidated debt of the form

$$\lim_{T \to \infty} E_t \left\{\beta^{T-t} \frac{U_C\left(C_{W,T}\right)}{\Pi_T} \left[n b_{H,T-1} + (1-n) b_{F,T-1}\right]\right\} = 0. \tag{36}$$

At the union level, given the absence of the nominal exchange rate as an automatic stabilizer, the definition of the terms of trade implies a one to one correspondence between GDP inflation rate differentials and variations of the terms of trade itself

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}}. \tag{37}$$

Finally, from the definition of the CPI, the relation between CPI inflation, GDP inflation and relative prices is

$$\Pi_t^{1-\theta} = n\left(\Pi_{H,t} p_{H,t-1}\right)^{1-\theta} + (1-n)\left(\Pi_{F,t} p_{F,t-1}\right)^{1-\theta}. \tag{38}$$

I can now define an equilibrium for this economy.

Definition 1 *An imperfectly competitive equilibrium is a sequence of stochastic processes $\mathcal{X}_t \equiv \{C_{W,t}, Y_{i,t}, p_{i,t}, \Delta_{i,t}, \Pi_{i,t}, K_{i,t}, k_{i,t}, F_{i,t}, f_{i,t}, B_{i,t}/P_t, b_{i,t}, T_t, \Pi_t\}$ that satisfy conditions [26]-[38] plus the definition of $b_{i,t}$, given fiscal and monetary policies $\mathcal{P}_t \equiv \{\tau_{i,t}, R_t\}$, exogenous processes $\xi_t \equiv \{a_{i,t}, \mu_{i,t}^w, G_{i,t}\}$ and initial conditions $\mathcal{I}_{-1} \equiv \{b_{i,-1}, B_{i,-1}/P_{-1}, \Delta_{i,-1}, T_{-1}\}$, for $i = \{H, F\}$ and $t \geq 0$.*

In Definition 1, it is assumed that the instrument of the fiscal authorities is the tax rate $(\tau_{i,t})$ and that the instrument of the monetary authority is the nominal interest rate (R_t). Obviously, this choice is purely arbitrary insofar the number of variables considered as instruments is equal to the number of policies to be specified[21].

3   The Optimal Policy Problem

Total welfare per-capita in country H and F is given by the sum of individual utility divided by the population size of each country, as in

$$u_{H,0} \equiv E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_{W,t}) - \frac{1}{n} \int_0^n V(y_t(h), a_{H,t}) \, dh \right] \right\}, \tag{39}$$

$$u_{F,0} \equiv E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_{W,t}) - \frac{1}{1-n} \int_n^1 V(y_t(f), a_{F,t}) \, df \right] \right\}. \tag{40}$$

The functional form of the utility of consumption and of the disutility of production[22] is assumed to be isoelastic

$$U(C_{W,t}) \equiv \frac{C_{W,t}^{1-\rho}}{1-\rho}, \qquad V(y_t(j), a_{i,t}) \equiv (a_{i,t})^{-(1+\eta)} \frac{(y_t(j))^{1+\eta}}{1+\eta}.$$

I define the objective of the optimal policy problem to be the population-weighted average of the two welfare criteria [39] and [40]

$$u_{W,0} \equiv n u_{H,0} + (1-n) u_{F,0}. \tag{41}$$

Using the demand equations [6], the disutility of production in [39] and [40] can be expressed as a function of aggregate output and the index of price dispersion [25]

$$\frac{1}{n} \int_0^n V(y_t(h), a_{H,t}) \, dh = \frac{(Y_{H,t}/a_{H,t})^{1+\eta}}{1+\eta} \Delta_{H,t} = V(Y_{H,t}, a_{H,t}) \Delta_{H,t},$$

and

$$\frac{1}{1-n} \int_n^1 V(y_t(f), a_{F,t}) \, df = \frac{(Y_{F,t}/a_{F,t})^{1+\eta}}{1+\eta} \Delta_{F,t} = V(Y_{F,t}, a_{F,t}) \Delta_{F,t},$$

Combining the new expressions for the disutility of production with the definition of the welfare criterion [41], the policy objective can be cast as

$$u_{W,0} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_{W,t}) - n V(Y_{H,t}, a_{H,t}) \Delta_{H,t} - (1-n) V(Y_{F,t}, a_{F,t}) \Delta_{F,t} \right] \right\}. \tag{42}$$

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[21]Indeed, the simple rules analyzed in section 6 will be specified in terms of the inflation rate Π_t and the real debt $B_{i,t}/P_t$.

[22]From $y_t(h) = a_{H,t} \ell_t(h)$, one can substitute out labor and express the disutility of labor in terms of output and productivity.

In a centralized solution, the central planner maximizes [42] subject to the constraints [26]-[38] and the definition of $b_{i,t}$, given the exogenous shocks ξ_t and the initial conditions $\mathcal{I}_{-1}$.

In the absence of further constraints, the time-zero (Ramsey) solution of the policy problem implies time-inconsistency of the optimal plan[23]. In the current example, the presence of predetermined prices (for some fraction of firms in each country) and debt implies that the policymaker would have the incentive to reduce the real value of debt and close the output gap by generating high inflation at time zero and then commit to low inflation thereafter. This policy is time-inconsistent because at any later stage the policymaker would face the same incentive as at time zero. Should reoptimization be allowed, the optimal policy would be again a one period high inflation and a commitment to low inflation in the future.

In order to obtain a time-consistent policy, some form of commitment is required. For this matter, I follow Woodford [1999] and characterize the optimal policy plan from a '*timeless perspective*'. This approach imposes on the problem a set of state-contingent commitments that prevent the policymaker from exploiting future private expectations along the path of the endogenous variables implied by the optimal policy plan[24]. The resulting set of first order conditions can then be interpreted as the policy rules that a traditional Ramsey solution would eventually follow, whereas the initial (time zero) policy prescription would differ insofar the Ramsey planner does not internalize the effects of previous expectations on the initial policy.

The next definition formalizes the requirements of the optimal policy plan.

Definition 2 *The optimal plan is a sequence of policies $\mathcal{P}_t$ that maximize $u_{W,0}$ such that the implied allocation $\mathcal{X}_t$ constitutes an equilibrium and the additional constraints of the timeless perspective are satisfied, for given exogenous processes ξ_t and initial conditions $\mathcal{I}_{-1}$.*

The non-linear optimal policy problem cannot be solved in closed form. Here, I choose to study the associated approximate optimal policy problem. To this extent, in section $A.1$ of the appendix, I show the existence of a well defined non-stochastic symmetric steady state supported by an optimal policy plan with no inflation and constant debt. I will then proceed to analyze the solution for small enough disturbances[25] such that, if the economy starts in a neighborhood of the steady state, it always stays close to it thereafter.

Before turning to that characterization, it is important to clarify the nature of fiscal policy in the model. In particular, it has been left unsolved the possibility of the existence of equilibria with explosive paths of government debt that do not violate the transversality condition on consolidated debt. Indeed, these paths cannot occur in the approximate equilibrium considered here. The reason is that the approximate

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[23]This problem is a well established result in the optimal fiscal and monetary policy literature, starting with Lucas and Stokey [1983].

[24]More practically, the optimal policy problem is solved under the additional constraint that the resulting first order conditions are time-invariant.

[25]The exact meaning of "small" disturbances will be qualified further below.

problem (and hence any induced approximate equilibrium) requires the path of the endogenous variables to stay close to the deterministic steady state, which is shown to be characterized by constant and equal debt for the two countries. Hence, one can safely assume that the transversality condition for either countries is never violated, at least in the approximate problem. In other words, the class of equilibria considered here will always be characterized by "(locally) Ricardian" fiscal policies in both countries, in the sense of Woodford [1998] and Loyo [1997], or "(locally) responsible", as in Bergin [2000].

4   Calibration

I calibrate the key distinctive parameters of the model, such as the degree of price rigidity and the steady state fiscal stances, in order to match the features of several countries belonging to the EMU. The remaining values are chosen consistently with similar studies within the international business cycles literature.

Based on country estimates of the New-Keynesian Phillips curve, Benigno and López-Salido [2005] divide the five major economies of the EMU (France, Germany, Italy, Netherlands and Spain, accounting for 88% of the total GDP of the Euro area) in two groups[26]. Germany displays a degree of price rigidity substantially lower than the other four countries, which are, on the contrary, quite homogeneous according to this dimension. Here, I follow that criterion to identify the two economies that constitute the currency union. Country H represents France, Italy, Netherlands and Spain, a total of 53% of the Euro area GDP, which implies that $n = 0.6$. In those countries, the average duration of price contracts $((1 - \alpha_H)^{-1})$ is equal to 8 quarters, which gives a value of α_H of 7/8. On the other hand, country F represents Germany, where the average price duration is of 5 quarters, determining a value of α_F equal to 4/5.

Table 1 displays the recent evolution of government surplus (or deficit) and government debt as a percentage of GDP for the twelve countries that constitute the EMU[27]. The calibration of fiscal variables is bound to satisfy the steady state government budget constraint $(1 - \beta)\bar{b} = \bar{\tau}\bar{Y} - \bar{G}$. Given the choice of the discount factor, there are three variables left to be determined but only two of them are independent.

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[26]The estimates of Benigno and Lopez-Salido [2005] are based on a "hybrid" specification of the New-Keynesian Phillips curve, including backward looking price setters not present in the model presented here. It is shown that the four countries with higher degree of price rigidity also display a significant fraction of backward looking price setters. Hence, the estimates of the degree of price stickiness inferred from the average duration of price contracts in the two zones cannot be considered a one to one map to the specification adopted in this paper. Nonetheless, the main results presented in the rest of the paper are robust to alternative assumptions on the value of α^* (around the benchmark), under the maintained assumption that the size of country H is 0.6 (that is, country H represents the same group of four countries) and that country H is characterized by a relatively higher degree of nominal stickiness,

[27]The first line of the table also shows population adjusted averages of both variables for the whole Euro area. The figures in bold represent a violation of the Maastricht criteria for either deficits (3% of GDP) or debt (60% of GDP).

Table 1: Government surplus (+) or deficit (-) and debt (% of GDP).

	2000		2001		2002		2003	
	Surplus	Debt	Surplus	Debt	Surplus	Debt	Surplus	Debt
Euro Area	-0.9	**70.4**	-1.7	**69.4**	-2.3	**69.2**	-2.7	**70.4**
Belgium	0.2	**109.1**	0.4	**108.1**	0.1	**105.8**	0.2	**100.5**
Germany	-1.2	**60.2**	-2.8	59.4	**-3.5**	**60.8**	**-3.9**	**64.2**
Greece	-2.0	**106.2**	-2.0	**106.9**	-1.4	**104.7**	-1.7	**102.4**
Spain	-1.0	**61.2**	-0.4	57.5	0.0	54.6	0.3	50.8
France	-1.4	57.2	-1.6	56.8	**-3.3**	58.6	**-4.1**	**63.0**
Ireland	4.4	38.4	1.1	36.1	-0.4	32.3	0.2	32.0
Italy	-1.8	**111.2**	-2.6	**110.6**	-2.3	**108.0**	-2.4	**106.2**
Luxembourg	6.3	5.5	6.3	5.5	2.7	5.7	-0.1	4.9
Netherlands	1.5	55.9	0.0	52.9	-1.9	52.6	-3.0	54.8
Austria	-1.9	**67.0**	0.2	**67.1**	-0.2	**66.6**	-1.1	**65.0**
Portugal	**-3.2**	53.3	**-4.4**	55.6	-2.7	58.1	-2.8	59.4
Finland	7.1	44.6	5.2	43.9	4.3	42.6	2.3	45.3

Source: ECB Annual Report [2003], p. 56 (available at www.ecb.int)

I choose a steady state tax level $\bar{\tau} = 30\%$, which approximately corresponds to the average value of direct and indirect tax revenues as a percentage of GDP of 31.6% among the EMU members in 2003 (source: OECD Economic Outlook Database). Further, the steady state ratio of debt to year GDP $\bar{b}/\left(4\bar{Y}\right)$ is assumed to be 60% (which gives $\bar{b}/\bar{Y} = 2.4$). This value is lower than the average Euro area debt to GDP ratio in the last four years (which is around 70%, according to OECD Economic Outlook Database) but it is consistent with the the upper bound in the Maastricht Treaty[28]. It then follows that the public spending to GDP ratio $\bar{G}/\bar{Y}$ is 27.6% and that the consumption to GDP ratio is $s_c \equiv \bar{C}/\bar{Y} = 72.4\%$.

Table 2 and 3 report the calibrated values for the remaining parameters of the model and the implied steady state quantities.

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[28]As it emerges clearly from inspection of Table 1, in the last four years, the vast majority of countries in the Euro area experienced substantial deficits, the average ranging between 0.9% and 2.7%, with more than one country violating the deficit/GDP threshold of 3% indicated by the Maastrich Treaty. One feature of the model is that the approximation is performed around a steady state with positive and constant debt. The assumption of positive steady state debt in turn requires a positive steady state primary surplus (equal to 2.4% of output, given the choice of $\bar{b}/\bar{Y}$ and β) which is counterfactual to current data but can be thought as a long run objective consistent with the choice of the debt to GDP ratio expressed in the Maastricht Treaty and assumed here too.

Table 2: Baseline calibration.

n	$=$	0.6	SIZE OF COUNTRY H
β	$=$	0.99	DISCOUNT FACTOR
η	$=$	0.47	INVERSE FRISCH ELASTICITY
α_H	$=$	$7/8$	DEGREE OF PRICE RIGIDITY IN COUNTRY H
α_F	$=$	$4/5$	DEGREE OF PRICE RIGIDITY IN COUNTRY F
$\bar{\tau}$	$=$	30%	S.S. TAX RATE
$\bar{b}/\left(4\bar{Y}\right)$	$=$	60%	S.S. DEBT-OUTPUT RATIO
ρ	$=$	3	COEFFICIENT OF RISK AVERSION
θ	$=$	4.5	ELASTICITY OF INTRATEMPORAL SUBSTITUTION
σ	$=$	11	ELASTICITY OF SUBSTITUTION AMONG GOODS
$\bar{\mu}^w$	$=$	1.05	S.S. GROSS WAGE MARKUP

Table 3: Implied steady state values.

$(1-\alpha_H)^{-1}$	$=$	8 qrt.s	AVG. DURATION OF PRICE CONTRACTS IN COUNTRY H
$(1-\alpha_F)^{-1}$	$=$	5 qrt.s	AVG. DURATION OF PRICE CONTRACTS IN COUNTRY F
$\bar{G}/\bar{Y}$	$=$	27.6%	S.S. PUBLIC SPENDING-OUTPUT RATIO
$\bar{s}/\bar{Y}$	$=$	2.4%	S.S. PRIMARY SURPLUS-OUTPUT RATIO
$\bar{C}/\bar{Y}$	$=$	72.4%	S.S. CONSUMPTION-OUTPUT RATIO
$\bar{\mu}$	$=$	1.65	S.S. GROSS TOTAL MARKUP

As it is common in the literature, the discount factor β is set to 0.99, that is, a four percent steady state annualized rate of return. Following Benigno and Benigno [2003], the inverse of the Frisch elasticity of labor supply to the real wage η, the coefficient of risk aversion ρ and the elasticity of substitution between the Home and Foreign bundles of goods θ are assumed to be 0.47, 3 and 4.5 respectively. As the elasticity of intratemporal substitution θ is larger than the elasticity of intertemporal substitution $1/\rho$, the bundles $C_{H,t}$ and $C_{F,t}$ are substitutes. The elasticity of substitution among differentiated goods σ is assumed to be 11, which implies that the component of the total price markup due to demand is equal to 10%, slightly lower than the value commonly adopted in the literature[29]. The only quantity that remains to be determined is the steady state gross wage markup $\bar{\mu}^w$. I calibrate the net steady state total markup to be 65%, close to the midpoint between the value in BW, who consider a total markup of 50%, and the value in Erceg, Henderson and

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[29]When the markup depends on monopolistic competition in the goods market only, the literature generally assumes values of the elasticity of substitution lower or equal to 10. In order to avoid excessive steady state distortions, I adopt a conservative calibration, just below the standard values.

Levin [2000], where the steady state total markup takes value equal to 77%. From the formula $\bar{\mu} = \sigma\bar{\mu}^w/(\sigma-1)(1-\bar{\tau}) = 1.65$, which measures the product of the gross price markup and the wedges introduced by labor market rigidities and tax distortions, given the previous choices of σ and $\bar{\tau}$, it then follows that $\bar{\mu}^w = 1.05$. Finally, following existing studies in the international business cycle literature[30], the log-linear deviations of average and relative shocks to productivity, wage markup and government spending, are assumed to follow an uncorrelated VAR(1) process with common persistence, set to 0.9, and standard deviation of the innovations equal to 0.01.

5 The LQ Approximate Problem

As mentioned, the optimal policy problem illustrated above cannot be solved in closed form. I follow BW and adopt an analytical approximation that relies on Taylor expansions around the deterministic symmetric steady state (derived in $A.1$). I wish to characterize the optimal fiscal and monetary policy under a timeless commitment and from a centralized perspective[31] for a local approximation of the non-linear stochastic problem detailed in section 3. More specifically, I study a first-order approximation around the deterministic steady state of the optimal plan. As described in BW, a log-linear approximation of the optimal policy can be obtained as the solution of a LQ problem composed by a second order Taylor expansion of the welfare objective and a first order expansion of the set of constraints[32].

As shown in section $A.2$ of the appendix, the derivation of the second order approximation of the welfare objective displays some non-zero linear terms in the endogenous variables. Woodford [2003] discusses how the presence of linear terms generally leads to evaluate welfare incorrectly by using a first order approximation to the equilibrium relations[33]. In certain cases, the problem can be solved by assuming that the policymaker uses the tax rate as a subsidy to eliminate the monopolistic distortions in steady state (see, for instance, Rotemberg and Woodford [1997]). The same approach cannot be used here because the existence of a non-negative stock of debt and of a non-negative level of public spending in steady state requires a non-negative tax rate $\bar{\tau}$ to satisfy the government budget constraint. Instead, I follow the method first proposed by Sutherland [2002] in a static context and extended by BW to a dynamic framework[34]. By taking a second order Taylor expansion of the equilibrium

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[30]See, for instance, Backus, Kehoe and Kydland [1992].

