EUROPEAN CENTRAL BANK

WORKING PAPER SERIES

NO. 474 / APRIL 2005

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A FORWARD LOOKING MODEL OF RATIONAL INFLATION INERTIA

by Kristoffer P. Nimark

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1 The author thanks Giuseppe Bertola, Giancarlo Corsetti, Fabio Ghironi, Michael Woodford, Stephen Wright and seminar participants at the ECB, European University Institute, Birkbeck College and Nuffield College for valuable comments on earlier drafts. The author is also grateful to the research department of the ECB for their hospitality during the period when this project was completed.

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ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

CONTENTS

Abstract

In this paper we derive a Phillips curve with a role for higher order expectations of marginal cost and future inflation. We introduce a small idiosyncratic component in firms' marginal costs and let the economywide average marginal cost be unobservable to the individual firm. The model can then replicate the backward looking component found in estimates of the 'Hybrid' New Keynesian Phillips Curve, even though the pricing decision of the firm is entirely forward looking. The Phillips curve derived here nests the standard New-Keynesian Phillips Curve as a special case. We take a structural approach to imperfect common knowledge that allow us to infer whether the assumed information imperfections necessary to replicate the data are quantitatively realistic or not. We also provide an algorithm for solving a class of models involving dynamic higher order expectations of endogenous variables.

Keywords: Calvo pricing, Higher order expectations, Imperfect Common Knowledge, New-Keynesian Phillips Curve

JEL classification: E00, E31 and E32.

Non-Technical Summary

In standard New-Keynesian models firms set prices to equal a mark up over expected marginal cost. The real marginal cost is determined by both exogenous and endogenous factors, where the exogenous factors are assumed to be common among all firms. While convenient from a modelling perspective, this assumption is clearly unrealistic. In this paper we relax the assumption of only common exogenous factors, by assuming a firm specific component in marginal costs. This does not only improve the realism of the model, but it can also help explain the stylized fact that in estimated so called 'hybrid' Phillips curves, i.e. when current inflation is regressed on expected and lagged inflation as well as current marginal cost, the coefficient on lagged inflation is usually found to be larger than zero and statistically significant. Inflation thus appears to be more persistent than marginal cost, its driving variable. The inertial character of inflation is at odds with the baseline New-Keynesian model, that predicts that the coefficient on the lagged inflation rate should be zero.

The apparent failure of the baseline New-Keynesian model to fit the data is well documented and has spurred economists to suggest explanations, often involving some type of mechanical indexation to past prices. Explanations of inflation inertia based on indexing are attractive since they admit relatively parsimonious representations of realistic inflation dynamics, but they all imply some type of non-rational behavior on the behalf of some firms. In the model presented here, the inertial behavior of inflation will be driven by *optimizing* pricesetters. The mechanism is the following. When there are firm specific components in marginal cost, individual firms do not know the marginal costs of other firms with certainty. This implies that firms cannot compute the aggregate price level before they choose their own price. The optimal nominal price of an individual good depends on the aggregate price level, so firms need to form an estimate of the aggregate price level to set the optimal relative price. Through the Calvo mechanism, there is a positive probability that a firm's price may be effective for more than one period. In addition to forming an estimate of the current aggregate price level, firms thus also need to form expectations of future aggregate price levels to set prices optimally. Firms have two sources of information: Their own marginal cost and the lagged price level. The larger the firm specific component of marginal cost is, the less accurate is a firm's own marginal cost as an indicator of current and future price levels and the more important is the observation of lagged inflation. With a persistent marginal cost process and strategic complementarities in nominal prices, lagged inflation will have a positive impact on current inflation though this 'information channel ', even when all firms are entirely forward looking. The variance of the firm specific component that is necessary to replicate the observed inflation inertia in the U.S. is about 1/5 (and smaller for the Euro area) of the overall variance of marginal cost, which we argue is not conspicuously unrealistic.

Apart from providing an explanation of inflation inertia, we also present a method to solve models where agents have private information and make dynamic choices in the presence of strategic complementarities. This may be of independent interest.

1. Introduction

In standard New-Keynesian models firms set prices to equal a mark up over expected marginal cost. The real marginal cost is determined by both exogenous and endogenous factors, where the exogenous factors are assumed to be common among all firms. While convenient from a modelling perspective, this assumption is clearly unrealistic. In this paper we relax the assumption of only common exogenous factors, by assuming an idiosyncratic component in firms' marginal costs. This does not only improve the realism of the model, but it can also help explain the stylized fact that in estimated 'hybrid' Phillips curves of the form

$$\pi_t = \gamma_f E_t \pi_{t+1} + \lambda mc_t + \gamma_b \pi_{t-1} \tag{1.1}$$

the coefficient on lagged inflation is usually found to be larger than zero and statistically significant. Inflation thus appears to be more persistent than marginal cost, its driving variable. The inertial character of inflation is at odds with the baseline New-Keynesian model, that predicts that γ_b should be zero and γ_f should be equal to the discount factor of the representative agent.[1]