[31]It is worth stressing that national governments have independent budget constraints, hence, debt and taxes are chosen separately for each country. However, the objective for fiscal policy is unique, namely aggregate welfare $u_{W,0}$. In this sense, the fiscal policy regime can also be thought as if the two independent fiscal authorities cared about the welfare of the whole union.

[32]Benigno and Woodford [2005] discuss why this approach is not suitable to study the solution of the problem under discretion. The basic reason is that, in case of large steady state distortions, discretionary policy entails high inflation even in the absence of shocks. Hence, fluctuactions around the steady state (which has zero inflation) cannot be considered 'small', violating the nature of the approximation.

[33]A simple enlightening example of this result is proposed by Kim and Kim [2003].

[34]An alternative approach would be based on the numerical procedures developed by Kim, Kim,

conditions, it is possible to eliminate the linear terms in the approximation of [41]. The result is a purely quadratic welfare objective that allows to evaluate welfare correctly using a first order approximations of the equilibrium conditions.

The resulting approximate welfare objective for the union as a whole can then be written as

$$u_{W,0} = -\frac{1}{2} U_C \bar{C} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_{W,t} \right\} + J_{W,0} + t.i.p. + \circ \left(\|\xi_t\|^3 \right), \qquad (43)$$

where $t.i.p.$ is an acronym for '*terms independent of policy*' and the element $\circ \left(\|\xi_t\|^3 \right)$ stands for terms of order three or higher. The per-period loss function is defined as

$$L_{W,t} \equiv \lambda_y y_t^2 + n (1-n) \lambda_q q_t^2 + n \lambda_{\pi_H} \pi_{H,t}^2 + (1-n) \lambda_{\pi_F} \pi_{F,t}^2. \qquad (44)$$

For any variable X_t, I define the log-deviation from its steady state value $\bar{X}$ as $\hat{X}_t \equiv \ln \left(X_t / \bar{X} \right)$. The variable $y_t \equiv \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t} \right)$ represents the deviation of average output in the union from its target level $\tilde{Y}_{W,t}$ compatible with full stabilization (defined in section $A.2$ of the appendix as a linear combination of the exogenous shocks). Similarly, the variable $q_t \equiv \left(\hat{T}_t - \tilde{T}_t \right)$ stands for the deviation of the terms of trade from its target level[35], also defined in $A.2$. Finally, GDP inflation rates are defined as $\pi_{i,t} \equiv \ln \left(P_{i,t} / P_{i,t-1} \right)$, which implies that the target level for GDP inflation is zero. The parameters λ_i are combinations of the structural parameters[36] defined in $A.2$. The term $J_{W,0}$ in [44] depends on the special nature of time zero. The presence of the additional constraints for policy to be optimal from a timeless perspective implies that the function $J_{W,0}$ (a linear combination of the average initial price dispersion and the initial real value of debt) can be taken as given. It follows that the policy that maximizes total welfare is the policy that minimizes the present discounted value of the loss function $L_{W,t}$ subject to the constraints implied by the equilibrium conditions of the model and by the additional timeless perspective requirements[37], given initial conditions on the terms of trade and on debt in each country.

Eliminating the aggregate demand equations and substituting for each country's relative price with the appropriate function of the terms of trade, one can reduce the equilibrium conditions to six equations. Along the simplification, it is also convenient to express consumption as the difference between average output and average public

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Schamburg and Sims [2003], Juillard [2003] and Schmitt-Grohé and Uribe [2004].

[35]The the target level for average output $\tilde{Y}_{W,t}$ and for the terms of trade $\tilde{T}_t$, as well as the target levels defined below for the tax rates, are not in general the "natural" levels commonly referred to in the literature, that is, the levels of variables under the assumption of complete price flexibility. The target levels would coincide with the natural levels only in the case of $\bar{\mu} = 1$, which is not possible in the current framework.

[36]The concavity of the loss function L_t^W is not generally guaranteed. Conditions for $\lambda_i > 0$ are not obvious to derive. In section $A.2$ of the appendix (see Figure $A.1$), I check numerically that the loss function is concave for different values of the steady state tax rate $\bar{\tau}$.

[37]In the context of this model, such additional constraints are imposed on the inflation rates (i.e., $\pi_{H,0} = \bar{\pi}_H$, $\pi_{F,0} = \bar{\pi}_F$ and $\pi_0 = \bar{\pi}$) and on the output gap at the union level (i.e., $y_0 = \bar{y}$) and have the effect of making the first order conditions time-invariant.

spending from the resource constraint. It follows that the average output level is the only endogenous quantity that conveys all the required information for optimal policy from the demand side.

The first two log-linear constraints are the aggregate supply curves of the two countries. The additional terms, with respect to the closed economy model of BW, are implied by the expenditure-switching effect of the terms of trade (see Svensson [2000])

$$\pi_{H,t} = \kappa_H \left[\delta_y y_t + \omega_\tau \left(\hat{\tau}_{H,t} - \tilde{\tau}_{H,t}\right) + (1-n)\,\delta_q q_t\right] + \beta E_t \pi_{H,t+1}, \qquad (45)$$

and

$$\pi_{F,t} = \kappa_F \left[\delta_y y_t + \omega_\tau \left(\hat{\tau}_{F,t} - \tilde{\tau}_{F,t}\right) - n\delta_q q_t\right] + \beta E_t \pi_{F,t+1}. \qquad (46)$$

The variables $\tilde{\tau}_{H,t}$ and $\tilde{\tau}_{F,t}$ are the target levels of taxation in the two countries compatible with full stabilization[38]. Those terms and the parameters δ_y, ω_τ and δ_q are defined in section $A.3$ of the appendix.

The next relations are the intertemporal budget constraints for the two fiscal authorities[39]

$$\hat{B}_{H,t} = (1-\beta)\left[b_y y_t + (1-n)\,b_q q_t + (1+\omega_g)\left(\hat{\tau}_{H,t} - \tilde{\tau}_{H,t}\right)\right] + \beta E_t \hat{B}_{H,t+1}, \qquad (47)$$

and

$$\hat{B}_{F,t} = (1-\beta)\left[b_y y_t - n b_q q_t + (1+\omega_g)\left(\hat{\tau}_{F,t} - \tilde{\tau}_{F,t}\right)\right] + \beta E_t \hat{B}_{F,t+1} \qquad (48)$$

where

$$\hat{B}_{H,t} \equiv \hat{b}_{H,t-1} - \rho s_c^{-1} y_t - \pi_t + \psi_{H,t},$$

and

$$\hat{B}_{F,t} \equiv \hat{b}_{F,t-1} - \rho s_c^{-1} y_t - \pi_t + \psi_{F,t}.$$

The parameters b_y, b_q and ω_g are defined in $A.3$. The left-hand side of the government budget constraint represents the real value of debt at maturity in units of marginal utility plus a "fiscal stress" term ($\psi_{i,t}$), defined in $A.3$ as a composite of exogenous disturbances. The current component of the right-hand side is the adjusted expression for the time t level of surplus, also in units of marginal utility.

The relation between total inflation (or CPI inflation) and country-specific inflation (or GDP inflation) can be approximated as

$$\pi_t = n\pi_{H,t} + (1-n)\,\pi_{F,t}. \qquad (49)$$

The depreciation of the terms of trade and the cross-country GDP inflation differential are linked by

$$q_t = q_{t-1} + \pi_{F,t} - \pi_{H,t} - \Delta \tilde{T}_t, \qquad (50)$$

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[38] Alternatively, the terms $-\omega_\tau \tilde{\tau}_{H,t}$ and $-\omega_\tau \tilde{\tau}_{F,t}$ can be interpreted as "cost-push" shocks that preclude simultaneous stabilization of GDP inflation, the welfare-relevant output gap and the terms of trade.

[39] As stressed before, the intertemporal budget constraints for the two governments can be written only for the approximate problem, given initial conditions close enough to the steady state and given that the magnitude of the shocks is bounded in such a way that variables never depart too far from the starting values.

where $\Delta \tilde{T}_t \equiv \tilde{T}_t - \tilde{T}_{t-1}$ is the one-period change in the target level of the terms of trade. Equations [49] and [50] constitute the last two constraints arising from the set of equilibrium conditions.

Compared to the closed economy of BW, the presence of structural (the different degree of price rigidities) and exogenous (the country-specific nature of the shocks) asymmetries between the two countries introduce one additional dimension to be taken into account by the optimal policy plan. The cooperative solution needs to seek an optimal balance between stabilization objectives in the two areas and the optimal solution of the traditional output-inflation tradeoff. Moreover, differently from a model with flexible exchange rates, like Benigno and Benigno [2003], the policymaker needs to directly internalize the distortions due the terms of trade. With monetary independence, the nominal exchange rate is residually determined through the PPP relation in first differences. In a currency union, this channel is shut down and any differentials in the GDP inflation rates are reflected in the terms of trade without absorption by nominal variables. Therefore, the dynamics of the terms of trade become an explicit constraint in the optimal policy problem. Indeed, in closed economy, it is only the fiscal stress that prevents contemporaneous stabilization of output gap and inflation in presence of sticky prices. In a currency union, even in the absence of any fiscal stress, full stabilization cannot be achieved because the nominal exchange rate does not absorb the fluctuations induced by terms of trade shocks.

The optimal stabilization policy can be determined following a Lagrangian method. The policymaker chooses the sequence $\{y_t,\, q_t,\, \pi_{H,t},\, \pi_{F,t},\, \pi_t,\, \hat{\tau}_{H,t},\, \hat{\tau}_{F,t},\, \hat{b}_{H,t},\, \hat{b}_{F,t}\}_{t=0}^{\infty}$ to maximize [43] subject to the infinite sequence of constraints [45]-[50], plus the additional initial requirements implied by the timeless perspective, given the set of initial conditions on debt and the terms of trade.

The first condition for the output gap is

$$\lambda_y y_t = -\omega_y \varphi_{2,t}^W - \rho s_c^{-1} \varphi_{2,t-1}^W, \tag{51}$$

where $\varphi_{2,t}^W$ is the population-weighted average of the Lagrange multipliers on equations [47] and [48]. The first order condition for the terms of trade gap is

$$\lambda_q q_t = -\omega_q \varphi_{2,t}^R - \varphi_{4,t} + \beta E_t \varphi_{4,t+1}, \tag{52}$$

where $\varphi_{2,t}^R$ is the difference between the Lagrange multipliers on equations [47] and [48] and $\varphi_{4,t}$ is the multiplier on [50]. The first order conditions for GDP inflation rates can be combined to give

$$n\kappa_H \lambda_{\pi_H} \pi_{H,t} + (1-n)\kappa_F \lambda_{\pi_F} \pi_{F,t} = (\omega_f + \kappa_W)\left(\varphi_{2,t}^W - \varphi_{2,t-1}^W\right) - n(1-n)\kappa_R \varphi_{4,t}, \tag{53}$$

and

$$\kappa_H \lambda_{\pi_H} \pi_{H,t} - \kappa_F \lambda_{\pi_F} \pi_{F,t} = \omega_f\left(\varphi_{2,t}^R - \varphi_{2,t-1}^R\right) + \kappa_R\left(\varphi_{2,t}^W - \varphi_{2,t-1}^W\right) - \tilde{\kappa}\varphi_{4,t}. \tag{54}$$

Finally, the first order conditions for the real value of debt can be combined as

$$\varphi_{2,t}^W = E_t \varphi_{2,t+1}^W, \tag{55}$$

and

$$\varphi_{2,t}^{R} = E_t \varphi_{2,t+1}^{R}. \tag{56}$$

The set of optimality conditions is then completed by the constraints [45]-[50]. The Lagrange multipliers on equations [45]-[46] and [49] have been eliminated by appropriate substitutions. The parameters ω_y, ω_q, ω_f, κ_W, κ_R and $\tilde{\kappa}$ are defined in section A.4 of the appendix.

The main feature of the optimal policy plan is the random walk behavior of the Lagrange multipliers on the value of debt in terms of marginal utility (equations [55] and [56]). As a consequence, the output gap, the terms of trade gap, the tax gap and the real value of debt all inherit a unit root. Under the assumption that the target levels of these variables are stationary (that is, under stationary processes for the exogenous shocks), the correspondent levels also contain a unit root[40]. Differently from the standard analysis of optimal fiscal policy (Barro [1979]), the tax rate is not a pure random walk but follows a more complicated integrated process. Yet, the message is similar. The optimal stabilization plan requires smoothing of the distortions induced by taxation via a permanent adjustment of the debt level to guarantee sustainability of the intertemporal government budget constraint.

On the other hand, inflation is stationary. If the degree of price rigidity is the same across countries, the optimal policy plan calls for stabilization of the expected CPI inflation rate ($E_t \pi_{t+1} = 0$). This result of stabilization of expected inflation depends crucially on the assumption of sticky prices. If prices were fully flexible[41], the optimal plan would call instead for stabilization of the output gap. Still, differently from BW, it would not be optimal to achieve a zero output gap in every period but rather smooth the gap around a constant[42]. In the general case, the weights on each country's GDP inflation rates need to be adjusted according to the a measure of price stickiness

$$\left(\frac{n}{\kappa_H}\right) E_t \pi_{H,t+1} + \left(\frac{1-n}{\kappa_F}\right) E_t \pi_{F,t+1} = 0.$$

Since κ_H and κ_F are inversely related to the degree of structural rigidities α_H and α_F, optimal policy attaches more weight to the stabilization of the expected GDP inflation rate in the country higher degree of price stickiness.

Finally, the terms of trade has a double nature. On the one hand, it helps achieving complete intratemporal smoothing because of the unit root property derived from fiscal adjustment. On the other hand, its future evolution is constrained by the adoption of the common currency, that is, by the absence of the nominal exchange rate acting as an automatic stabilizer.

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[40]Given the permanent effect of stationary shocks on the endogenous variables, a bounded solution exists only under the assumption that the disturbances occur in a finite number of periods. However, neither the total number of periods nor the specific date of occurrence affects the optimal solution. For any given number of periods when shocks occur, a larger number can be allowed by assuming a tighter bound on the size of the shocks for the optimal paths of the endogenous variables to remain within a given neighborhood of the steady state.

[41]This case corresponds to assuming $\alpha_i = 0$, which in turn implies $\kappa_i^{-1} = 0$ and $\lambda_{\pi_i} = 0$.

[42]A zero output gap would require CPI inflation to respond to variations of the fiscal stress in both countries which is clearly impossible.

In general, the optimal policy plan is the solution of the system of linear stochastic difference equations composed by the set of optimality conditions [45]-[56], which has general representation

$$A_1 E_t x_{t+1} + A_0 x_t + A_{-1} x_{t-1} = B \xi_t. \tag{57}$$

In the system of equations [57], x_t represents a vector of endogenous variables and Lagrange multipliers

$$x_t' \equiv \begin{bmatrix} y_t & q_t & \pi_{H,t} & \pi_{F,t} & \pi_t & \hat{\tau}_{W,t} & \hat{\tau}_{R,t} & \hat{b}_{W,t} & \hat{b}_{R,t} & \varphi_{2,t}^W & \varphi_{2,t}^R & \varphi_{4,t} \end{bmatrix},$$

where, for convenience, fiscal variables have been expressed in terms of average and relative stances. The vector of shocks ξ_t contains average and relative composites of exogenous disturbances (target tax rates and fiscal stress terms) plus the one-period variation of the terms of trade target

$$\xi_t' \equiv \begin{bmatrix} \tilde{\tau}_{W,t} & \tilde{\tau}_{R,t} & \psi_{W,t} & \psi_{R,t} & \Delta \tilde{T}_t \end{bmatrix}.$$

Finally, the objects A_1, A_0, A_{-1} and B are non-stochastic conformable matrices of coefficients defined in $A.4$. In general, the full analytical characterization of the optimal stabilization plan is highly complicated, mainly due to the contemporaneous presence of the two government budget constraints and of the evolution of the terms of trade, plus the structural asymmetries in the degree of price rigidity. Below, I will mainly refer to the optimal policy plan as a benchmark for evaluation of alternative (suboptimal) fiscal and monetary rules[43].

6   Simple Rules

This section introduces the simple policy rules that are assumed to represent the behavior of the fiscal and monetary authorities in the currency union. I express the monetary rule in terms of CPI inflation targeting and the fiscal rule in terms of a constraint on real debt.

The rule for the single central bank takes the form of a "flexible" inflation targeting[44]

$$\Pi_t \left(\frac{Y_t^{gap}}{Y_{t-1}^{gap}} \right)^\gamma = C_m, \tag{58}$$

where Y_t^{gap} is implicitly defined as the non-linear version of the output gap y_t, γ is a parameter bigger or equal than zero and C_m is a constant chosen so that the rule

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[43]For that purpose, I will solve the model numerically, employing the algorithm by Anderson and Moore [1985], which provides an efficient method to compute the reduced form solution of system [57] and to check whether the determinacy conditions are satisfied. In section $A.5$ of the appendix, I study analytically the determinacy properties of the optimal policy plan under the assumption of equal degree of price rigidity across countries.

[44]Svensson [1999] discusses the advantages of targeting rules (rather than instrument rules) for monetary policy. Benigno [2004] studies the optimality of inflation targeting rules in a currency union.

[58] is compatible with the steady state of the model. If $\gamma = 0$, then, it must be the case that $C_m = 1$ and the rule becomes a "strict" inflation targeting.

On the other hand, the fiscal authority of country i is subject to a constraint on real debt that can be cast as

$$\frac{B_{i,t}}{P_t} \left(Y_{i,t}^{gap}\right)^{\phi} = C_f, \tag{59}$$

where $Y_{i,t}^{gap}$ is implicitly defined as the non-linear version of the output gap for country i ($y_{i,t}$ defined in section $A.2$ of the appendix), ϕ is a parameter bigger or equal than zero and C_f is a constant chosen so that the rule [59] is compatible with the steady state of the model[45]. If $\phi = 0$, then, consistency requires $C_f = \beta \bar{b}$ and the rule requires that the fiscal authority of each country maintains the real debt constant over time[46].

The central point of my analysis is the comparison of strict versus flexible policy rules in terms of stabilization and welfare[47]. I consider the baseline specification of the policy framework in terms of strict inflation targeting ($\gamma = 0$) and constant real debt ($\phi = 0$) and compare the outcome, in terms of response to exogenous shocks and welfare, with a regime characterized by optimal flexible rules[48]. The baseline specification of a "rigid" policy regime emphasizes the limitations, for stabilization purposes, of the current framework in the European Monetary Union. While being an obvious abstraction, and perhaps an excessively tight formulation of actual policymaking, it still seems a correct stylized formalization of the institutional features associated to the EMU in the context of this model.

As far as monetary policy is concerned, article 105.1 of the Maastricht Treaty states that *"The primary objective of the European System of Central Banks shall be to maintain price stability"*. The Governing Council of the ECB further formalized quantitatively the definition of its mandate announcing that *"price stability shall be defined as a year-on-year increase of the Harmonized Index of Consumer Prices (HICP) of below 2%"* (quotes are from the ECB Monthly Bulletin [1999]). More recently, the Governing Council of the ECB has announced that *"in the pursuit of price stability it aims to maintain inflation rates at levels close to 2% over the medium term"* (ECB Monthly Bulletin [2003]). Hence, the institutional mandate is indeed compatible with a narrow inflation targeting regime, at least over the medium term. The difference in the level of the target is rather innocuous. It can be accounted for by the necessity in practice to include the possibility of measurement errors (not considered here) and hence to avoid deflationary monetary policies. A more relevant criticism might derive from the fact that the model includes an equality constraint in place of an upper bound. The latest announcement of the Governing Council

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[45]The parameter ϕ and the constant C_f are in fact equal for both countries. The equality of the parameter ϕ defines the nature of "rule", as to give an objective criterion identical for both countries. The equality of the constant C_f depends on the symmetry of the steady state of the model.

[46]A similar constant debt rule has been used in recent works by Schmitt-Grohé and Uribe [2004c] and by Benhabib and Eusepi [2004]

[47]The determinacy properties of the model under simple rules are discussed in $A.5$.

[48]Optimality refers to the choice of the coefficients γ and ϕ.

of the ECB, however, limits this argument. A more practical issue concerns the computational difficulties of solving the model under inequality constraints. Given those considerations, an inflation targeting rule at equality seems adequate to describe the mandate of the European Central Bank to maintain price stability.

A constant debt rule, on the other hand, implies from the government budget constraint that a sufficient real surplus must be created in each period to cover net interest rate spending on outstanding real debt

$$s_{i,t} = \left(\frac{R_{t-1}}{\Pi_t} - 1 \right) \beta \bar{b}. \tag{60}$$

It coincides with the budget requirement embedded in the Maastricht Treaty, once variables are defined in real terms. As discussed for the strict CPI inflation targeting regime above, constant debt rules (and balanced budget rules such as [60]) might be generally more restrictive than the fiscal rules prescribed in the Stability and Growth Pact (deficit to GDP ratio not in excess of 3%, debt to GDP ratio not in excess of 60%). The absence of steady state growth precludes the possibility of meaningfully relating the effects of a debt/GDP upper bound on the economies of the EMU members. This caveat, together with the simpler computational implementation of equality constraints, play a significant role in favor of the specification adopted here[49]. Moreover, a quick look at Table 1 should reinforce the impression that modeling a balanced budget rule with strict equality might not be inappropriate, as suggested also by the literature on the deficit bias in the fiscal policymaking process (see Corsetti and Roubini [1992]) and by Stockman [2001] in a related work on balanced budget rules and optimal fiscal policy.

7   Dynamics and Welfare under Simple Rules

The objective of this section is twofold. First, I will analyze the dynamic response of the relevant endogenous variables to two examples of exogenous shocks. This exercise provides the basic intuition of the stabilization mechanisms at work and starkly illustrates the departures from optimality, depending on the different nature of the shocks. The second purpose is the quantification of those departures in welfare terms. One advantage of using a model with microfoundations is that the welfare objective for policy analysis is clearly defined as a transformation of the individual utility function. The welfare implications of alternative policies can then be measured coherently based on the expression for $u_{W,0}$ previously derived.

7.1   Response to Exogenous Shocks

In this section, the stabilization properties associated with the fiscal-monetary regime described by constant debt rules ($\phi = 0$ in [59]) and strict inflation targeting ($\gamma = 0$

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[49]See also Canzoneri, Cumby and Diba [2005] for a similar point on fiscal rules in a currency union. It is worth stressing that a rule that allows a 3% ceiling on the deficit to GDP ratio at equality not only would imply just a level effect but also would be at odds with the steady state of the model, which requires the positive level of debt $\bar{b}$ to be financed by a surplus $\bar{s} > 0$.

in [58]) are compared to the outcome under the optimal policy plan, as characterized by the set of first order conditions [51]-[56].

As an example of impulse responses, I focus on the stabilization of average and relative public spending shocks. The experiments consist of unit innovations in the processes for $\hat{G}_{W,t}$ and $\hat{G}_{R,t}$. The former has a direct impact on the average fiscal stress as well as on the target levels for the output gap and the average tax rate. The latter affects directly the relative fiscal stress as well as the terms of trade and the relative tax rate targets. In order to sharpen the intuition about the mechanism of propagation of the shocks, the experiments are derived under the assumption of no persistence in the exogenous disturbances. I report the responses of union-wide variables (the output gap y_t, the nominal interest rate r_t, the CPI inflation rate π_t and the terms of trade $\hat{T}_t$) as well as of Home and Foreign variables (output gaps $y_{H,t}$ and $y_{F,t}$, GDP inflation rates $\pi_{H,t}$ and $\pi_{F,t}$, tax rates $\hat{\tau}_{H,t}$ and $\hat{\tau}_{F,t}$ and real value of debt at maturity $\hat{b}_{H,t}$ and $\hat{b}_{F,t}$) for 20 periods (quarters) after a blip in $\hat{G}_{W,t}$ (Figure 1) and in $\hat{G}_{R,t}$ (Figure 2). In both graphs, the continuous blue line represents the response of the endogenous variables under optimal policy while the red dashed line is the response under the alternative regime.

From inspection of Figure 1 it appears that, under the optimal policy plan, the positive shock to $\hat{G}_{W,t}$ is financed mostly by higher government borrowing, which increases by almost 1.5% in its value at maturity, and by a one period increase in the tax rate[50], of slightly less than 4%. As pointed out in section 5, the nature of a change in the value of government debt at maturity $\hat{b}_{i,t}$ under the optimal policy plan is always permanent, due to the unit root in the Lagrange multipliers associated to the two government budget constraints[51]. The reaction of monetary policy consists of a one period hike in the nominal interest rate of about 1.5%. The contractionary effect of higher tax rates and higher interest rates counterbalances the expansionary effect of the shock to government spending so that the output gap and the inflation rates stay at their target levels. Given the aggregate nature of the shock, the response of the terms of trade is negligible. The reason why it is not exactly zero is that the optimal plan attaches different weights to GDP inflation rates in different countries, due to the different degree of price rigidities (the relevant measure of the different costs of inflation across countries).

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[50]The adjustment of fiscal variables are likely to be upward biased by the timing protocol which allows for contemporaneous responses in a model calibrated to quarterly frequence. However, this consideration concerns the optimal policy plan as well as the solution under alternative rules. It seems safe to conjecture that the relative ranking of alternative policies should not be affected.

[51]The stationary nature of the nominal interest rate implies that the stock of government debt is $I(1)$ too.
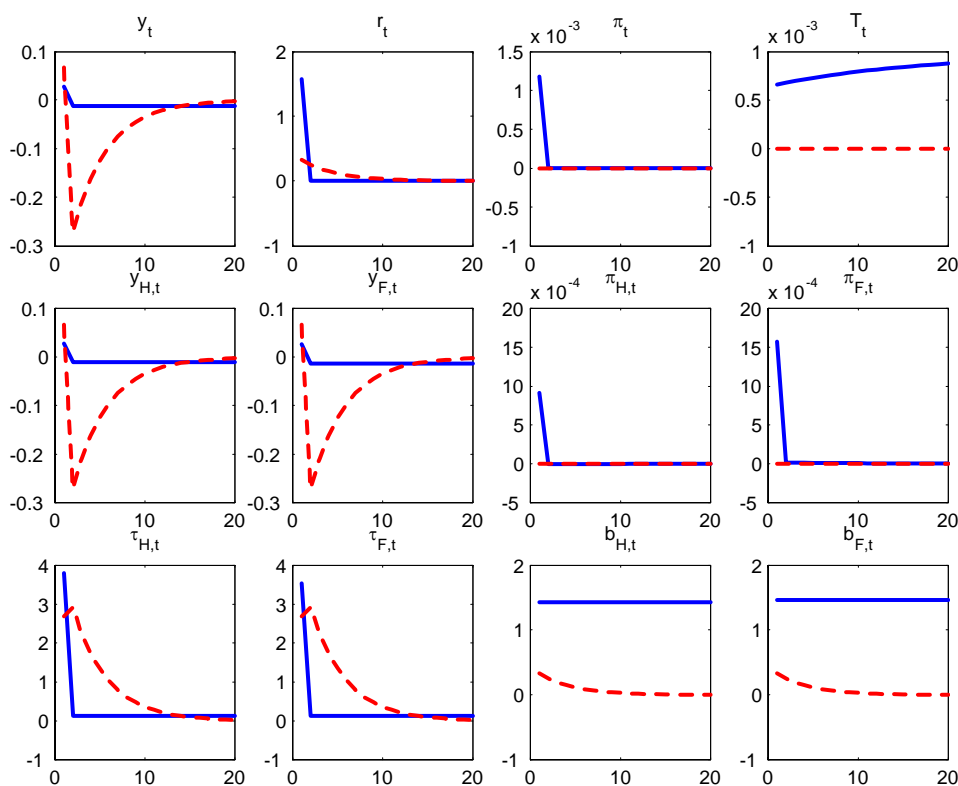
Figure 1: Response to a unit innovation in $\hat{G}_{W,t}$.

Under the suboptimal regime, on impact, the response is qualitatively similar. The main difference resides in the temporary adjustment of the debt variable. While the stock of real debt is constant, the real value of debt at maturity varies to the extent that it mimics variations in the nominal interest rate[52]. Taxes must respond to the shock to compensate for the constraint on debt. Given that the real value of debt at maturity reverts back to its initial steady state value, it is optimal for the government to smooth the higher tax rate over a prolonged period. While the output gap increases more than under optimal policy on impact (because the response in tax rates is about 1% less), it displays a significant drop in the second period and reverts back to target as the tax rates return to zero. Given the prominent role played by fiscal stances in this case, the action of monetary policy is limited to a smaller increase in the first period of 25 to 30 basis points, although characterized by more persistence, similar to what happens for the tax rates. As required by the monetary rule, CPI inflation is maintained exactly at zero and, given the symmetric nature of the shock, so are the GDP inflation rates, with no adjustment required in the terms of trade.

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[52]This can be easily seen from the log-linear approximation of the fiscal rule [59], which delivers the average relation $\hat{b}_{W,t} = r_t$, when $\phi = 0$.
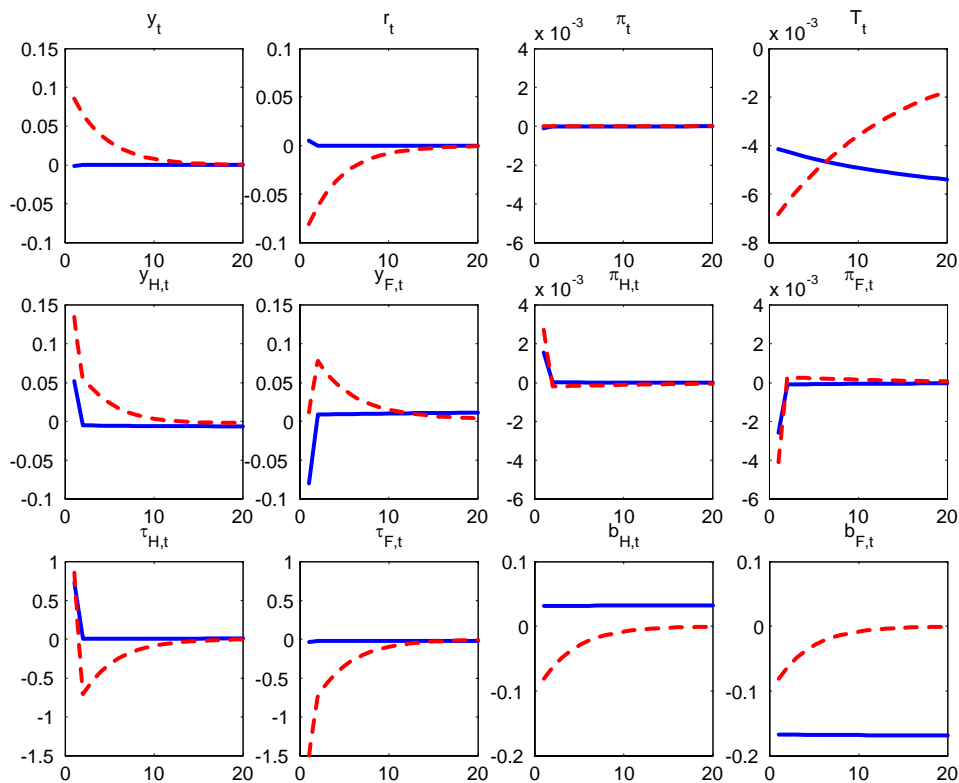
Figure 2: Response to a unit innovation in $\hat{G}_{R,t}$.

Next, I consider the effects of a shock to relative public spending $\hat{G}_{R,t}$ (Figure 2). The differences between the optimal stabilization plan and the alternative regime are possibly more significant than before, due to the additional degree of symmetry imposed by the debt rules[53]. The main feature of optimal policy is that now the adjustment occurs almost entirely via fiscal variables. Optimality requires that in country H a mix of higher tax rates (slightly less than 1%) and higher debt (a permanent increase of less than 0.05%) finance the temporary shock to government expenditure. The output gap increases by 0.05% for one period and reverts back to trend afterwards. The response of variables in country F does not exactly mirror the response of country H variables. A permanent decrease in government debt of 0.1% is sufficient to satisfy the government budget constraint at the new value of spending without imposing additional adjustments in the tax rates. The output gap drops for one period as the objective of optimal policy is the average output gap, that stays constant. The impact on GDP and CPI inflation rates and on the terms of trade is again quantitatively small, with a more significant adjustment of country F GDP inflation, because of the relative lower degree of price rigidity. The outcome is a small but permanent appreciation of the terms of trade.

On the other hand, the presence of strict debt rules prevents debt from adjusting

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[53]Indeed, the log-linear version of [59] when $\phi = 0$ reads as $\hat{b}_{R,t} = 0$, which precludes any asymmetric movements of debt across countries.

in opposite directions. A more relevant stabilization role is therefore played by tax rates. Namely, the tax rate increases by almost 1% in country H and drops by 1.5% in country F. The higher tax rate in the Home country partially dampens the effect of the positive shock to government spending. The nominal interest rate reduces by about 10 basis points with a certain degree of persistence (the half-life is slightly more than a year), inducing an analogous effect on the real value of debt at maturity. As relative debt at maturity must be zero, the reduction in the interest rate generates lower debt in country F too. At the same time, though, variations of the interest rate are consistent with zero CPI inflation only through adjustment in opposite directions of the GDP inflation rates, which, in turn, generate a temporary but persistent appreciation of the terms of trade. In the following periods, variables start reverting to their long run equilibrium, although it is interesting to notice that, even with white noise shocks, debt rules generate some endogenous persistence in the output gap and in the tax rates similar to that present in the dynamics of the terms of trade. In particular, the tax cut that occurred in country F at the moment of the shock leads to a temporary but persistent increase of the output gap in the following periods. On the other hand, the combination of higher taxes and higher output gap in country H leaves room for a reduction of the tax rate.

I now move to the analysis of the alternative regimes in terms of welfare. After identifying the optimal coefficients of the flexible fiscal and monetary rules, I will show how a policy regime which involves some concerns for output stabilization can improve upon the strict formulation considered so far, mimicking closely the optimal policy response to shocks and leading to substantial welfare gains.

7.2 Welfare Analysis

I construct two welfare measures based on transformations of the objective $u_{W,0}$ in [43]. Conditional on the system being in a steady state before time 0, the objective is invariant with respect to alternative fiscal and monetary policy rules, allowing direct comparisons of different regimes with the optimal policy plan and among themselves[54].

I define the welfare index d_p as

$$d_p \equiv -2\left(1-\beta\right)\left[\frac{E\left\{(u_{W,0})^p\right\} - E\left\{(u_{W,0})^{opt}\right\}}{U_C \bar{C}}\right], \tag{61}$$

where $(u_{W,0})^{opt}$ is welfare under the optimal policy plan and $(u_{W,0})^p$ is welfare under any alternative policy plan, indexed by p. The operator $E\left\{\cdot\right\}$ defines the expectation over the distribution of shocks at time zero. The welfare index d_p measures the permanent shift in steady state consumption that is lost under policy p with respect to the benchmark optimal plan and is independent of additive terms.

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[54]This is one important advantage of the approximation method employed in this paper, as discussed in Benigno and Woodford [2005]. In section $A.6$ of the appendix, I also show how the reduced form solution of the model, which constitutes the output of the Anderson and Moore [1985] algorithm, can be used to calculate welfare given the objective $u_{W,0}$ defined above.

It is then possible to compare welfare under two alternative suboptimal regimes and evaluate the welfare costs, in percentage terms, of moving from the fiscal-monetary regime p_1 to the alternative regime p_2 using the ratio

$$g\left(p_1, p_2\right) \equiv \frac{d_{p_1} - d_{p_2}}{d_{p_1}} = \frac{E\left\{(u_{W,0})^{p_1}\right\} - E\left\{(u_{W,0})^{p_2}\right\}}{E\left\{(u_{W,0})^{p_1}\right\} - E\left\{(u_{W,0})^{opt}\right\}}. \tag{62}$$

In $g\left(p_1, p_2\right)$, the denominator measures the cost of policy p_1 with respect to the optimal policy whereas the numerator measures the reduction (increase) of this cost in moving from policy p_1 to policy p_2.