The apparent failure of the baseline New-Keynesian model to fit the data is well documented and has spurred economists to suggest explanations, often involving some type of mechanical indexation to past prices.[2] For instance, Gali and Gertler (2001) suggest that a fraction of firms set the price of their own good equal to the previous periods average reset price plus the lagged inflation rate, while Christiano, Eichenbaum and Evans (2003) let a fraction of firms increase their own good prices with the lagged inflation rate. Both of these explanations of inflation inertia are attractive since they admit relatively parsimonious representations of realistic inflation dynamics, but they have been criticized for being ad hoc. In the present paper the inertial behavior of inflation will be driven by *optimizing* pricesetters. Under the assumption of imperfect common knowledge, individual firms find it optimal to use lagged inflation as an indicator of current and future prices of goods produced by competing firms. A strategic complementarity in prices then induces a positive relation between lagged and current inflation, even when the pricing decision of the firm is entirely forward looking.

The idea that incomplete adjustment of prices could be explained by information imperfections dates back to the Phelps-Lucas island model of the 1970's.[3] Recent papers by Mankiw and Reis (2002) and Woodford (2002) revive this idea, and show how limited information availability, or limited information processing capacities, can produce persistent real effects of nominal disturbances.[4] The model presented here adds two novel contributions to the literature that are worth highlighting.

First, through the Calvo mechanism of price adjustment the model can be made consistent with observed average price durations. As a consequence, expectations of future inflation will play a prominent role in determining today's inflation since there is a positive probability that a firm's price may be effective for more than one period. The dynamic structure of the pricing problem makes existing solution methods non-applicable and we derive a new algorithm to solve a class of dynamic linear models with imperfect common knowledge and strategic interaction. Second, while introducing information imperfections we explicitly specify what quantities that are observed by the individual firm. This allows us to make a first pass at

[1] See for instance Clarida, Gali and Gertler (1999).

[2] See for instance Fuhrer and Moore (1995), Gali and Gertler (2001), Gali, Gertler and Lopez-Salido (2003a, 2003b).

[3] See Phelps (1970) and Lucas (1972), (1973) and (1975).

[4] Variants of the Woodford (2002) framework include Amato and Shin (2003), Adam (2004) and Hellwig (2004).

the question of whether the assumed information imperfections are quantitatively realistic or not.

In the next section we derive a Phillips curve under the assumptions of imperfect common knowledge and Calvo pricing. In Section 3 we use two limit cases of marginal cost structures that preclude any private information, to show that the Phillips curve derived in Section 2 nests the standard New-Keynesian Phillips curve and how idiosyncratic components in firms' marginal cost introduces a backward looking element in the Phillips curve. In Section 4 we define the concept of *hierarchies of expectations* and the assumptions that will be imposed on these to solve the model. Section 5 presents and discusses the implied dynamics of inflation and higher order expectations under Calvo pricing and imperfect common knowledge when average marginal cost follows an exogenous process. Section 6 derives a simple general equilibrium model and compares the implied dynamics of the theoretical model with actual U.S. and Euro area estimates of the Hybrid New-Keynesian Phillips Curve. Section 7 concludes.

2. The Phillips Curve under Imperfect Common Knowledge and Calvo Pricing

In most (perhaps in all interesting) economies, one agent's optimal decision depends on the decisions of others. In an economy where all firms and agents are symmetric and all exogenous disturbances are common across firms and agents, knowing the actions of others is a trivial task. An agent can, by observing his own exogenous disturbance, infer the disturbances faced by everybody else and take action based on that information knowing that in equilibrium all agents will choose the same action. This is not possible in an economy with idiosyncratic exogenous shocks. Instead, each agent have to form an expectation of the other agents' actions based on what he can observe directly and on *collected* information. The expectation will be imperfect if the collection process adds noise to the observation or if it takes time. In the model below, monopolistically competitive firms set prices to average a constant mark up over real marginal cost and, as in Calvo (1983), there is an exogenous probability that the price might be effective for more than one period. Unlike standard New-Keynesian models though, firms face idiosyncratic marginal costs and thus cannot compute the current aggregate price level before it sets its own optimal price. Instead, firms will form an expectation of the price level based on the directly observable own marginal cost, and the lagged observation of the price level. In what follows, all variables are in log deviations from steady state values. Denote the real marginal cost of firm j at time t $mc_t(j)$ and let it be the sum of the economywide average marginal cost mc_t and the idiosyncratic component $\varepsilon_t(j)$

$$\begin{aligned} mc_t(j) &= mc_t + \varepsilon_t(j) \qquad (2.1) \\ \varepsilon_t(j) &\sim N\left(0, \sigma_\varepsilon^2\right) \quad \forall j \in (0,1). \end{aligned}$$

Average marginal cost mc_t follows some known, but unobservable, process. Firms cannot by direct observation distinguish between the idiosyncratic component $\varepsilon_t(j)$ and the economy wide average component mc_t, but will use their knowledge of the average marginal cost process

$$mc = \varrho(\cdot)$$

to filter out the economy wide component from the individual observation. The filtering problem faced by the individual firm is thus similar to the problem faced by the inhabitants of the market 'islands' in the Lucas (1975) paper, but with some differences. In the Lucas Island model, information is shared among agents between periods so that all agents share the same preobservation prior on the expected aggregate price change, while in our model no such information sharing ever occurs. Firms in our model will thus have different preobservation priors while in Lucas' model all islands share the same prior. Below, we will show that this complicates the filtering problem of the agents considerably.