First, I investigate the welfare losses induced by the regime characterized by strict inflation targeting and constant real debt. In absolute terms, the welfare cost of optimal policy in units of steady state marginal utility of consumption is equal to 0.0919%. On the other hand, the cost of the strict policy regime is equal to 0.2559%. It follows that the measure d_s (where s stands for 'strict' policy regime) takes value equal to 0.003281, which means a permanent loss of 0.3281% in steady state consumption from adopting CPI inflation targeting and constant debt rules rather than the optimal plan.

Perhaps, it is even more remarkable to note that almost 90% of the total cost is due to the lack of stabilization in the case of an asymmetric shock. This experiment complements quantitatively the description of the large and persistent deviations of the endogenous variables from their target under the suboptimal fiscal-monetary regime documented in Figure 2. While the optimal plan accomplishes stabilization mostly through fiscal policy, the constraints imposed on the use of debt by constant debt rules strengthen the effects on inflation differentials due to the absence of the nominal exchange rate acting as an automatic stabilizer. When shocks are highly persistent, as it is the case in the baseline calibration, the costs of constraining fiscal policy become highly relevant in welfare terms.

Next, I study to what extent the introduction of some degree of flexibility in either monetary or fiscal policy improves upon the strict formulation described in the previous section. In order to isolate the effects of flexibility in either forms of policy, I begin with the implications of conducting monetary policy following a flexible inflation targeting when fiscal policy is subject to constant debt requirements. Then, I analyze the potential benefits of introducing flexibility in fiscal policy rules, maintaining the assumption of strict inflation targeting for monetary policy. I set to zero one coefficient at a time in [58] and [59] and search for the value of the non-zero coefficient which maximizes the welfare criterion $u_{W,0}$ subject to the remaining equilibrium conditions. Given the optimal flexible rule, I can then compute the loss d_p associated to that specific regime and the gain $g\left(s, x_p\right)$ of moving from the strict benchmark regime s to the flexible regime x_p, where $p = \{m, f\}$ stands for 'monetary' or 'fiscal'.

First, I consider the flexible monetary rule [58] which can be written up to a first order log-linear approximation as

$$\pi_t + \gamma \Delta y_t = 0. \tag{63}$$

As discussed in section $A.5$ of the appendix, determinacy requirements pose no limitation on the search for the optimal γ. By assumption, I consider only positive values of the parameter to guarantee a standard countercyclical formulation of monetary policy. Hence, the rule permits deviations from the main objective of price stability to the extent that output is off target, in a way similar to the flexible inflation targeting rules in Svensson [1999]. In this case, the optimal value of γ is 0.05, which implies a relatively small weight on the stabilization of the real activity indicator.

The absolute welfare cost of the suboptimal policy is equal to 0.2477%, quite close to the case of strict inflation targeting. As compared to the optimal plan, the measure d_m takes value 0.3116% while the gains of moving to the more flexible form of CPI inflation targeting in [63], when fiscal policy is bound to obey to strict debt rules, can be quantified by $g(s, x_m) = 5.0337\%$. Both in absolute and relative terms, the gains from flexibility in the monetary rule appear to be quantitatively small.

The result is quite robust to alternative specification of the indicator of real activity. If the level of the output gap were to be included, optimality would require a pure CPI inflation targeting ($\gamma = 0$). Similarly, if the real variable included was output growth, the optimal coefficient would be highly biased towards inflation stabilization ($\gamma = 0.01$ with an associated welfare gain of 0.8583%).

The second step consists of seeking the optimal flexibility coefficients in the fiscal rules [59] which can be approximated up to the first order by

$$\hat{b}_{i,t} - r_t + \phi y_{i,t} = 0. \tag{64}$$

A rule like [64] provides a margin of action for national fiscal policies to respond to fluctuations in real activity. In particular, governments can deviate from maintaining constant debt and a balanced budget when output is off target. The rule can indeed be recast in terms of current surplus as

$$(1 - \beta)\,\hat{s}_{i,t} = \hat{b}_{i,t-1} - \pi_t + \beta\phi y_{i,t}, \tag{65}$$

where $\hat{s}_{i,t}$ is the first order approximation of the primary surplus. According to [65], fiscal authorities run a balanced budget whenever output is at its desired level and are required to adjust the current surplus countercyclically viceversa. It is important to stress that the rule is symmetric, that is, it not only allows active fiscal policies in bad times but also requires virtuous public finance during booms. As noted, it is assumed that the rule induces countercyclical fiscal policies ($\phi > 0$). On the other hand, the search of the optimal value for ϕ is limited above by the condition for determinacy under flexible fiscal rules derived in $A.5$ ($\phi < 10.46$). A grid search over the interval $(0, 10.46)$ returns an optimal value for the feedback coefficient on the output gap equal to 9.55, implying a quite aggressive response of debt to variations in real activity.

Table 4: Gains from flexible policy rules

Rules	Coefficient	Welfare	
	γ, ϕ (optimal)	Loss d_p (w.r.t. optimality)	Gains $g(s, x_p)$ (from flexibility)
Strict (s)	–	0.3281%	–
Flexible Monetary (x_m)	$\gamma = 0.05$	$d_m = 0.3116\%$	$g(s, x_m) = 5.0337\%$
Flexible Fiscal (x_f)	$\phi = 9.55$	$d_f = 0.0709\%$	$g(s, x_f) = 78.3835\%$

The welfare loss under flexible debt rules is equal to 0.1274%, which corresponds to about one half of the loss under the strict policy regime and the flexible monetary regime. Compared to the optimal policy benchmark, the welfare measure associated with the flexible fiscal regime is given by $d_f = 0.0709\%$. Most importantly, the percentage gains of moving from strict to flexible debt rules are given by $g(s, x_f) = 78.3835\%$. Substantial gains would also be obtained with smaller coefficients. If $\phi = 5$, the gain is still higher than 50% (equal to 64.67%) and still bigger than 15% (equal to 16.03%) if $\phi = 1$. It is paramount to stress the fact that the welfare gains from a flexible formulation of fiscal rules are large both in absolute and relative terms. The departure from optimality is greatly reduced and the benefits of moving to a regime which entails strict inflation targeting and flexible debt targeting are indeed substantial. Table 4 summarizes the welfare results associated with flexibility in monetary and fiscal rules.

As a partial caveat, one should also point out that the welfare gains obtained from a flexible fiscal regime are less robust to alternative specifications of the indicator of real activity than the results for monetary policy rules. If such an indicator was the variations of the output gap, the resulting optimal coefficient would be much smaller (equal to 0.23) and, most notably, the welfare gains would be reduced to 3%. A similar result would obtain with the level of output, rather than of the output gap, in the fiscal rule [64] (in this case, the optimal coefficient would be equal to 0.26 and the welfare gains equal to 1.2508%). The level of the output gap appears to be the natural measure to include in the rule for consistency in the formulation of the model. Nonetheless, the appropriate design of fiscal rules in practice calls for a careful choice of the indicator to include in a flexible formulation.
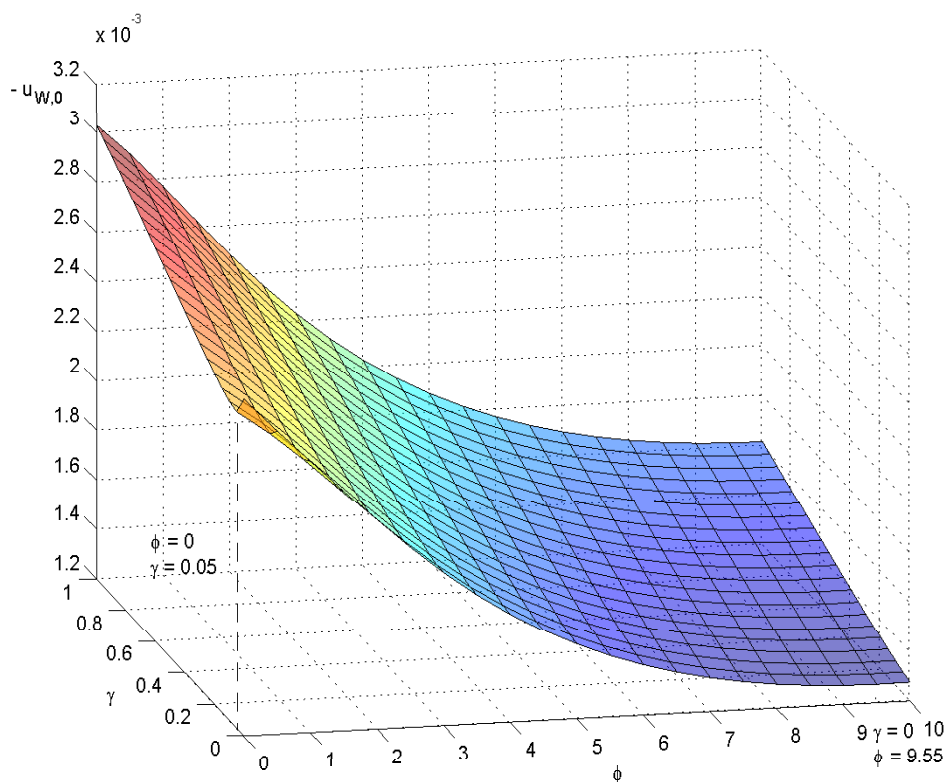
Figure 3: Optimal coefficients with flexible rules.

The search for the optimal coefficients in [63] and [64] clarifies that the major source of welfare gains is related to the appropriate design of fiscal policy rules. Indeed, when both coefficients are allowed to depart from zero at the same time, the optimal coefficient for monetary policy is $\gamma = 0$ and the optimal coefficient for fiscal policy is $\phi = 9.55$. In other words, the flexible fiscal regime discussed above constitutes the optimal choice contingent on monetary and fiscal rules belonging to the family [63] - [64] (Figure 3).

Flexible fiscal rules that allow debt to respond to deviations of output from target generate substantial welfare gains because a high degree of persistence is induced in the adjustment of the real value of debt at maturity. The dynamic evolution of $\hat{b}_{H,t}$ and $\hat{b}_{F,t}$ under flexible debt rules tracks the pattern followed by these variables under the optimal policy plan for a prolonged period, thus mimicking, for a substantial horizon, the unit root nature which characterizes the optimal stabilization. The persistent adjustment of debt minimizes the distortions related to variations of the tax rates and the associated deviations of output from its target. Similarly, the response of the terms of trade induces a substantial degree of smoothing in the international transmission of the shocks, close to what achieved under the optimal plan. Figure 4 plots the impulse response function to a unit innovation in $\hat{G}_{R,t}$ under flexible rules with $\gamma = 0$ and $\phi = 9.55$ and under optimal policy (the horizon is 50 quarters).
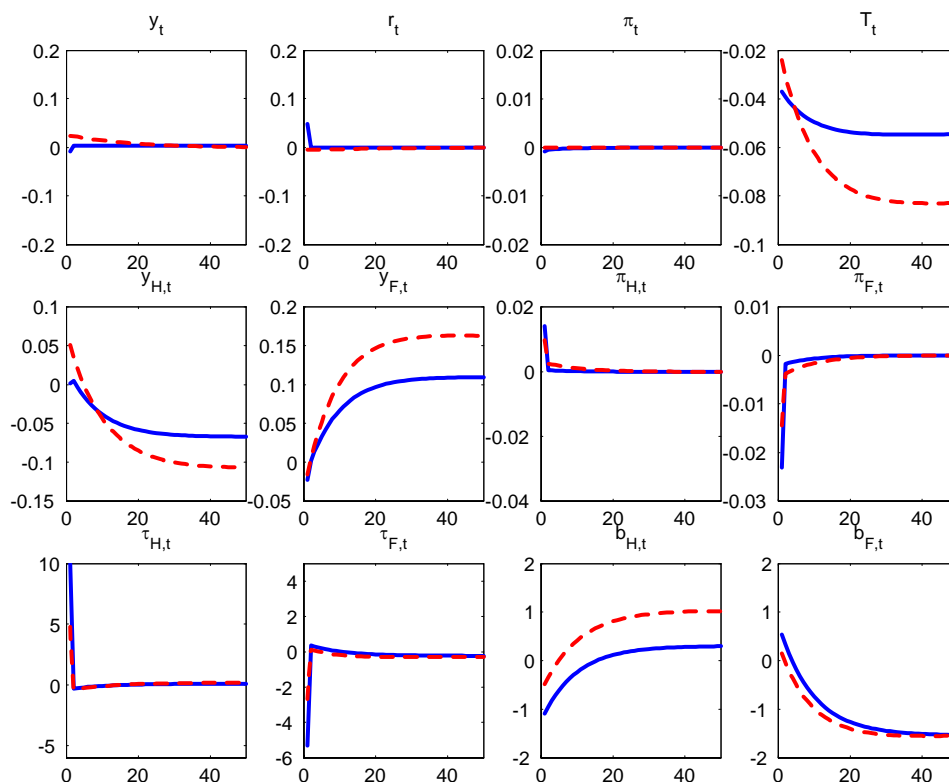
Figure 4: Response to a unit innovation in $\hat{G}_{R,t}$ under optimal flexible rules.

On the other hand, flexibility in monetary policy becomes important only if fiscal policy rules are sufficiently rigid. Otherwise, a pure inflation targeting is the appropriate rule to assign to the monetary authority of a currency union. As discussed above, the role of sticky prices is central to generate optimality of price stability. If prices were almost fully flexible[55], optimal policy would imply stabilization of the output gap around a constant. In such circumstances, a rule which generates high inflation volatility and an aggressive response to departures of output from target would approximate fairly well the optimal plan. Instead, price rigidity implies that variations of the inflation rates are highly costly. It follows that larger welfare gains are obtained if the main stabilization role is played by permanent (or at least very persistent) variations in the level of debt.

The results of the analysis can be compared with other studies of fiscal and monetary policy in a currency union. Similar to Beetsma and Jensen [2004], the findings of this paper confirm the central role of fiscal policy in stabilization of relative shocks. However, fiscal policy is active also in response to aggregate shocks. The difference depends upon the interaction between fiscal and monetary policy when lump-sum instruments are not available. When monetary policy reacts to an aggregate shock,

---

[55]This consideration should be read as a limiting result, for the optimal plan under fully flexible prices entails indeterminacy of the debt dynamics.

it generates variations in the real burden of debt that need to be balanced by fiscal adjustments, in order to satisfy the intertemporal government budget constraint or any other stricter rule. The model displays also several similarities with the analysis of Galí and Monacelli [2004]. In particular, the countercyclical nature of fiscal policy is confirmed in the setup presented here, both under the optimal plan and with simple rules. Finally, the combined result of price stability as central mandate for the monetary authority and the activism of fiscal policy to stabilize asymmetric shocks is shared with Canzoneri, Cumby and Diba [2005]. Remarkably, such similarities hold despite the differences in the frameworks, which concern the menu of instruments that fiscal authorities can access, the presence of additional rigidities and capital and the form of the monetary rule.

Overall, the central message of the paper is that it is possible to design rules that limit the discretionality of government policies but at the same time allow appropriate fiscal stabilizations of relative shocks. Such rules do not depart too far from those included in the Stability and Growth Pact but prescribe a more aggressive response of national fiscal stances to fluctuations in the indicator of real activity. The objective of price stability appears to be the appropriate mandate for the centralized monetary authority.

8   Conclusions

This paper has investigated the question of the optimal fiscal and monetary stabilization policies in a currency union where a benevolent policymaker seeks to maximize an average welfare criterion for the whole area. Fiscal and monetary policy interact with each other given the prominent role of nominal rigidities and given the wedge introduced by distortionary taxation. The approximate policy problem can be cast in a linear-quadratic framework where standard optimization techniques can be applied. The policy objective consists of a second order approximation of individual utility that allows to evaluate welfare correctly using a first order approximation to the equilibrium conditions. The optimal plan induces a unit root in all the real variables, hence achieving a high degree of smoothing mainly obtained through adjustment of government debt. The methodology employed in the paper further permits a ranking of alternative suboptimal rules for both monetary and fiscal policy. The benchmark is represented by a strict CPI inflation targeting rule for monetary policy and constant debt rules for fiscal policy. Numerical simulations of the model show that granting more flexibility to both forms of policy will increase welfare. However, the welfare gains from higher monetary flexibility are very small when compared to the gains from higher flexibility in fiscal rules and the order of magnitude of the ratio is about sixteen under the baseline calibration. Overall, the results strengthen the argument for further research on the role of fiscal policy in a currency union, with specific focus on the careful design of appropriate rules that conjugate the needs of discipline and stabilization.

Along the analysis, the benchmark has been represented by the optimal plan that a centralized policymaker would implement by maximizing a union-wide welfare ob-

jective. This solution represents a natural starting point to evaluate the performance of alternative policy rules. It might be interesting however to understand the differences in terms of stabilization and policy interaction under a more realistic regime where fiscal authorities maximize national welfare and the monetary authority cares about welfare of the whole currency union. In such a setting, strategic considerations for the use of fiscal policy are likely to emerge and to create some tensions with stabilization motives. This step is left for future research.

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A   Appendix

This appendix describes some of the derivations of the results in the paper. Further details are contained in a mathematical supplement available at http://homepages.nyu.edu/˜apf210/research/research.htm.

A.1   The Steady State

The purpose of this section is to show that the model has a well-defined steady state with zero inflation and constant values for all variables. I will consider the deterministic optimal policy problem where each component of the vector of exogenous disturbances takes a constant value. I will assume that these values are equal across countries[56] so that $\bar{a}_{H,t} = \bar{a}_{F,t} = \bar{a}$, $\bar{\mu}^w_{H,t} = \bar{\mu}^w_{F,t} = \bar{\mu}^w$ and $\bar{G}_{H,t} = \bar{G}_{F,t} = \bar{G}$. I also impose that $\bar{a}$, $\bar{\mu}^w > 1$ and $\bar{G} > 0$. Finally, the steady state debt level in per-capita terms is also common for both countries, that is, $\bar{b}_{H,t} = \bar{b}_{F,t} = \bar{b} > 0$. While the value of $\bar{b}$ is arbitrary, I will discuss below how it is nonetheless subject to an upper bound. Let X_0 be the set of initial commitments that make the solution of the problem optimal from a timeless perspective. Given the initial degree of price dispersion in the two countries $\Delta_{H,-1} = \Delta_{F,-1} = 1$, the initial level of per-capita debt $b_{H,-1} = b_{F,-1} = \bar{b}$ and the initial value for the terms of trade $T_{-1} = 1$, I wish to find a solution that involves a constant policy and a constant set of commitments, price dispersions, debt levels equal to the initial ones and terms of trade. The centralized planner chooses the following variables:

$$\{Y_{i,t}, \Delta_{i,t}, K_{i,t}, F_{i,t}, \Pi_{i,t}, p_{i,t}, W_{i,t}, b_{i,t}, \tau_{i,t}, C_{W,t}, T_t, \Pi_t\}_{t=0}^{\infty}, \qquad i = \{H, F\}$$

$$to \ \max u_{W,0} = \sum_{t=0}^{\infty} \beta^t \left[U\left(C_{W,t}\right) - nV\left(Y_{H,t}, a_{H,t}\right)\Delta_{H,t} - (1-n)V\left(Y_{F,t}, a_{F,t}\right)\Delta_{F,t} \right],$$

subject to the following set of constraints for country $i = \{H, F\}$:

$$K_{i,t} p\left(\Pi_{i,t}\right)^{\frac{1+\sigma\eta}{\sigma-1}} = F_{i,t},$$

$$K_{i,t} = k\left(Y_{i,t}, \bar{a}, \bar{\mu}^w\right) + \alpha_i \beta \Pi_{i,t+1}^{\sigma(1+\eta)} K_{i,t+1},$$

$$F_{i,t} = (1 - \tau_{i,t}) f\left(C_{W,t}, Y_{i,t}\right) p_{i,t} + \alpha_i \beta \Pi_{i,t+1}^{\sigma-1} F_{i,t+1},$$

$$W_{i,t} = \frac{U_C\left(C_{W,t}\right) b_{i,t-1}}{\Pi_t},$$

$$W_{i,t} = U_C\left(C_{W,t}\right) p_{i,t} \left(\tau_{i,t} Y_{i,t} - \bar{G}\right) + \beta W_{i,t+1},$$

$$\Delta_{i,t} = \alpha_i \Delta_{i,t-1} \Pi_{i,t}^{\sigma(1+\eta)} + (1-\alpha_i) p\left(\Pi_{i,t}\right)^{\frac{\sigma(1+\eta)}{\sigma-1}},$$

$$Y_{i,t} = p_{i,t}^{-\theta} C_{W,t} + \bar{G},$$

---

[56]Notice that the requirement for public spending is that the steady state per-capita level is equal across countries.

$$p_{H,t}^{\theta-1} = n + (1-n)\,T_t^{1-\theta}, \qquad p_{F,t}^{\theta-1} = nT_t^{\theta-1} + (1-n),$$

and the constraints at the union level

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}},$$

$$\Pi_t^{1-\theta} = n\Pi_{H,t}^{1-\theta} p_{H,t-1}^{1-\theta} + (1-n)\,\Pi_{F,t}^{1-\theta} p_{F,t-1}^{1-\theta},$$

and given the initial conditions X_0, $\Delta_{i,-1}$, $b_{i,-1}$ and T_{-1}. Also, the relative price for country i is $p_{i,t} \equiv P_{i,t}/P_t$ and the following function has been defined

$$p\left(\Pi_{i,t}\right) \equiv \left(\frac{1-\alpha_i\Pi_{i,t}^{\sigma-1}}{1-\alpha_i}\right).$$

I attach Lagrange multipliers $\phi_{1,t}^i$ through $\phi_{8,t}^i$ to the constraints[57] of the country i. I attach the multipliers $\phi_{9,t}^W$ and $\phi_{10,t}^W$ to the the two union-wide constraints. As pointed out by BW, additional Lagrange multipliers are needed for the initial conditions X_0 to impose constant commitments ($X_0 = \bar{X}$). These multipliers are normalized in such a way that the first order conditions for $t = 0$ look the same as the first order conditions at a generic period $t > 0$. As discussed in the previous section, the stationarity constraints for fiscal policy can only be imposed on the consolidated real value of debt at maturity $nW_{H,0} + (1-n)\,W_{F,0}$. Such commitments prevent the policymaker to exploit the predetermined nature of debt via one-shot high inflation but do not imply that the transversality condition for the real value of debt is satisfied a priori in both countries[58]. While the multipliers on the flow government budget constraints are $\phi_{5,t}^H$ and $\phi_{5,t}^F$, the additional multiplier on the stationarity constraint is denoted by $\phi_{5,-1}^W$ and in principle differs from the previous two. The resulting Lagrangian as well as the first order conditions for the choice variables listed above are derived in the mathematical supplement.

I wish to consider here a steady state where variables are constant[59] (denoted by an upperbar) and where inflation is zero ($\bar{\Pi} = \bar{\Pi}_i = 1$). This steady state would imply also $\bar{\Delta} = \bar{\Delta}_i = 1$, $\bar{K} = \bar{F} = \bar{K}_i = \bar{F}_i$, $\bar{p} = \bar{p}_i = 1$ and $p(1) = 1$. Moreover, one can also see that

$$p_\pi\left(\bar{\Pi}_i = 1\right) = \frac{\alpha_i\,(1-\sigma)}{1-\alpha_i}.$$

From the definition of $f\left(\cdot,\cdot\right)$, it follows that

$$f\left(\bar{Y},\bar{C}\right) = U_C\left(\bar{C}\right)\bar{Y}.$$

The pricing equation gives

$$k\left(\bar{Y},\bar{a},\bar{\mu}^w\right) = (1-\bar{\tau})\,f\left(\bar{C},\bar{Y}\right),$$

---

[57]The Lagrange multiplier on each country's government budget constraint is normalized by the size of its country, which is equivalent to writing the budget constraint in terms of total rather than per-capita levels.

[58]In fact, this will be true in the symmetric deterministic steady state considered here (provided that it exists) because $b_{i,t} = \bar{b}$ so that

$$\lim_{s\to\infty} \beta^{s-t}\frac{U_C\left(\bar{C}\right)\bar{b}}{\bar{\Pi}} = \lim_{s\to\infty}\beta^{s-t} = 0.$$

[59]Notice that the requirement here is also that the steady state values of the Foreign variables correspond to those of the Home country.

which can be rewritten using the definition of the two functions $k(\cdot)$ and $f(\cdot,\cdot)$ as

$$(1-\bar{\tau})U_C(\bar{C}) = \left(\frac{\sigma}{\sigma-1}\right)\bar{\mu}^w V_y(\bar{Y},\bar{a}).$$

The government budget constraint gives

$$(1-\beta)\bar{b} = \bar{\tau}\bar{Y} - \bar{G}.$$

Since I assumed a positive steady state level of government purchases and debt, it follows that the steady state tax rate must be positive. Finally, from the demand equation, it follows that

$$\bar{Y} = \bar{C} + \bar{G}.$$

Combining the last equation with the optimal pricing relation yields steady state output as a negative function of the steady state tax rate and the steady state markup and a positive function of steady state government purchases and productivity ($\bar{Y} = Y(\bar{\tau}, s_g, \bar{a}, \bar{\mu}^w)$ where $s_g \equiv \bar{G}/\bar{Y}$ is taken as given from the benchmark calibration). In any solution, $\bar{\tau}$ must be less than unity, otherwise output would be zero and so would be revenues for the government. Hence, over the $(0,1)$ interval, revenues are bounded above (as in a standard Laffer curve argument) and so must be $\bar{b}$ for a steady state to exist consistent with that initial condition.

In order to verify that this is indeed a solution to the policy problem defined above, in the mathematical supplement I also check that the first order conditions above are satisfied for time-invariant Lagrange multipliers. This step completes the proof of the existence of a well-defined symmetric steady state with zero inflation and positive debt. In the paper, any reference to the steady state is to be understood in relation with the steady state just characterized.

A.2 Second Order Approximation of the Utility Function

The derivation of the second order approximation of the utility function is analogous to the closed economy case of BW (see the mathematical supplement for details) and yields the following expression for country H

$$
\begin{aligned}
u_{H,t} &= U_C\bar{C}E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[\hat{C}_{W,s} + \frac{1}{2}(1-\rho)\hat{C}_{W,s}^2\right.\right. \\
&\quad \left.\left. - (s_c\bar{\mu})^{-1}\left(\hat{Y}_{H,s} + \frac{1}{2}(1+\eta)\hat{Y}_{H,s}^2 - (1+\eta)\hat{a}_{H,t}\hat{Y}_{H,s} + \frac{1}{2}\sigma\kappa_H^{-1}\pi_{H,s}^2\right)\right]\right\} + t.i.p. + \circ\left(\left\|\hat{\xi}_{H,t}\right\|^3\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\kappa_H &\equiv \frac{(1-\alpha_H)(1-\alpha_H\beta)}{\alpha_H(1+\sigma\eta)}, \\
\bar{\mu} &\equiv \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{\bar{\mu}^w}{1-\bar{\tau}}\right) = \frac{U_C}{V_y} > 1,
\end{aligned}
$$

and $s_c \equiv \bar{C}/\bar{Y}$. I define the vector of endogenous variables in deviations from the steady state as

$$x_t' \equiv \begin{bmatrix} \hat{Y}_{H,t} & \hat{p}_{H,t} & \hat{\tau}_{H,t} & \hat{Y}_{F,t} & \hat{p}_{F,t} & \hat{\tau}_{F,t} & \hat{T}_t & \hat{C}_{W,t} \end{bmatrix}'.$$

I denote the single vector of exogenous shocks by $\xi_t = \begin{bmatrix} \hat{\xi}_{H,t} & \hat{\xi}_{F,t} \end{bmatrix}$. The six entries are

$$\xi_t' \equiv \begin{bmatrix} \hat{a}_{H,t} & \hat{\mu}_{H,t}^w & \hat{G}_{H,t} & \hat{a}_{F,t} & \hat{\mu}_{F,t}^w & \hat{G}_{F,t} \end{bmatrix}'.$$

I can then express utility in matrix notation as

$$u_{H,t} = U_C\bar{C}E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[z_{H,x}'x_s - \frac{1}{2}x_s'Z_{H,x}x_s - x_s'Z_{H,\xi}\xi_s - \frac{1}{2}z_{\pi_H}\pi_{H,s}^2\right]\right\} + t.i.p. + \circ\left(\|\xi_t\|^3\right),$$

where

$$z'_{H,x} \equiv \begin{bmatrix} -\left(s_c\bar{\mu}\right)^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Z_{H,x} \equiv \begin{bmatrix} \left(s_c\bar{\mu}\right)^{-1}\left(1+\eta\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\left(1-\rho\right) \end{bmatrix},$$

$$Z_{H,\xi} \equiv \begin{bmatrix} \left(s_c\bar{\mu}\right)^{-1}\left(1+\eta\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$z_{\pi_H} \equiv \sigma\left(s_c\bar{\mu}\kappa_H\right)^{-1}.$$

The Foreign counterpart is

$$u_{F,t} = U_C\bar{C}E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[z'_{F,x}x_s - \frac{1}{2}x'_s Z_{F,x}x_s - x'_s Z_{F,\xi}\xi_s - \frac{1}{2}z_{\pi_F}\pi_{F,s}^2\right]\right\} + t.i.p. + \circ\left(\|\xi_t\|^3\right),$$

where

$$z'_{F,x} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & -\left(s_c\bar{\mu}\right)^{-1} & 0 & 0 & -\left(1-\rho\right) \end{bmatrix},$$

$$Z_{F,x} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(s_c\bar{\mu}\right)^{-1}\left(1+\eta\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\left(1-\rho\right) \end{bmatrix},$$

$$Z_{F,\xi} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\left(s_c\bar{\mu}\right)^{-1}\left(1+\eta\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$z_{\pi_F} \equiv \sigma\left(s_c\bar{\mu}\kappa_F\right)^{-1}.$$

As explained in Woodford [2003], a second order approximation to the equilibrium conditions allows to eliminate the linear terms in the second order approximation of the welfare criterion above and derive a purely quadratic criterion suitable to study optimal policy and welfare under alternative regimes in a standard LQ framework. The complete derivations are available in the mathematical supplement. The result can be cast in terms of an average objective $u_{W,0} \equiv n u_{H,0} + (1 - n) u_{F,0}$ which reads as

$$u_{W,0} = -\frac{1}{2} U_C \bar{C} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_{W,t} \right\} + J_{W,0} + t.i.p. + \circ \left(\|\xi_t\|^3 \right),$$

where

$$L_{W,t} \equiv \lambda_y \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t} \right)^2 + n (1 - n) \lambda_q \left(\hat{T}_t - \tilde{T}_t \right)^2 + n \lambda_{\pi_H} \pi_{H,t}^2 + (1 - n) \lambda_{\pi_F} \pi_{F,t}^2, \quad (66)$$

The term $J_{W,0}$ summarizes the elements peculiar to time zero that can be taken as given in the timeless perspective approach. The target level for output is defined as

$$\tilde{Y}_{W,t} \equiv - (n \lambda_y)^{-1} \left[\left(\tilde{Q}_{W,\xi} \right)_{11} \hat{a}_{W,t} + \left(\tilde{Q}_{W,\xi} \right)_{12} \hat{\mu}_{W,t}^w + \left(\tilde{Q}_{W,\xi} \right)_{13} \hat{G}_{W,t} \right].$$

The target level for the terms of trade is defined as

$$\tilde{T}_t \equiv - (\lambda_q)^{-1} \left[\left(\tilde{Q}_{W,\xi} \right)_{41} \hat{a}_{R,t} + \left(\tilde{Q}_{W,\xi} \right)_{42} \hat{\mu}_{R,t}^w + \left(\tilde{Q}_{W,\xi} \right)_{43} \hat{G}_{R,t} \right].$$

The new parameters in the objective functions are

$$\lambda_y \equiv \left(\tilde{Q}_{W,x} \right)_{11}, \qquad \lambda_q \equiv n^{-1} (1 - n)^{-1} \left(\tilde{Q}_{W,x} \right)_{44},$$

$$\lambda_{\pi_H} \equiv n^{-1} q_{W,\pi_H}, \qquad \lambda_{\pi_F} \equiv (1 - n)^{-1} q_{W,\pi_F}.$$

Finally, for any component of the vector of shock $\xi_{i,t}$, I have defined

$$\xi_{W,t} \equiv n \xi_{H,t} + (1 - n) \xi_{F,t}, \qquad \xi_{R,t} \equiv \xi_{H,t} - \xi_{F,t}.$$

The output gap coefficient depends on

$$\left(\tilde{Q}_{W,x} \right)_{11} = Q_{w,x}^1 + s_c^{-2} Q_{w,x}^2 + 2 s_c^{-1} Q_{w,x}^3,$$

where

$$
\begin{aligned}
Q_{w,x}^1 &= (s_c \bar{\mu})^{-1} (1 + \eta) + \eta (2 + \eta) (\zeta_{W,1} + \zeta_{W,2}) + (1 + \omega_g) (\zeta_{W,3} + \zeta_{W,4}), \\
Q_{w,x}^2 &= - (1 - \rho) - \rho^2 \left[(\zeta_{W,1} + \zeta_{W,2}) - (\zeta_{W,3} + \zeta_{W,4}) \right] + s_c (1 - s_c) (\zeta_{W,5} + \zeta_{W,6}), \\
Q_{w,x}^3 &= \rho \left[(\zeta_{W,1} + \zeta_{W,2}) - (1 + \omega_g) (\zeta_{W,3} + \zeta_{W,4}) \right],
\end{aligned}
$$

and $\omega_g \equiv s_g / (\bar{\tau} - s_g)$.