Following Calvo (1983) there is a constant probability $(1-\theta)$ that a firm will reset its price in any given period. The price level then follows

$$p_t = \theta p_{t-1} + (1-\theta)p_t^* \tag{2.2}$$

where p_t^* is the average price chosen by firms resetting their price in period t

$$p_t^* = \int p_t^*(j) \quad dj$$

Firm j's optimal reset price is a discounted sum of firm j's expected future nominal marginal costs given by

$$p_t^*(j) = (1-\beta\theta)\, E_t(j) \left[\sum_{i=0}^{\infty} (\beta\theta)^i \left(p_{t+i} + mc_{t+i}(j)\right) \right] \tag{2.3}$$

where β is the firm's discount factor and $E_t(j) \equiv E_t\left[\cdot \mid I_t(j)\right]$ is firm j's expectations operator conditional on firm j's information set at time t I_t

$$I_t(j) = \left\{ mc_s(j), p_{s-1}, \varrho, \beta, \theta, \sigma_\varepsilon^2, \sigma_v^2 \mid s \leq t \right\}. \tag{2.4}$$

Each firm can observe its own marginal cost and the lagged aggregate price level. The structural parameters $\left\{\varrho, \beta, \theta, \sigma_\varepsilon^2, \sigma_v^2\right\}$ and the lagged price level p_{s-1} are common knowledge.

We can rewrite (2.3) as

$$p_t^*(j) = (1-\beta\theta)\, E_t(j) \left[p_t + mc_t(j)\right] + \beta\theta E_t(j) p_{t+1}^*(j) \tag{2.5}$$

and substitute the price level (2.2) into (2.5) to get

$$p_t^*(j) = (1-\beta\theta)\, E_t(j) \left[\theta p_{t-1} + (1-\theta)p_t^* + mc_t(j)\right] + \beta\theta E_t(j) p_{t+1}^*(j) \tag{2.6}$$

which shows that the optimal reset price of firm j depends on the firm j's expectation of the average current reset price. Firms thus have to form an expectation of the average reset price, which in turn depends on the average expectation of the average reset price, which in turn depends on the average expectation of the average expectation of the average reset price... and so on until infinity. We can apply this logic to the optimal reset price of the firm by taking averages across firms of the optimal reset price expression (2.5) and substituting it into (2.6) and then alternatingly substituting the price level (2.2) and the average reset price into each other. Rearranging the resulting expression allows us to write current inflation as a function of average higher order expectations (i.e. expectations of expectations of expectations...and so on) of current marginal cost and future inflation, plus a term that is a weighted difference between the higher order expectations of the price level and the actual price level.

$$\begin{aligned}
\pi_t \quad = \quad & \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(1-(1-\beta\theta)(1-\theta)\right) \sum_{k=0}^{\infty} \left((1-\beta\theta)(1-\theta)\right)^k mc_t^{(k)} \\
& +\beta\left(1-(1-\beta\theta)(1-\theta)\right) \sum_{k=0}^{\infty} \left((1-\beta\theta)(1-\theta)\right)^k \pi_{t+1}^{(k+1)} \\
& + (\theta-1)\,\beta \left[p_t - \left(1-(1-\beta\theta)(1-\theta)\right) \sum_{k=0}^{\infty} \left((1-\beta\theta)(1-\theta)\right)^k p_t^{(k+1)} \right].
\end{aligned} \tag{2.7}$$

A complete derivation of (2.7) is in the Appendix. We used the following notation convention for higher order expectations

$$\begin{aligned} x_t^{(0)} &\equiv x_t \\ x_t^{(1)} &\equiv \overline{E}_t [x_t] \\ x_t^{(2)} &\equiv \overline{E}_t \left[\overline{E}_t x_t\right] \\ x_t^{(k)} &\equiv \underbrace{\overline{E}_t[\overline{E}_t...\overline{E}_t}_{k\ times} [x_t]] \end{aligned}$$

where $\overline{E}_t$ is the average expectations operator

$$\overline{E}_t [x_t] \equiv \int E [x_t \mid I_t(j)] \; dj. \tag{2.8}$$

Note that the law of iterated expectations does not hold for expectations of average expectations when information sets differ across firms. When the kth order average expectation of a variable in period s is held in period t, we denote this by $x_{s|t}^{(k)}$, i.e.

$$\underbrace{\overline{E}_t[\overline{E}_t...\overline{E}_t}_{k\ times} [x_s]] \equiv x_{s|t}^{(k)}. \tag{2.9}$$

In (2.7) estimates of order k is weighted by $((1-\beta\theta)(1-\theta))^k$. Since $(1-\beta\theta)(1-\theta)$ is smaller than unity, the impact of expectations is decreasing as the order of expectation increases. One should also note that $(1-\beta\theta)(1-\theta)$ is decreasing in θ, i.e. higher order expectations are less important when prices are very sticky: When fewer firms change their prices in a given period, i.e. when θ is large, average expectations are less important for the firms that actually do change prices.