The terms of trade coefficient depends on

$$\left(\tilde{Q}_{W,x} \right)_{44} = (\theta s_c)^2 Q_{w,x}^4 + Q_{w,x}^5 + Q_{w,x}^7 - 2 \theta s_c Q_{w,x}^6,$$

where

$$
\begin{aligned}
Q_{w,x}^4 &= n (1 - n) Q_{w,x}^1, \\
Q_{w,x}^5 &= n (1 - n) \left[- (\zeta_{W,1} + \zeta_{W,2}) + (\zeta_{W,3} + \zeta_{W,4}) + \theta^2 s_c (\zeta_{W,5} + \zeta_{W,6}) \right], \\
Q_{w,x}^6 &= -n (1 - n) \rho^{-1} \left[(Q_{W,x})_{12} + (Q_{W,x})_{56} \right], \\
Q_{w,x}^7 &= n (1 - n) (\theta - 1) (\zeta_{W,7} + \zeta_{W,8}).
\end{aligned}
$$

The parameters $\zeta_{W,j}$, for $j = \{1, ..., 8\}$ are such that

$$
\begin{aligned}
\zeta_{W,1} + \zeta_{W,2} &= (\bar{\mu} - 1)(1 + \omega_g)(\bar{\mu}d_\zeta)^{-1}, \\
\zeta_{W,3} + \zeta_{W,4} &= -\omega_\tau (\bar{\mu} - 1)(\bar{\mu}d_\zeta)^{-1}, \\
\zeta_{W,5} + \zeta_{W,6} &= \varphi (s_c \bar{\mu} d_\zeta)^{-1}, \\
\zeta_{W,7} + \zeta_{W,8} &= -\vartheta (\bar{\mu} d_\zeta)^{-1},
\end{aligned}
$$

where $\omega_\tau \equiv \bar{\tau}/(1 - \bar{\tau})$ and

$$
\begin{aligned}
d_\zeta &\equiv s_c (1 + \omega_g)(\rho s_c^{-1} + \eta - \omega_\tau) + \rho \omega_\tau, \\
\varphi &\equiv \bar{\mu} s_c (1 + \omega_g)(\eta - \omega_\tau) + \rho (1 + \omega_g + \omega_\tau), \\
\vartheta &\equiv (\bar{\mu} - 1)(1 + \omega_g + \omega_\tau) + \theta \varphi.
\end{aligned}
$$

In the target level for output, the weights on the exogenous shocks are given by

$$
\left(\tilde{Q}_{W,\xi} \right)_{11} = -n (s_c \bar{\mu})^{-1}(1 + \eta) - (1 + \eta)^2 \zeta_{W,1}, \qquad \left(\tilde{Q}_{W,\xi} \right)_{14} = n^{-1}(1 - n) \left(\tilde{Q}_{W,\xi} \right)_{11}.
$$

$$
\left(\tilde{Q}_{W,\xi} \right)_{12} = (1 + \eta) \zeta_{W,1}, \qquad \left(\tilde{Q}_{W,\xi} \right)_{15} = n^{-1}(1 - n) \left(\tilde{Q}_{W,\xi} \right)_{12}.
$$

$$
\left(\tilde{Q}_{W,\xi} \right)_{13} = s_c^{-1} Q_{w,\xi}^1 - n s_c^{-2} Q_{w,x}^2 + n s_c^{-1} Q_{w,x}^3, \qquad \left(\tilde{Q}_{W,\xi} \right)_{16} = n^{-1}(1 - n) \left(\tilde{Q}_{W,\xi} \right)_{13},
$$

where

$$
Q_{w,\xi}^1 = \rho s_d \zeta_{W,3} - s_c \zeta_{W,5}.
$$

In the target level for the terms of trade, the weights on the exogenous shocks are given by

$$
\left(\tilde{Q}_{W,\xi} \right)_{41} = (1 - n) \theta s_c \left(\tilde{Q}_{W,\xi} \right)_{11}, \qquad \left(\tilde{Q}_{W,\xi} \right)_{44} = - \left(\tilde{Q}_{W,\xi} \right)_{41}.
$$

$$
\left(\tilde{Q}_{W,\xi} \right)_{42} = (1 - n) \theta s_c \left(\tilde{Q}_{W,\xi} \right)_{12}, \qquad \left(\tilde{Q}_{W,\xi} \right)_{45} = - \left(\tilde{Q}_{W,\xi} \right)_{42}.
$$

$$
\left(\tilde{Q}_{W,\xi} \right)_{43} = -(1 - n) \tilde{Q}_{w,\xi}^2 - Q_{w,x}^6 + \theta s_c Q_{w,x}^1, \qquad \left(\tilde{Q}_{W,\xi} \right)_{46} = - \left(\tilde{Q}_{W,\xi} \right)_{43},
$$

where

$$
\tilde{Q}_{w,\xi}^2 = -s_d \zeta_{W,3} + \theta s_c \zeta_{W,5}.
$$

The components $\zeta_{W,1}$, $\zeta_{W,3}$ and $\zeta_{W,5}$ are given by

$$
\begin{aligned}
\zeta_{W,1} &= n (\bar{\mu} - 1)(1 + \omega_g)(\bar{\mu}d_\zeta)^{-1}, \\
\zeta_{W,3} &= -n \omega_\tau (\bar{\mu} - 1)(\bar{\mu}d_\zeta)^{-1}, \\
\zeta_{W,5} &= n \varphi (s_c \bar{\mu} d_\zeta)^{-1},
\end{aligned}
$$

The coefficients of the inflation terms are

$$
q_{W,\pi_H} = n (\kappa_H \bar{\mu})^{-1} \sigma \left[s_c^{-1} + (\bar{\mu} - 1)(1 + \omega_g)(1 + \eta) d_\zeta^{-1} \right],
$$

and

$$
q_{W,\pi_F} = (1 - n)(\kappa_F \bar{\mu})^{-1} \sigma \left[s_c^{-1} + (\bar{\mu} - 1)(1 + \omega_g)(1 + \eta) d_\zeta^{-1} \right].
$$

Given the complicated functional form of the $\lambda_i's$, it is not obvious to derive conditions that ensure concavity of [66]. Here, I report numerical evaluations of those coefficients as a function of the steady state tax rate, keeping the other parameters at their benchmark value discussed in the text. It can be seen that for any value of $\bar{\tau} \in (0, 1)$, the $\lambda_i's$ are strictly bigger than zero (see Figure A.1). It is also interesting to notice that the main costs in terms of welfare arise from price dispersion. The coefficients on GDP inflation rate are very high as compared to those on the output gap and the terms of trade gap.
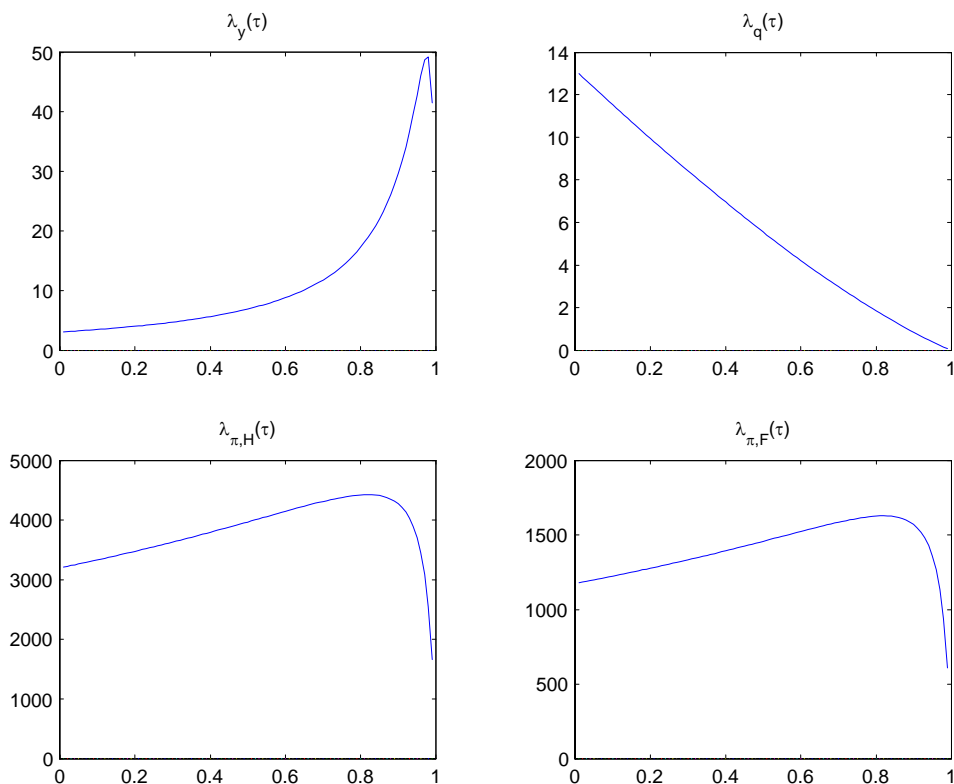
Figure A.1: Coefficients of the loss function.

The target levels for Home and Foreign output as

$$\tilde{Y}_{H,t} \equiv \tilde{Y}_{W,t} + (1-n)\,\theta s_c \tilde{T}_t + (1-n)\,\hat{G}_{R,t},$$

and

$$\tilde{Y}_{F,t} \equiv \tilde{Y}_{W,t} - n\theta s_c \tilde{T}_t - n\hat{G}_{R,t}.$$

Subtracting the targets just defined from their level counterparts, the Home and Foreign output gaps result as

$$\hat{Y}_{H,t} - \tilde{Y}_{H,t} = \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t}\right) + (1-n)\,\theta s_c \left(\hat{T}_t - \tilde{T}_t\right),$$

and

$$\hat{Y}_{F,t} - \tilde{Y}_{F,t} = \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t}\right) - n\theta s_c \left(\hat{T}_t - \tilde{T}_t\right).$$

A.3   The Equilibrium Conditions

In this section, I write the log-linear approximation of the equilibrium relations, where variables are expressed in deviations from targets. Home and Foreign output, consumption and relative prices are eliminated from the demand block of the model by appropriate substitutions.

The aggregate supply equation for the Home country is

$$\pi_{H,t} = \kappa_H \left[\delta_y \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t}\right) + \omega_\tau \left(\hat{\tau}_{H,t} - \tilde{\tau}_{H,t}\right) + (1-n)\,\delta_q \left(\hat{T}_t - \tilde{T}_t\right) \right] + \beta E_t \pi_{H,t+1},$$

where the target for the tax rate is defined as

$$\tilde{\tau}_{H,t} \equiv -\omega_\tau^{-1} \left[\delta_y \tilde{Y}_{W,t} + (1-n) \delta_q \tilde{T}_t - (1+\eta) \hat{a}_{H,t} + \hat{\mu}_{H,t}^w + (1-n) \eta \hat{G}_{R,t} - \rho s_c^{-1} \hat{G}_{W,t} \right].$$

The parameters are defined as $\delta_y \equiv \eta + \rho s_c^{-1}$ and $\delta_q \equiv 1 + \eta \theta s_c$. Similarly, the aggregate supply equation for the Foreign country is

$$\pi_{F,t} = \kappa_F \left[\delta_y \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t} \right) + \omega_\tau \left(\hat{\tau}_{F,t} - \tilde{\tau}_{F,t} \right) - n \delta_q \left(\hat{T}_t - \tilde{T}_t \right) \right] + \beta E_t \pi_{F,t+1},$$

where

$$\tilde{\tau}_{F,t} \equiv -\omega_\tau^{-1} \left[\delta_y \tilde{Y}_{W,t} - n \delta_q \tilde{T}_t - (1+\eta) \hat{a}_{F,t} + \hat{\mu}_{F,t}^w - n \eta \hat{G}_{R,t} - \rho s_c^{-1} \hat{G}_{W,t} \right].$$

Up to the first order, the flow version of the government budget constraint is

$$\hat{b}_{H,t-1} - \rho s_c^{-1} \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t} \right) - \pi_t + \psi_{H,t}$$
$$= (1-\beta) \left[b_y \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t} \right) + (1-n) b_q \left(\hat{T}_t - \tilde{T}_t \right) + (1+\omega_g) \left(\hat{\tau}_{H,t} - \tilde{\tau}_{H,t} \right) \right]$$
$$+ \beta E_t \left[\hat{b}_{H,t} - \rho s_c^{-1} \left(\hat{Y}_{W,t+1} - \tilde{Y}_{W,t+1} \right) - \pi_{t+1} + \psi_{H,t+1} \right].$$

where the fiscal stress is

$$\psi_{H,t} \equiv -\rho s_c^{-1} \left(\tilde{Y}_{W,t} - \hat{G}_{W,t} \right) - (1-\beta) E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} v_{H,s} \right\}.$$

and

$$v_{H,t} \equiv b_y \tilde{Y}_{W,t} + (1-n) b_q \tilde{T}_t + (1+\omega_g) \tilde{\tau}_{H,t} - s_d \hat{G}_{H,t} + \rho s_c^{-1} \hat{G}_{W,t} + (1-n)(1+\omega_g) \hat{G}_{R,t}.$$

The parameters are $b_y \equiv (1+\omega_g) - \rho s_c^{-1}$ and $b_q \equiv (1+\omega_g) \theta s_c - 1$. The correspondent equation for the Foreign country is

$$\hat{b}_{F,t-1} - \rho s_c^{-1} \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t} \right) - \pi_t + \psi_{F,t}$$
$$= (1-\beta) \left[b_y \left(\hat{Y}_{W,t} - \tilde{Y}_{W,t} \right) - n b_q \left(\hat{T}_t - \tilde{T}_t \right) + (1+\omega_g) \left(\hat{\tau}_{F,t} - \tilde{\tau}_{F,t} \right) \right]$$
$$+ \beta E_t \left[\hat{b}_{F,t} - \rho s_c^{-1} \left(\hat{Y}_{W,t+1} - \tilde{Y}_{W,t+1} \right) - \pi_{t+1} + \psi_{F,t+1} \right].$$

where

$$\psi_{F,t} \equiv -\rho s_c^{-1} \left(\tilde{Y}_{W,t} - \hat{G}_{W,t} \right) - (1-\beta) E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} v_{F,s} \right\},$$

and

$$v_{F,t} \equiv b_y \tilde{Y}_{W,t} - n b_q \tilde{T}_t + (1+\omega_g) \tilde{\tau}_{F,t} - s_d \hat{G}_{F,t} + \rho s_c^{-1} \hat{G}_{W,t} - n (1+\omega_g) \hat{G}_{R,t}.$$

From the definition of the price index, one can write a relation between the CPI (overall) inflation rate π_t and the GDP (country-specific) inflation rates $\pi_{H,t}$ and $\pi_{F,t}$

$$\pi_t = n \pi_{H,t} + (1-n) \pi_{F,t}.$$

Finally, from the definition of the terms of trade, one can also see that the percentage change in the terms of trade is determined by the cross-country GDP inflation differential

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t}.$$

or

$$\left(\hat{T}_t - \tilde{T}_t \right) = \left(\hat{T}_{t-1} - \tilde{T}_{t-1} \right) + \pi_{F,t} - \pi_{H,t} - \left(\tilde{T}_t - \tilde{T}_{t-1} \right).$$

A.4 The Optimal Policy Problem

Let $2n\varphi_{1,t}^H$ and $2(1-n)\varphi_{1,t}^F$ be the Lagrange multipliers on the two aggregate supply relations, $2n\varphi_{2,t}^H$ and $2(1-n)\varphi_{2,t}^F$ the Lagrange multipliers on the two intertemporal government budget constraints and $2\varphi_{3,t}$ and $2n(1-n)\varphi_{4,t}$ be the Lagrange multipliers on the remaining two constraints for the relations between CPI inflation and GDP inflation rates and between terms of trade and GDP inflation differentials. The resulting Lagrangian is

$$
\begin{aligned}
L_0 \;=\; & \frac{1}{2}E_0\Bigg\{\sum_{t=0}^{\infty}\beta^t\Big[\lambda_y y_t^2 + n(1-n)\lambda_q q_t^2 + n\lambda_{\pi_H}\pi_{H,t}^2 + (1-n)\lambda_{\pi_F}\pi_{F,t}^2 \\
& +2n\varphi_{1,t}^H\left(\kappa_H^{-1}\pi_{H,t} - \delta_y y_t - \omega_\tau(\hat{\tau}_{H,t}-\tilde{\tau}_{H,t}) - (1-n)\delta_q q_t - \kappa_H^{-1}\beta\pi_{H,t+1}\right) \\
& +2(1-n)\varphi_{1,t}^F\left(\kappa_F^{-1}\pi_{F,t} - \delta_y y_t - \omega_\tau(\hat{\tau}_{F,t}-\tilde{\tau}_{F,t}) + n\delta_q q_t - \kappa_F^{-1}\beta\pi_{F,t+1}\right) \\
& +2n\varphi_{2,t}^H\Big(\hat{b}_{H,t-1} - \rho s_c^{-1}y_t - \pi_t + \psi_{H,t} \\
& \quad -(1-\beta)[b_y y_t + (1-n)b_q q_t + (1+\omega_g)(\hat{\tau}_{H,t}-\tilde{\tau}_{H,t})] \\
& \quad -\beta\left(\hat{b}_{H,t} - \rho s_c^{-1}y_{t+1} - \pi_{t+1} + \psi_{H,t+1}\right)\Big) \\
& +2(1-n)\varphi_{2,t}^F\Big(\hat{b}_{F,t-1} - \rho s_c^{-1}y_t - \pi_t + \psi_{F,t} \\
& \quad -(1-\beta)[b_y y_t - nb_q q_t + (1+\omega_g)(\hat{\tau}_{F,t}-\tilde{\tau}_{F,t})] \\
& \quad -\beta\left(\hat{b}_{F,t} - \rho s_c^{-1}y_{t+1} - \pi_{t+1} + \psi_{F,t+1}\right)\Big) \\
& +2\varphi_{3,t}\left(\pi_t - n\pi_{H,t} - (1-n)\pi_{F,t}\right) \\
& +2n(1-n)\varphi_{4,t}\left(q_t - q_{t-1} - \pi_{F,t} + \pi_{H,t} + \Delta\tilde{T}_t\right)\Big]\Bigg\} \\
& -2n\varphi_{1,-1}^H\kappa_H^{-1}\pi_{H,0} - 2(1-n)\varphi_{1,-1}^F\kappa_F^{-1}\pi_{F,0} \\
& +2\left[n\varphi_{2,-1}^H + (1-n)\varphi_{2,-1}^F\right]\left(\pi_0 + \rho s_c^{-1}y_0\right)
\end{aligned}
$$

where the last two lines of the Lagrangian contain the stationarity constraints for the optimal policy from a timeless perspective discussed in the text.

The first order conditions of the optimal plan reported in the text can be written compactly as

$$
A_1 E_t x_{t+1} + A_0 x_t + A_{-1}x_{t-1} = B\xi_t.
$$

The vector of endogenous variables and Lagrange multipliers is

$$
x_t' \equiv \left[\begin{array}{cccccccccccc} y_t & q_t & \pi_{H,t} & \pi_{F,t} & \pi_t & \hat{\tau}_{W,t} & \hat{\tau}_{R,t} & \hat{b}_{W,t} & \hat{b}_{R,t} & \varphi_{2,t}^W & \varphi_{2,t}^R & \varphi_{4,t} \end{array}\right],
$$

where, for convenience, fiscal variables have been expressed in terms of average and relative stances. The vector of shocks can be written compactly as

$$
\xi_t' \equiv \left[\begin{array}{ccccc} \tilde{\tau}_{W,t} & \tilde{\tau}_{R,t} & u_{W,t}^\psi & u_{R,t}^\psi & \Delta\tilde{T}_t \end{array}\right],
$$

where $u_{i,t}^\psi \equiv \psi_{i,t} - \beta E_t\psi_{i,t+1}$, for $i = \{W,R\}$. Finally, the objects A_1, A_0, A_{-1} and B are non-

stochastic conformable matrixes of coefficients defined as

$$
A_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -\beta n \kappa_H^{-1} & -\beta(1-n)\kappa_F^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\beta \kappa_H^{-1} & \beta \kappa_F^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta \rho s_c^{-1} & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
A_0 \equiv \begin{bmatrix}
\lambda_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_y & 0 & 0 \\
0 & \lambda_q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_q & 1 \\
0 & 0 & n\kappa_H \lambda_{\pi_H} & (1-n)\kappa_F \lambda_{\pi_F} & 0 & 0 & 0 & 0 & 0 & -\omega_f - \kappa_W & 0 & n(1-n)\kappa_R \\
0 & 0 & \kappa_H \lambda_{\pi_H} & -\kappa_F \lambda_{\pi_F} & 0 & 0 & 0 & 0 & 0 & -\kappa_R & -\omega_f & \tilde{\kappa} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
-\delta_y & 0 & n\kappa_H^{-1} & -(1-n)\kappa_F^{-1} & 0 & -\omega_\tau & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\delta_q & \kappa_H^{-1} & -\kappa_F^{-1} & 0 & 0 & -\omega_\tau & 0 & 0 & 0 & 0 & 0 \\
-\nu_y & 0 & 0 & 0 & -1 & -\omega_f \omega_\tau & 0 & -\beta & 0 & 0 & 0 & 0 \\
0 & -\nu_q & 0 & 0 & 0 & 0 & -\omega_f \omega_\tau & 0 & -\beta & 0 & 0 & 0 \\
0 & 0 & -n & -(1-n) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
A_{-1} \equiv \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho s_c^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\beta)\omega_f + \kappa_W & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_R & (1-\beta)\omega_f & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
B \equiv \begin{bmatrix}
& & \underset{(6\times 5)}{0} & & \\
-\omega_\tau & 0 & 0 & 0 & 0 \\
0 & -\omega_\tau & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1
\end{bmatrix},
$$

where $\omega_f \equiv (1-\beta)(1+\omega_g)\omega_\tau^{-1}$, $\nu_y \equiv (1-\beta)b_y + \rho s_c^{-1}$, $\omega_y \equiv \omega_f \delta_y - \nu_y$, $\nu_q \equiv (1-\beta)b_q$, $\omega_q \equiv \omega_f \delta_q - \nu_q$, $\kappa_W \equiv n\kappa_H + (1-n)\kappa_F$, $\kappa_R \equiv \kappa_H - \kappa_F$, $\tilde{\kappa} \equiv (1-n)\kappa_H + n\kappa_F$.

A.5 Determinacy

In this section, I study analytically the determinacy properties of the model under the assumption that the degree of price rigidity is the same across countries ($\alpha_H = \alpha_F$). I will refer to this assumption as A1. The mathematical supplement extends this basic analysis by evaluating the sensitivity of the determinacy properties to alternative parametrizations. I begin with the proof of the following proposition.

Proposition 3 *Under assumption A1, the equilibrium induced by the optimal policy plan can be described by an aggregate block (in terms of average variables) and a relative block (in terms of difference variables). The solution of each block can be determined independently.*
Proof. *Since the degree of price rigidity is the same, it follows that $\kappa_H = \kappa_F = \kappa$ and $\lambda_{\pi_H} = \lambda_{\pi_F} = \lambda_\pi$. One can then combine the two first order conditions for GDP inflation rates [53] and [54] to get*

$$\kappa \lambda_\pi \pi_t = (\omega_f + \kappa) \left(\varphi_{2,t}^W - \varphi_{2,t-1}^W \right), \tag{67}$$

and

$$\kappa \lambda_\pi \pi_{R,t} = \omega_f \left(\varphi_{2,t}^R - \varphi_{2,t-1}^R \right) - \kappa \varphi_{4,t} \tag{68}$$

Similarly, I also take the average and the difference of the Phillips curves of the two countries [45] and [46] to obtain

$$\pi_t = \kappa \left[\delta_y y_t + \omega_\tau \left(\hat{\tau}_{W,t} - \tilde{\tau}_{W,t} \right)_t \right] + \beta E_t \pi_{t+1}, \tag{69}$$

and

$$\pi_{R,t} = \kappa \left[\omega_\tau \left(\hat{\tau}_{R,t} - \tilde{\tau}_{R,t} \right) + \delta_q q_t \right] + \beta E_t \pi_{R,t+1}, \tag{70}$$

The two government budget constraints [47] and [48] deliver

$$\begin{aligned}
\hat{b}_{W,t-1} - \rho s_c^{-1} y_t - \pi_t + \psi_{W,t} &= (1-\beta) \left[b_y y_t + (1+\omega_g) \left(\hat{\tau}_{W,t} - \tilde{\tau}_{W,t} \right) \right] \\
&\quad + \beta E_t \left(\hat{b}_{W,t} - \rho s_c^{-1} y_{t+1} - \pi_t + \psi_{W,t+1} \right),
\end{aligned} \tag{71}$$

and

$$\hat{b}_{R,t-1} + \psi_{R,t} = (1-\beta) \left[b_q q_t + (1+\omega_g) \left(\hat{\tau}_{R,t} - \tilde{\tau}_{R,t} \right) \right] + \beta E_t \left(\hat{b}_{R,t} + \psi_{R,t+1} \right). \tag{72}$$

The solution of the average block can then be cast in terms of the sequence $\{ y_t, \pi_t, \hat{\tau}_{W,t} - \tilde{\tau}_{W,t}, \hat{b}_{W,t}, \varphi_{2,t}^W \}$ which solves the system of equations composed by [51], [67], [55], [69], and [71]. On the other hand, the solution of the relative block can be found by solving for the sequence $\{ q_t, \pi_{R,t}, \hat{\tau}_{R,t} - \tilde{\tau}_{R,t}, \hat{b}_{R,t}, \varphi_{2,t}^R, \varphi_{4,t} \}$ the system of equations composed by [52], [68], [56], [70], [72] and [50]. The two systems are obviously independent, hence, completing the proof of the proposition. ∎

Before moving to analyze the determinacy properties of the model, I will state and prove a second proposition which will be used extensively in the subsequent analysis.

Proposition 4 *Let $P(\lambda) \equiv \lambda^2 + A_1 \lambda + A_0 = 0$ and let λ_1 and λ_2 be the roots of $P(\lambda)$. Then, the absolute values of λ_1 and λ_2 split across the unit circle if and only if $P(1) > 0$ and $P(-1) < 0$ or viceversa.*
Proof. *First, notice that one can always rewrite the polynomial $P(\lambda)$ as*

$$P(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) \tag{73}$$

I first show that $P(1) > 0$ and $P(-1) < 0$ imply that the absolute values of the two roots λ_1 and λ_2 split across the unit circle. From the right hand side of [73], it is easy to see that

$$P(1) = (1 - \lambda_1)(1 - \lambda_2) > 0, \tag{74}$$

and

$$P\left(-1\right) = \left(1 + \lambda_1\right)\left(1 + \lambda_2\right) < 0. \tag{75}$$

If $P\left(1\right) > 0$, it means that λ_1 and λ_2 are on the same side of 1. Similarly, if $P\left(-1\right) < 0$, it means that λ_1 and λ_2 are on opposite sides of -1. It then follows that one root must lie inside the unit circle and the other outside. The case $P\left(1\right) < 0$ and $P\left(-1\right) > 0$ is totally symmetric. Next, I show that if $|\lambda_1|$ and $|\lambda_2|$ lie on opposite sides of 1, it must be the case that $P\left(1\right)$ and $P\left(-1\right)$ lie on opposite sides of 0. Without loss of generality, suppose $|\lambda_1| > 1$ and $|\lambda_2| < 1$. There are two cases two be considered. First, if $\lambda_1 > 1$, then, one can see from [74] that $P\left(1\right) < 0$ and from [75] that $P\left(-1\right) > 0$, which confirms the claim. Second, if $\lambda_1 < -1$, then, again from [74] and [75], $P\left(1\right) > 0$ and $P\left(-1\right) < 0$. The case $|\lambda_1| < 1$ and $|\lambda_2| > 1$ is symmetric. ∎

Proposition 4 complements Proposition $C.1$ in Woodford [2003] which gives necessary and sufficient conditions for the two roots of $P\left(\lambda\right)$ to be both outside the unit circle.

A.5.1 Determinacy of the Optimal Policy Plan

Given assumption A1 and Proposition 3, I can analyze the relative and aggregate block separately. I start with the relative block. From [68], I solve for inflation differentials as to obtain

$$\pi_{R,t} = \frac{\omega_f}{\kappa\lambda_\pi}\varphi_{2,t}^R - \frac{1}{\lambda_\pi}\varphi_{4,t}.$$

I plug the result into the evolution of the terms of trade [50] to obtain

$$q_t = q_{t-1} - \frac{\omega_f}{\kappa\lambda_\pi}\varphi_{2,t}^R + \frac{1}{\lambda_\pi}\varphi_{4,t} - \Delta\tilde{T}_t.$$

I solve the last expression for the Lagrange multiplier $\varphi_{4,t}$

$$\varphi_{4,t} = \lambda_\pi\left(q_t - q_{t-1}\right) + \frac{\omega_f}{\kappa}\varphi_{2,t}^R + \lambda_\pi\Delta\tilde{T}_t. \tag{76}$$

I update the last expression, take expectations at time t, multiply it by β and subtract it from [76] to get

$$\begin{aligned}\varphi_{4,t} - \beta E_t\varphi_{4,t+1} &= \lambda_\pi\left(q_t - q_{t-1}\right) - \beta\lambda_\pi\left(E_t q_{t+1} - q_t\right) \\ &\quad + \frac{\omega_f}{\kappa}\varphi_{2,t}^R - \frac{\beta\omega_f}{\kappa}E_t\varphi_{2,t+1}^R + \lambda_\pi\left(\Delta\tilde{T}_t - \beta E_t\Delta\tilde{T}_{t+1}\right).\end{aligned} \tag{77}$$

I then plug [77] into [52] and obtain

$$\begin{aligned}\lambda_q q_t = -\omega_q\varphi_{2,t}^R - \lambda_\pi\left(q_t - q_{t-1}\right) + \beta\lambda_\pi\left(E_t q_{t+1} - q_t\right) \\ - \frac{\omega_f}{\kappa}\varphi_{2,t}^R + \frac{\beta\omega_f}{\kappa}E_t\varphi_{2,t+1}^R - \lambda_\pi\left(\Delta\tilde{T}_t - \beta E_t\Delta\tilde{T}_{t+1}\right),\end{aligned}$$

I solve the last expression for the Lagrange multiplier $\varphi_{2,t}^R$ to get

$$\begin{aligned}\left(\omega_q + \frac{\omega_f}{\kappa}\right)\varphi_{2,t}^R &= -\lambda_q q_t - \lambda_\pi\left(q_t - q_{t-1}\right) + \beta\lambda_\pi\left(E_t q_{t+1} - q_t\right) \\ &\quad + \frac{\beta\omega_f}{\kappa}E_t\varphi_{2,t+1}^R - \lambda_\pi\left(\Delta\tilde{T}_t - \beta E_t\Delta\tilde{T}_{t+1}\right).\end{aligned} \tag{78}$$

From [56], I know that $\varphi_{2,t}^R$ follows a random walk. Hence, the right hand side of [78] must follow a random walk too

$$-\lambda_q q_t - \lambda_\pi\left(q_t - q_{t-1}\right) + \beta\lambda_\pi\left(E_t q_{t+1} - q_t\right) + \frac{\beta\omega_f}{\kappa}E_t\varphi_{2,t+1}^R - \lambda_\pi\left(\Delta\tilde{T}_t - \beta E_t\Delta\tilde{T}_{t+1}\right) =$$

$$E_t\left[-\lambda_q q_{t+1} - \lambda_\pi\left(q_{t+1} - q_t\right) + \beta\lambda_\pi\left(E_{t+1}q_{t+2} - q_{t+1}\right) + \frac{\beta\omega_f}{\kappa}E_{t+1}\varphi_{2,t+2}^R - \lambda_\pi\left(\Delta\tilde{T}_{t+1} - \beta E_{t+1}\Delta\tilde{T}_{t+2}\right)\right].$$

By the law of iterated expectations, $E_t \left(E_{t+1} \varphi^R_{2,t+2} \right) = E_t \varphi^R_{2,t+2}$. Moreover, expression [56] implies also that $E_t \varphi^R_{2,t+1} = E_t \varphi^R_{2,t+2}$ so that the Lagrange multipliers disappear from the condition above. Remember also that, by definition, the terms of trade gap $q_t = \hat{T}_t - \tilde{T}_t$. It follows that $\Delta q_t + \Delta \tilde{T}_t = \Delta \hat{T}_t$. I can then rewrite the previous expression as

$$\lambda_q E_t \Delta q_{t+1} = \lambda_\pi \left(\Delta \hat{T}_t - E_t \Delta \hat{T}_{t+1} \right) + \beta \lambda_\pi E_t \left(\Delta \hat{T}_{t+1} - E_{t+1} \Delta \hat{T}_{t+2} \right).$$

In order to express the last expression more compactly, it will be useful to define

$$\mathcal{T}_t \equiv \Delta \hat{T}_t - E_t \Delta \hat{T}_{t+1}.$$

Finally, I obtain a single forward looking first order difference equation in the new variable $\mathcal{T}_t$ which summarizes the optimality conditions of the relative block under A1

$$\mathcal{T}_t = \lambda_T E_t \Delta q_{t+1} + \beta E_t \mathcal{T}_{t+1}, \tag{79}$$

where $\lambda_T \equiv \lambda_q / \lambda_\pi$. The remaining conditions to study the determinacy properties come from the equilibrium relations. From the relative Phillips curve [70] I solve for the relative tax gap to obtain

$$\hat{\tau}_{R,t} - \tilde{\tau}_{R,t} = \left(\frac{1}{\kappa \omega_\tau} \right) (\pi_{R,t} - \beta E_t \pi_{R,t+1}) - \frac{\delta_q}{\omega_\tau} q_t.$$

I substitute this result into the relative government budget constraint [72] to obtain

$$\hat{b}_{R,t-1} + \psi_{R,t} = \frac{\omega_f}{\kappa} (\pi_{R,t} - \beta E_t \pi_{R,t+1}) - \omega_q q_t + \beta E_t \left(\hat{b}_{R,t} + \psi_{R,t+1} \right).$$

From [50], I express inflation rate differentials as function of variations in the terms of trade gap

$$\pi_{R,t} = - \left(\Delta q_t + \Delta \tilde{T}_t \right).$$

I substitute the latter result into the government budget constraint to get

$$\begin{aligned} \hat{b}_{R,t-1} + f_{R,t} &= -\frac{\omega_f}{\kappa} \left(\Delta q_t - \beta E_t \Delta q_{t+1} \right) - \omega_q q_t + \beta E_t \left(\hat{b}_{R,t} + f_{R,t+1} \right) \\ &\quad -\frac{\omega_f}{\kappa} \left(\Delta \tilde{T}_t - \beta E_t \Delta \tilde{T}_{t+1} \right). \end{aligned} \tag{80}$$

I can now study the determinacy properties of the system constituted by the definition of Δq_t

$$\Delta q_t \equiv q_t - q_{t-1},$$

the relative government budget constraint [80], the condition [79] and the definition

$$\mathcal{T}_t \equiv \Delta q_t - E_t \Delta q_{t+1} + \left(\Delta \tilde{T}_t - E_t \Delta \tilde{T}_{t+1} \right).$$

Such system of four equations in four unknowns can be written in matrix notation as

$$A E_t z_{t+1} = B z_t + C \varepsilon_{R,t},$$

where the vector of endogenous variables is

$$z_t \equiv \begin{bmatrix} q_{t-1} & \hat{b}_{R,t-1} & \Delta q_t & \mathcal{T}_t \end{bmatrix}',$$

the vector of exogenous shocks is

$$\varepsilon_{R,t} \equiv \begin{bmatrix} u^\psi_{R,t} & u^{\tilde{T}}_{1,t} & u^{\tilde{T}}_{2,t} \end{bmatrix},$$

and the two new shocks are defined as $u^{\tilde{T}}_{1,t} \equiv \Delta \tilde{T}_t - \beta E_t \Delta \tilde{T}_{t+1}$ and $u^{\tilde{T}}_{2,t} \equiv \Delta \tilde{T}_t - E_t \Delta \tilde{T}_{t+1}$. The matrixes of the system are

$$A \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ \beta & \omega_q & \beta \omega_f / \kappa & 0 \\ 0 & 0 & \lambda_T & \beta \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B \equiv \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega_f/\kappa & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix},$$

$$C \equiv \begin{bmatrix} 0 & 0 & 0 \\ 1 & -\omega_f/\kappa & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The determinacy properties depend on the eigenvalues of the matrix $A^{-1}B$ which is given by:

$$A^{-1}B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -\beta/\omega_q & 1/\omega_q & (1-\beta)\,\omega_f/(\kappa\omega_q) - \beta/\omega_q & \beta\omega_f/(\kappa\omega_q) \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -\lambda_T/\beta & (1+\lambda_T)/\beta \end{bmatrix}.$$

Since $A^{-1}B$ is block triangular, it follows that the eigenvalues are $\{\lambda_1 = 1, \lambda_2 = \omega_q^{-1}, \lambda_3, \lambda_4\}$, where λ_3 and λ_4 are the eigenvalues of the 2×2 matrix

$$D \equiv \begin{bmatrix} 1 & -1 \\ -\lambda_T/\beta & (1+\lambda_T)/\beta \end{bmatrix}.$$

The relative block has two predetermined variables (q_t and $\hat{b}_{R,t}$). Determinacy requires two eigenvalues outside the unit circle (the solution possesses a unit root associated with λ_1). The characteristic equation associated to matrix D is

$$P(\lambda) = \lambda^2 - [1 + (1+\lambda_T)/\beta]\lambda + 1/\beta = 0.$$

It is straightforward to see that

$$P(1) = -\lambda_T/\beta, \qquad P(1) = [2(1+\beta) + \lambda_T]/\beta.$$

Given that $\lambda_T = \lambda_q/\lambda_\pi > 0$, it follows that the conditions of Proposition 4 are satisfied and the absolute values of λ_3 and λ_4 always lie on opposite sides of 1. Hence, determinacy depends crucially on the value of ω_q. Under the baseline calibration, it is the case that $\omega_q = 0.341$ so that $\lambda_2 > 1$ and the relative block has a determinate solution.