3. Two Limit Cases without Private Information

In the previous section we saw that individual firms need to estimate the economywide average marginal cost (and higher order estimates of marginal cost) to set its own price optimally. To do this, the individual firm uses its knowledge of the structure of the economy and the observations of the lagged price level and of its own marginal cost. The size of the variance of the idiosyncratic component relative to the size of the variance of the average marginal cost innovation determines how accurate firms' estimates will be. Two limit cases of this variance ratio can help intuition. When the variance of the idiosyncratic component is set to zero, we show that (2.7) nests the standard New-Keynesian Phillips curve. In the second case the variance of the idiosyncratic component is very large, and this will demonstrate how imperfect common knowledge introduces a link between past and current inflation. Both cases preclude any private information, and hence admit analytical solutions. In this section as well as in Section 5, we will make the simplifying assumption that average marginal cost is driven by the exogenous AR(1) process

$$\begin{aligned} mc_t &= \rho mc_{t-1} + \nu_t \\ \nu_t &\sim N(0, \sigma_\nu^2). \end{aligned} \tag{3.1}$$

This will facilitate the exposition, and in a later section we will show that the implied dynamics also carry over to a simple general equilibrium setting where marginal cost are determined by both exogenous and endogenous factors.

3.1. **Common Marginal Costs.** If we set the variance of the idiosyncratic component of firms' marginal costs equal to zero, i.e. $\sigma_\varepsilon^2 = 0$, it follows that

$$mc_t(j) = mc_t \ : \forall \ j \tag{3.2}$$

Since firms know the structure of the economy, (3.2) implies that there is no uncertainty of any order. Formally

$$mc_t^{(k)} = mc_t : \ k = 0, 1, 2, ...\infty. \tag{3.3}$$

Firms can now compute the current price level perfectly by substituting (3.3) into (2.7). We thus have

$$p_t^{(k)} = p_t : \ k = 0, 1, 2, ...\infty \tag{3.4}$$

and (2.7) is reduced to the standard New-Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} mc_t \tag{3.5}$$

where inflation is completely forward looking, with marginal cost as the driving variable. By repeated forward substitution can (3.5) be written as

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}(1-\rho\beta)^{-1} mc_t \tag{3.6}$$

which shows that inflation is only as persistent as marginal cost when the individual firm's own marginal cost is a perfect indicator of the economy wide average.

3.2. **Large Variance of Idiosyncratic Marginal Cost Component.** In this section we want to illustrate the consequences for inflation dynamics when the observation of a firm's own marginal cost holds no information about the economywide average. This is strictly true only when the variance of the idiosyncratic marginal cost component reaches infinity, but shocks with infinite variance prevents us from invoking the law of large numbers to calculate average marginal cost. For illustrative purposes we will temporarily give up on some mathematical rigor. In the following example the variance of the idiosyncratic component of a firm's marginal cost is supposed to be 'large enough' for the firm to discard its own marginal cost as an indicator of the economywide average. Instead, each firm uses only the common observation of the lagged price level to form an imperfect expectation of the economy wide average marginal cost. This structure is common knowledge and implies that there is some first order uncertainty about average marginal costs, but since the firms know that all other firms condition on the same information set, there is no higher order uncertainty. Formally

$$mc_t^{(1)} \neq mc_t \tag{3.7}$$
$$mc_t^{(k)} = mc_t^{(l)} : k, l > 0 \tag{3.8}$$

Substituting (3.7) and (3.8) into (2.7) yields

$$\begin{aligned}\pi_t = \ & \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(1-(1-\beta\theta)(1-\theta)\right) mc_t \\ & + \frac{(1-\theta)(1-\beta\theta)^2}{\theta} mc_t^{(1)} \\ & + \beta \overline{E}_t \pi_{t+1} + (\theta - 1)\,\beta\left[p_t - p_t^{(1)}\right] \end{aligned} \tag{3.9}$$

The first order expectation of marginal cost, $mc_t^{(1)}$, can be found by lagging (3.9) one period and solving for mc_{t-1}

$$\begin{aligned} mc_{t-1} &= \Psi^{-1}\pi_{t-1} \\ &\quad -\frac{(1-\theta)(1-\beta\theta)}{1-(1-\beta\theta)(1-\theta)} mc_{t-1|t-1}^{(1)} \\ &\quad -\Psi^{-1}\beta\overline{E}_{t-1}\pi_t - \Psi^{-1}(\theta-1)\beta\left[p_{t-1}-p_{t-1|t-1}^{(1)}\right] \end{aligned} \tag{3.10}$$

where

$$\Psi = \left[\frac{(1-\theta)(1-\beta\theta)}{\theta}(1-(1-\beta\theta)(1-\theta))\right].$$

Since all terms on the right hand side of (3.10) are known to all firms in period t, firms can 'back out' the previous period's average marginal cost perfectly and we have

$$mc_{t-1|t}^{(1)} = mc_{t-1}. \tag{3.11}$$

When the idiosyncratic component of marginal cost is very large, there is no information about the current innovation in the average marginal cost process in the observation of firms j's own marginal cost and the best estimate of the current average marginal cost is simply

$$mc_t^{(1)} = \rho mc_{t-1}. \tag{3.12}$$

Substituting the expression for the first order estimate of marginal cost into the inflation equation (3.9) gives

$$\begin{aligned} \pi_t &= \frac{(1-\theta)(1-\beta\theta)}{\theta}(1-(1-\beta\theta)(1-\theta))\, mc_t \\ &\quad +\frac{(1-\theta)(1-\beta\theta)}{\theta}\left[(1-\theta)(1-\beta\theta)+(1-\rho\beta)^{-1}\rho\right]\rho mc_{t-1} \\ &\quad +\beta(\theta-1)(1-\theta)(1-\beta\theta)\nu_t \end{aligned} \tag{3.13}$$

where we used the following relationships

$$\overline{E}_t\pi_{t+1} = \frac{(1-\theta)(1-\beta\theta)}{\theta}(1-\rho\beta)^{-1}\rho^2 mc_{t-1} \tag{3.14}$$

$$E_t mc_{t+1} = E_t mc_{t+1}^{(1)} \tag{3.15}$$

$$p_t - p_t^{(1)} = (1-\theta)(1-\beta\theta)\nu_t \tag{3.16}$$

(3.14) and (3.15) follow from the law of iterated expectations (which we can apply to the common first order expectation) and (3.16) follows from (B.8) in Appendix B. Comparing (3.6) with (3.13) demonstrates how relaxing the assumption of only common shocks introduces an element of backward looking behavior in the Phillips curve. Apart from current average marginal cost, inflation now also depends on lagged average marginal cost.

To make the role played by lagged inflation explicit, we can substitute the expression for the lagged marginal cost (3.10) into (3.9) to get an expression of current inflation as a function of lagged inflation of the form

$$\pi_t = \widehat{\gamma}_f\overline{E}_t\pi_{t+1} + \lambda_1 mc_t - \lambda_2 mc_{t-2} + \widehat{\gamma}_b\pi_{t-1} + \delta_1\nu_t - \delta_2\nu_{t-1}. \tag{3.17}$$

Using again the relations (3.14)-(3.16) and simplifying yields the coefficients

$$\widehat{\gamma}_f = \beta, \ \ \widehat{\gamma}_b = \frac{(1-\theta)(1-\beta\theta)}{(1-(1-\beta\theta)(1-\theta))}\rho \tag{3.18}$$

$$\lambda_1 = \frac{(1-\theta)(1-\beta\theta)}{\theta}(1-(1-\beta\theta)(1-\theta)) \tag{3.19}$$

$$\lambda_2 = \left(\frac{(1-\theta)(1-\beta\theta)}{(1-(1-\beta\theta)(1-\theta))}\right)^2\rho \tag{3.20}$$

$$\delta_1 = (\theta-1)(1-\theta)(1-\beta\theta)\beta \tag{3.21}$$

$$\delta_2 = \frac{(1-\theta)(1-\beta\theta)(\theta-1)}{(1-(1-\beta\theta)(1-\theta))}\beta\rho \tag{3.22}$$

The Phillips curve (3.17) demonstrates that lagged inflation matters for current inflation when the variance of the idiosyncratic marginal cost shocks are large enough for individual firms ' own marginal costs to be uninformative about the economy-wide average. Lagged inflation is then firms ' only source of information about the the marginal costs faced by other firms and all firms thus condition on the same information, which allow us to solve the model analytically. In the general case, when $0 < \sigma_\varepsilon^2 < \infty$, neither the lagged price level, nor the observation of a firms own marginal cost completely reveals neither the average marginal cost nor other agents estimates of average marginal cost. Firms will then use both observations to form higher order estimates of marginal costs and due to the Calvo mechanism, higher order estimates of future inflation.

4. Expectations and Private Information

In the previous section firms had no private information and firms' first and higher order expectations thus coincided. This is not true in the general case, and we will have to treat first and higher order expectations as separate objects. We will impose some structure on the expectations in order to solve the model. First we define the concept of a *hierarchy of expectations.* Let firm j's hierarchy of expectations from order l to m of variable x_t be defined by

$$\left\{x_t^{(k)}(j)\right\}_{k=l}^{m} \equiv \left\{x_t^{(l)}(j),\ x_t^{(l+1)}(j), ..., x_t^{(m-1)}(j),\ x_t^{(m)}(j)\right\}.$$

In the solution strategy we will follow, the hierarchy of expectations of current marginal cost will be treated as the 'fundamental' variable, or the state, while the hierarchy of inflation expectations will be treated as endogenous to the expectations of the 'fundamental' marginal cost hierarchy. We impose two assumptions on firms' hierarchies of inflation expectations.