I now consider the average block. I rewrite the first order condition for the output gap [51] as

$$\lambda_y y_t = -\omega_y \varphi_{2,t}^W - \rho s_c^{-1} \varphi_{2,t-1}^W,$$

and the first order condition for average inflation [67] as

$$\kappa\lambda_\pi \pi_t = (\omega_f + \kappa)\left(\varphi_{2,t}^W - \varphi_{2,t-1}^W\right).$$

I further define the coefficients $m_\varphi \equiv -\omega_y/\lambda_y$, $n_\varphi \equiv -\rho s_c^{-1}/\lambda_y$ and $\omega_\varphi \equiv -(\omega_f + \kappa)/(\kappa\lambda_\pi)$. One can combine the two expressions above with the first order condition for average debt [55] to obtain

$$E_t \pi_{t+1} = 0, \tag{81}$$

and

$$\pi_t + \frac{n_\varphi}{m_\varphi}\pi_{t-1} + \frac{\omega_\varphi}{m_\varphi}\Delta y_t = 0. \tag{82}$$

Expressions [81] and [82] constitute a complete characterization of the optimality conditions for the average block. Most importantly, when combined with the remaining equilibrium relations (the average Phillips curve [69] and the average government budget constraint [71]), the resulting system

is identical to the closed economy model of BW. As shown by BW in the appendix, the solution of such system is always determinate. It follows that the condition $|1/\omega_q| > 1$ fully characterizes the determinacy properties of the model under the optimal policy plan. Sensitivity analysis (see the mathematical supplement) shows that determinacy occurs for wide (and relevant) regions of the parameter space.

A.5.2   Determinacy under Simple Rules

Proposition 3, suitably modified to take into account the new policy rules, retains its validity under simple rules too. The economy can still be characterized in terms of two blocks (aggregate and relative).

I start again with the analysis of the relative block, which is characterized by the rule for fiscal policy

$$\hat{b}_{R,t} = 0, \tag{83}$$

and by the three equilibrium equations [70], [72] and [50]. As in the previous section, I can solve for the tax gap from [70] and plug the result into the government budget constraint. I can also eliminate debt using [83] and inflation rate differentials from [50]. The outcome is a second order linear difference equation in the terms of trade given by

$$E_t q_{t+1} - \left(1 + \frac{1}{\beta} + \frac{\kappa \omega_q}{\beta \omega_f}\right) q_t + \frac{1}{\beta} q_{t-1} = \frac{\kappa}{\beta \omega_f} u_{R,t}^{\psi} + \frac{1}{\beta} u_{1,t}^{\tilde{T}}.$$

The associated characteristic equation is

$$P(\lambda) = \lambda^2 - \left(1 + \frac{1}{\beta} + \frac{\kappa \omega_q}{\beta \omega_f}\right) \lambda + \frac{1}{\beta} = 0.$$

One can easily see that

$$P(1) = -\frac{\kappa \omega_q}{\beta \omega_f}, \qquad P(-1) = \frac{1}{\beta}\left[2(1+\beta) + \frac{\kappa \omega_q}{\omega_f}\right].$$

From the definition of the parameters, it is easy to see that ω_f is always positive. Under the baseline calibration, $\omega_q > 0$, hence, $P(1) < 0$ and $P(-1) > 0$. From Proposition 4, it follows that the relative block is determinate under the baseline calibration.

The monetary rule for the average block is

$$\pi_t = 0. \tag{84}$$

The fiscal rule is

$$\hat{b}_{W,t} = r_t. \tag{85}$$

The equilibrium equations are the average Phillips curve [69], the average government budget constraint [71] and the log-linear approximation of the Euler equation [26] which reads as

$$r_t = \tilde{r}_t + E_t \pi_{t+1} + \rho s_c^{-1} \left(E_t y_{t+1} - y_t\right). \tag{86}$$

From [69], I solve for the tax gap to obtain

$$\hat{\tau}_{W,t} - \tilde{\tau}_{W,t} = -\frac{\delta_y}{\omega_\tau} y_t,$$

where I also used the monetary rule [84]. I replace the result into [71], together with [84] and get

$$\hat{b}_{W,t-1} + u_{W,t}^f = \omega_f \left(\omega_\tau - \delta_y\right) y_t + \beta \hat{b}_{W,t} - \beta \rho s_c^{-1} \left(E_t y_{t+1} - y_t\right),$$

where I have exploited the definition of b_y to simplify terms. I can then substitute from [86] to eliminate the last term in the previous expression and finally apply the fiscal policy rule [85]. The

result is a closed form solution for the output gap as a function of the existing stock of liabilities and exogenous shocks of the form

$$y_t = \frac{1}{\omega_f (\omega_\tau - \delta_y)} \left(\hat{b}_{W,t-1} + u^f_{W,t} - \beta \tilde{r}_t \right). \tag{87}$$

From [87] and [86], I can then derive the dynamic evolution of debt using the fiscal rule [85]. The resulting expression is

$$\hat{b}_{W,t} = \left[\frac{\rho s_c^{-1}}{\rho s_c^{-1} - \omega_f (\omega_\tau - \delta_y)} \right] \hat{b}_{W,t-1} + \epsilon_{W,t},$$

where $\epsilon_{W,t}$ is a composite shock whose definition is immaterial for the determinacy properties which, instead, depend on the coefficient in brackets. In particular, a non-explosive (and, hence, determinate) solution would require

$$\left| \frac{\rho s_c^{-1}}{\rho s_c^{-1} - \omega_f (\omega_\tau - \delta_y)} \right| < 1. \tag{88}$$

Under the baseline calibration, the coefficient governing the dynamics of debt is equal to 0.7724, thus, ensuring determinacy. Again, sensitivity analysis shows that determinacy is obtained for wide regions of the parameter space.

A.5.3 Determinacy under Flexible Rules

Once again, a modified version of Proposition 3 holds and the economy can be studied with respect to two separate blocks.

As usual, I begin with the relative block. The only difference with respect with the previous section is that now the relative fiscal rule is

$$\hat{b}_{R,t} + \theta s_c \phi q_t = 0. \tag{89}$$

I follow the same steps as before to derive a second order difference equation in the terms of trade gap given by

$$E_t q_{t+1} - \left(1 + \frac{1}{\beta} + \frac{\kappa \omega_q}{\beta \omega_f} + \frac{\kappa \theta s_c \phi}{\omega_f} \right) q_t + \frac{1}{\beta} \left(1 + \frac{\kappa \theta s_c \phi}{\omega_f} \right) + q_{t-1} = \frac{\kappa}{\beta \omega_f} u^\psi_{R,t} + \frac{1}{\beta} u^{\tilde{T}}_{1,t}.$$

From the associated characteristic polynomial $P(\lambda)$, it follows that

$$P(1) = \frac{\kappa}{\beta \omega_f} \left[(1 - \beta) \theta s_c \phi - \omega_q \right],$$

and

$$P(-1) = \frac{(1 + \beta) \theta s_c \kappa}{\beta \omega_f} + \frac{2(1 + \beta)}{\beta} + \frac{\kappa \omega_q}{\beta \omega_f}.$$

Given that under the baseline calibration ω_q and ω_f are positive, $P(-1) > 0 \; \forall \phi > 0$. Therefore, a necessary condition for determinacy of under flexible rules is given by $P(1) < 0$ which can be solved for ϕ as to yield

$$\phi < \frac{\omega_q}{(1 - \beta) \theta s_c} = 10.4676. \tag{90}$$

Next, I turn to analyzing the average block. The flexible rule for monetary policy which substitutes [84] is

$$\pi_t + \gamma \Delta y_t = 0. \tag{91}$$

On the other hand, the flexible fiscal rule is

$$\hat{b}_{W,t} + \phi y_t = r_t. \tag{92}$$

As before, I eliminate taxes from the average Phillips curve [69]. This time, however, CPI inflation is not zero. Hence, I substitute for that variable from [91] to obtain

$$\hat{\tau}_{W,t} - \tilde{\tau}_{W,t} = -\frac{\gamma}{\kappa \omega_\tau} \left(\Delta y_t - \beta E_t \Delta y_{t+1} \right) - \frac{\delta_y}{\omega_\tau} y_t.$$

I replace the result into the average government budget constraint, together with the monetary

$$\hat{b}_{W,t-1} + \gamma \Delta y_t + u_{W,t}^{\psi} = \omega_f \left(\omega_\tau - \delta_y\right) y_t - \frac{\omega_f \gamma}{\kappa} \left(\Delta y_t - \beta E_t \Delta y_{t+1}\right)$$
$$+ \beta \hat{b}_{W,t} - \beta \rho s_c^{-1} E_t \Delta y_{t+1} + \beta \gamma E_t \Delta y_{t+1}$$

I can collect terms and express the last expression as

$$\beta \hat{b}_{W,t} + \beta \left[\gamma \left(1 + \frac{\omega_f}{\kappa}\right) - \rho s_c^{-1}\right] E_t \Delta y_{t+1} + \omega_f \left(\omega_\tau - \delta_y\right) y_t \quad = \tag{93}$$
$$\hat{b}_{W,t-1} + \gamma \left(1 + \frac{\omega_f}{\kappa}\right) \Delta y_t + u_{W,t}^{\psi}.$$

I can also substitute the Euler equation [86] and the monetary rule [91] into the fiscal rule [92] to get

$$\hat{b}_{W,t} + \left(\gamma - \rho s_c^{-1}\right) E_t \Delta y_{t+1} + \phi y_t = \tilde{r}_t. \tag{94}$$

The determinacy properties of the average block depend upon the system constituted by expressions [94] and [93], together with the definition

$$y_t = \Delta y_t + y_{t-1}. \tag{95}$$

The system can be written in the form

$$A E_t z_{t+1} = B z_t + C \varepsilon_{W,t},$$

where the vector of endogenous variables is

$$z_t \equiv \left[\begin{array}{ccc} \hat{b}_{W,t-1} & \Delta y_t & y_{t-1} \end{array}\right]',$$

the vector of exogenous shocks is

$$\varepsilon_{R,t} \equiv \left[\begin{array}{cc} u_{W,t}^{\psi} & \tilde{r}_t \end{array}\right]',$$

and the matrixes of the system are

$$A \equiv \left[\begin{array}{ccc} 1 & a_{12} & \phi \\ \beta & a_{22} & a_{23} \\ 0 & 0 & 1 \end{array}\right],$$

$$B \equiv \left[\begin{array}{ccc} 0 & 0 & 0 \\ 1 & b_{22} & 0 \\ 0 & 1 & 1 \end{array}\right],$$

$$C \equiv \left[\begin{array}{ccc} 0 & 0 & 0 \\ 1 & -\omega_f/\kappa & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right].$$

with $a_{12} \equiv \gamma - \rho s_c^{-1}$, $a_{22} \equiv \beta \left(b_{22} - \rho s_c^{-1}\right)$, $a_{23} \equiv \omega_f \left(\omega_\tau - \delta_y\right)$ and $b_{22} \equiv \gamma \left(1 + \omega_f/\kappa\right)$. Let $d \equiv \det(A) = a_{22} - \beta a_{12} = \beta \gamma \left(\omega_f/\kappa\right) > 0$. It is then possible to show that

$$A^{-1}B = \left[\begin{array}{ccc} c_{11} & c_{11}b_{22} + c_{13} & c_{13} \\ c_{21} & c_{21}b_{22} + c_{23} & c_{23} \\ 0 & 1 & 1 \end{array}\right],$$

where $c_{11} = -a_{12}/d$, $c_{13} = \left(a_{12}a_{23} - \phi a_{22}\right)/d$, $c_{21} = 1/d$, $c_{23} = \left(\beta \phi - a_{23}\right)/d$. It is also possible to prove that at least one eigenvalue of $A^{-1}B$ is equal to zero (the difference of the second and third column is equal to the first multiplied by b_{22}, hence, $A^{-1}B$ is singular and must have at least one null eigenvalue). Hence, determinacy requires that the two remaining eigenvalues lie on different sides of 1. The other two eigenvalues are the solution of the characteristic equation

$$P(\lambda) = \lambda^2 - \left(1 + c_{11} + b_{22}c_{11} + c_{23}\right) \lambda + \left(c_{11} + b_{22}c_{11} + c_{11}c_{23} - c_{21}c_{13}\right) = 0.$$

When $\phi = 0$ and $\gamma > 0$ (flexibility is granted to monetary policy only), little algebra leads to

$$P(1) = a_{23}/d,$$

and

$$P(-1) = 2\left(1 - \frac{a_{12}}{d} + \frac{b_{22}}{d}\right) - \frac{a_{23}}{d}.$$

From the definitions of the parameters, one can see that under the baseline calibration $P(1) < 0$ and $P(-1) > 0 \ \forall \gamma > 0$. Hence, by Proposition 4, it follows that in this case the average block has always a determinate solution. On the other hand, when $\phi > 0$ and $\gamma = 0$ (flexibility is granted to fiscal policy only), it is possible to obtain a closed form solution for output similar to [87] and derive the dynamic evolution of debt which is governed by the coefficient

$$\delta_b \equiv \frac{\rho s_c^{-1} - \phi}{\rho s_c^{-1} + \beta\phi - \omega_f (\omega_\tau - \delta_y)}. \tag{96}$$

Comparing the expression of [96] with [88], one can see that the effect of a positive ϕ is to reduce the coefficient. Hence, under the baseline calibration, a restriction on ϕ can be derived by solving the inequality $\delta_b > -1$ which leads to

$$\phi < \frac{2\rho s_c^{-1} - a_{23}}{1 - \beta} = 950.794.$$

In the more general case when both flexibility parameters are positive, the two conditions above become

$$P(1) = -c_{23}(1 - c_{11}) - c_{21}c_{13},$$

and

$$P(-1) = 2(1 + c_{11} + b_{22}c_{21}) + c_{23}(1 + c_{11}) - c_{21}c_{13}.$$

I solve numerically for these two conditions to find the set of values for γ and ϕ that satisfy the conditions in Proposition 4. The results show that the binding constraint for determinacy derives always from [90].

A.6 Welfare and Solution of the Model

In this section, I show a simple and computationally feasible method to derive the value of welfare under a given policy rule. The exposition is slightly more general than in previous sections but it will be obvious that the exercises conducted in the text fits naturally in the setup described hereafter. It is assumed that the welfare objective can be written as

$$L(y_{-1}) \equiv (1 - \beta) E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\},$$

where $L_t \equiv y_t' Q y_t$ is the per-period loss function, the vector y_t has dimension $(n \times 1)$ and contains all the endogenous variables (predetermined and not predetermined) and Q is a given $(n \times n)$ symmetric matrix. It is further assumed that the solution of the model can be written in vector auto-regression representation

$$y_t = A y_{t-1} + B \varepsilon_t, \tag{97}$$

for initial conditions $y_{-1} = 0$ and $i.i.d.$ shocks ε_t with mean zero. Given the solution [97], the per-period loss function can then be rewritten as

$$
\begin{aligned}
L_t &= (A y_{t-1} + B \varepsilon_t)' Q (A y_{t-1} + B \varepsilon_t) \\
&= y_{t-1}' A' Q A y_{t-1} + 2 \varepsilon_t' B' Q A y_{t-1} + \varepsilon_t' B' Q B \varepsilon_t.
\end{aligned}
$$

It then follows that the welfare objective can be rewritten as

$$L(y_{-1}) = (1 - \beta) E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(y_{t-1}' A' Q A y_{t-1} + \varepsilon_t' B' Q B \varepsilon_t \right) \right\}, \tag{98}$$

where the expectation of the cross-product simplifies to zero because of the *i.i.d.* property of the shocks.

Welfare will be evaluated from an ex-ante perspective. Hence, I integrate over the distribution of shocks at time zero and denote by $E\{\cdot\}$ the corresponding expectation. I analyze the two components of the welfare objective separately. I begin from the second component which is easier to solve. Let $\Omega \equiv E\{\varepsilon_t \varepsilon_t'\}$ be the variance-covariance matrix of the shocks. Then, it is straightforward to see that

$$(1-\beta)E\left\{\sum_{t=0}^{\infty} \beta^t \left(\varepsilon_t' B' Q B \varepsilon_t\right)\right\} = tr\left(B'QB\Omega\right).$$

Now, I turn to the first component of expression [98] above. I can again apply the *trace* operator and write

$$
\begin{aligned}
(1-\beta)E\left\{\sum_{t=0}^{\infty} \beta^t \left(y_{t-1}' A' Q A y_{t-1}\right)\right\} &= (1-\beta)E\left\{\sum_{t=0}^{\infty} \beta^t \left[tr\left(A'QAy_{t-1}y_{t-1}'\right)\right]\right\} \\
&= (1-\beta)tr\left\{\sum_{t=0}^{\infty} \beta^t \left[A'QAE\left(y_{t-1}y_{t-1}'\right)\right]\right\} \\
&= tr\left(A'QAJ\right),
\end{aligned}
$$

where

$$J = J\left(y_{-1}\right) \equiv E\left\{(1-\beta)\sum_{t=0}^{\infty} \beta^t y_{t-1}y_{t-1}'\right\}.$$

I can then rewrite J in recursive form using the zero initial condition as

$$
\begin{aligned}
J &= (1-\beta)y_{-1}y_{-1}' + \beta E\left\{(1-\beta)\sum_{t=0}^{\infty} \beta^t y_t y_t'\right\} \\
&= \beta E\left\{(1-\beta)\sum_{t=0}^{\infty} \beta^t \left(Ay_{t-1} + B\varepsilon_t\right)\left(Ay_{t-1} + B\varepsilon_t\right)'\right\} \\
&= \beta\left(AJA' + B\Omega B'\right).
\end{aligned}
$$

Using the properties of the *vec* operator (see Hamilton [1994], p.265), it is then possible solving for $vec\left(J\right)$ from the last equality

$$vec\left(J\right) = \left[I_{n^2} - \beta\left(A \otimes A\right)\right]^{-1} vec\left(\beta B \Omega B'\right).$$

The command "reshape" in Matlab allows to obtain the matrix J back. The value of welfare is then given by

$$E\left\{L\left(y_{-1}\right)\right\} = tr\left(A'QAJ\right) + tr\left(B'QB\Omega\right).$$

Expression $E\left\{L\left(y_{-1}\right)\right\}$ constitutes the basis for the welfare analysis conducted in the text.

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