Assumption 1: $E_t(j)\pi_{t+s} = E\left[\pi_{t+s} \mid I_t(j)\right] : j \in (0,1),\ s = 0,1,2,3,...\infty.$

Assumption 1 states that firm j's first order expectation of inflation should be equal to the mathematical expectation of inflation, given firm j's information set. This is a standard rationality assumption, imposing that firms do not make systematic mistakes, given their information sets.

Assumption 2: Let $F_s(j) : \mathbb{R}^\infty \rightarrow \mathbb{R}$ be a mapping from firm j's hierarchy of expectations of marginal cost to firm j's first order expectation of inflation s periods ahead. Then $F_s(j)\left(\left\{mc_t^{(k)}(j)\right\}_{k=l}^{\infty}\right) = E_t(j)\pi_{t+s}^{(l)} \Rightarrow F_s(j)\left(\left\{mc_t^{(k)}(j)\right\}_{k=l+m}^{\infty}\right) = \pi_{t+s|t}^{(l+m)}(j)\ :\ k,l,s = 1,2,3,...\infty.$

Assumption 2 states that the mapping from firm j's marginal cost hierarchy from order l to infinity to firm j's l^{th} order expectation of inflation is equivalent to the mapping from firm j's marginal cost expectation hierarchy from order $l+m$ to infinity to firm j's $l+m$ order expectation of inflation. In other words, firms believe that if other firms shared their hierarchy of marginal cost expectations, they would also share their hierarchy of inflation expectations. Taken together, Assumption 1 and 2 implies that the structural model is common knowledge, i.e. all firms believe that all firms use the true model to form expectations. Since we will work with a linear model, we can also impose Assumption 1 and 2 on average expectation hierarchies which will allow us to substitute out all terms involving inflation expectations in the Phillips curve (2.7) and get inflation as a function solely of the hierarchy of average marginal cost expectations. We define a hierarchy of average expectations as

$$\left\{x_t^{(k)}\right\}_{k=l}^{m} \equiv \left\{\int x_t^{(k)}(j)\ dj\right\}_{k=l}^{m}.$$

In the Phillips curve under imperfect common knowledge (2.7) inflation is determined by expectations of up to infinite order, which is problematic since we cannot solve the model using an infinite dimensional state representation. To obtain an arbitrarily good approximation, we will exploit the fact that the impact of expectations is decreasing as the order of expectation increases. Intuitively, the magnitude of a price setter's response to a unit change in his expectation of marginal cost, current or future inflation is decreasing as the order of expectation increases. In (2.7) this can be seen from the fact that the term raised to the power of the order of expectation k, $(1-\beta\theta)(1-\theta)^k$, is smaller than one. As k becomes large, this term approaches zero.

To solve the model, we will conjecture that inflation is a linear function of the hierarchy of average expectations up to the $\overline{k}^{th}$ order of average marginal cost (including $k = 0$, i.e. actual average marginal cost) and by choosing $\overline{k}$ large enough, an arbitrarily accurate solution can be found. The conjectured solution is in the following form

$$\pi_t = \mathbf{c}\begin{bmatrix} mc_t^{(0)} \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} \tag{4.1}$$

$$\begin{bmatrix} mc_t^{(0)} \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} = M\begin{bmatrix} mc_{t-1}^{(0)} \\ \vdots \\ mc_{t-1}^{(\overline{k})} \end{bmatrix} + \mathbf{n}v_t \tag{4.2}$$

where $\mathbf{c}$ is a $1\times\left(\overline{k}+1\right)$ row vector, M a $\left(\overline{k}+1\right)\times\left(\overline{k}+1\right)$ matrix, $\mathbf{n}$ a $\left(\overline{k}+1\right)\times 1$ column vector and ν_t is the innovation in the average marginal cost process. At this stage, we know two things about M. The first element of the first row of M is the AR(1) coefficient ρ and all other elements of the first row is zero. We also know that the largest eigenvalue of M has to lie within the unit circle for the model to be stable.

We use an iterative version of the method of undetermined coefficients to solve for $\mathbf{c}$, M and $\mathbf{n}$. In the first step we treat M and $\mathbf{n}$ as given and derive the implied $\mathbf{c}$ by imposing Assumption 1 and 2 on firms' inflation expectation hierarchies. In the second step we take $\mathbf{c}$ as given and calculate the implied M and $\mathbf{n}$ by letting firms estimate the average marginal cost hierarchy as a hidden process using the Kalman filter. The Kalman filter will play a dual role.[5] Not only will it be used by firms to estimate the average marginal cost expectation hierarchy, but since this hierarchy

[5]The Kalman filter plays a similar dual role in Woodford (2002). However, due to the forward looking nature of inflation in the present paper, we are not able to reduce the state to a 2x1 vector.

is made up of the average of the very same estimates, it will also determine the law of motion of the hierarchy, M and $\mathbf{n}$.

Solving the model thus implies finding a fixed point for $\mathbf{c}$, M and $\mathbf{n}$ and the details are given in the Appendix. After a solution has been found, we check whether adding one more order of expectations, i.e. increasing $\overline{k}$ by one and re-solving the model, changes the impact of a shock to marginal cost on inflation enough to motivate including higher orders of expectations. Once we are satisfied with the accuracy of our solution, we can simulate the model using (4.1) and (4.2).

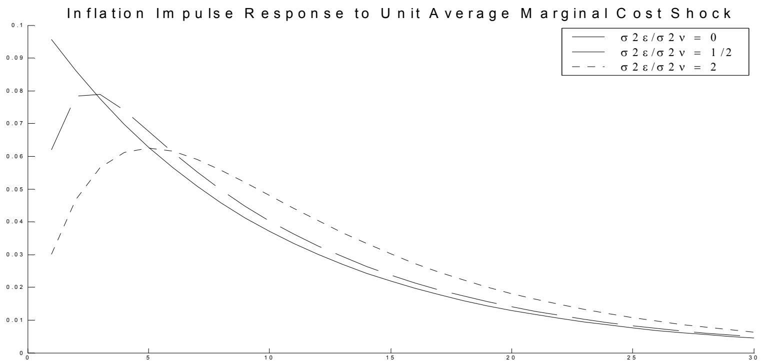
5. Inflation Dynamics with Private Information and Exogenous Marginal Cost

In the introduction we set out to show that relaxing the assumption of only common marginal costs can introduce endogenous inertia in inflation, i.e. that inflation becomes more persistent than marginal cost. In this section we will demonstrate this in a simple setting where marginal costs follows the exogenous process (3.1) and in the next section we will show that the dynamics implied by introducing idiosyncratic components will carry over to a simple general equilibrium setting where marginal cost is a function of both endogenous and exogenous factors.

The parameter determining how inertial inflation will be is the ratio between the variance of the idiosyncratic component of marginal cost and the variance of the innovation in the economy wide average marginal cost. This ratio determines how informative firms' own marginal cost is as an indicator for average marginal costs. In addition to these variances, we have to choose values for the parameters that govern the degree of price stickiness θ, the AR(1) coefficient of the average marginal cost process (3.1) ρ and firms' subjective discount factor β. Typically, data on CPI are available only with a one month delay and a period in our model should thus be interpreted as being one month. The model will be parameterized accordingly. Survey evidence suggests that average price duration is somewhere between 5 and 13 months.[6] In the benchmark specification, θ will be set to 0.9 which yields an average price duration of 10 months. For completeness, we will also show how inflation dynamics change as the degree of price stickiness is decreased. Firms subjective discount factor β is set to 0.995 and the persistence of marginal costs ρ is set to 0.9. Under this parameterization, $\overline{k} = 9$ is enough to achieve a precise solution: Adding another order of expectation changes the impact of a marginal cost shock on inflation by less than one thousandth of a percent.

5.1. **Inflation Dynamics and the Size of the Idiosyncratic Shocks.** Figure 1, where impulse responses of inflation subject to a unit shock in average marginal cost are plotted for different values of $\sigma_\varepsilon^2/\sigma_\nu^2$, illustrates how the ratio of the variances $\sigma_\varepsilon^2/\sigma_\nu^2$ affects the dynamics of inflation.

[6] See Carlton (1986) and Bils and Klenow (2002).



There are four things that are worth pointing out. First, with a zero idiosyncratic component variance, the model replicates the full information response, with monotonic convergence to the mean after the shock. Second, with non-zero idiosyncratic marginal cost component the response of inflation is hump shaped, with the peak of the hump appearing later the larger the ratio $\sigma_\varepsilon^2/\sigma_v^2$ is. Third, the larger this ratio is, the smaller is the first period impact of a marginal cost shock and the lower is inflation at the peak. Fourth, though inflation is lower initially and at the peak with a large $\sigma_\varepsilon^2/\sigma_v^2$, it is sufficiently more persistent to lie above the responses of inflation with lower variance ratios in the later periods of the graph. Since the underlying marginal cost shock in all three cases decreases monotonically, and in a shape identical to the inflation response with a zero variance ratio, the humps must be driven by the dynamics of the higher order estimates of marginal cost. Figure 2 and 3 below shows the dynamics of the hierarchy of marginal cost expectations up to $k = 3$ after a one unit shock to marginal cost for the two non-zero ratios of $\sigma_\varepsilon^2/\sigma_v^2$.

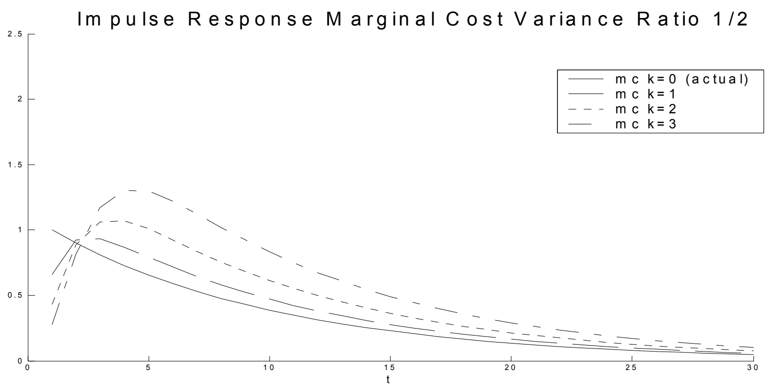


Figure 2

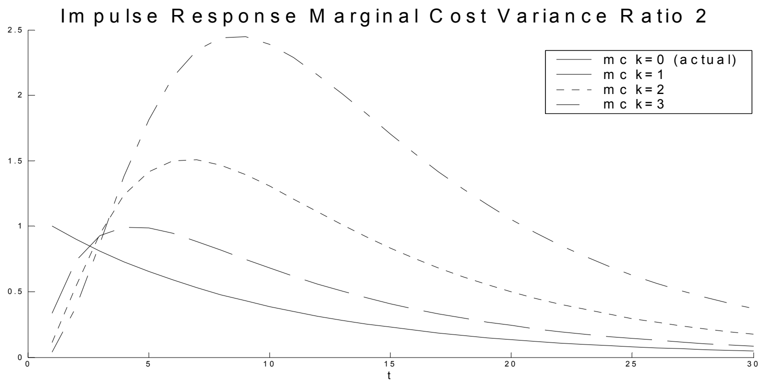


Figure 3

Figure 2 and 3 shows that first order estimates move less than the actual shock on impact, and that the larger the variance of the idiosyncratic component is, the smaller is the response of first order estimates on impact. The idiosyncratic component thus works as 'noise' in the filtering problem, that smooths out estimates of the innovations in the average marginal cost process. We can also see that higher order estimates move less than first order estimates. The key to understanding the dynamics of the higher order estimates is that *firms expect other firms to, on average, make the systematic mistake they do not believe that they made themselves.* Firms' first order estimate of average marginal cost is unbiased given their information set, but they know that on average, shocks are underestimated. Therefore, for a given change in first order expectations on impact, higher order expectations move less.

The delayed overshooting of higher over lower order expectations in Figure 2 and 3 can be understood with the same logic. In the second period, the impact period's price level becomes common knowledge and will be higher than expected, since the previous period shock was, on average, underestimated. This higher than expected lagged price level will be attributed partly to the impact of the higher order estimates but it will also cause firms to revise their estimates of the underlying marginal cost shock upwards. Again, firms will expect other firms to revise upwards more than the actual shock warrants, since they believe that others underestimated the impact period shock more than they did themselves. This effect would not be present without the endogenous signal that the lagged price level constitutes.

Of course, a negative unit shock to average marginal cost would trigger symmetric but negative responses to both inflation and expectations.

5.2. **Inflation Dynamics and the Degree of Price Stickiness.** As mentioned above, higher order expectations increases in importance as the we increase the number of firms that change prices in each period. Below we have plotted inflation impulse response to a unit average marginal cost shock for average price durations of 5 months (dashed line) and 10 months (solid line).

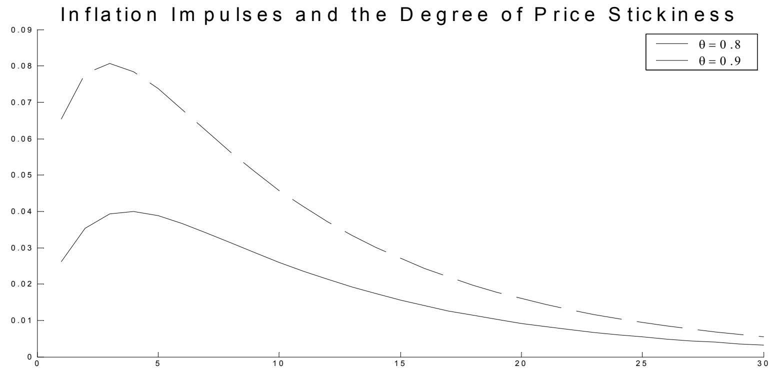


Figure 4

As a larger fraction of firms change their price in each period, inflation responds stronger to a shock. There are two effects at work. For a given individual price change, the fact that more firms change their prices of course causes a larger movement in the aggregate price level. Secondly, and less obvious, the magnitude of the individual firm's price change will also increase, since there is a strategic complementarity between the price of the individual good and the aggregate pricelevel.

6. Inflation Dynamics in a Simple General Equilibrium Model

In this section we will set up a simple general equilibrium model where marginal cost is determined by both endogenous and exogenous factors. The economy consists of households who supply labor and consume goods, firms that produce differentiated goods and set prices and a monetary policy authority that sets the nominal interest rate. The model is standard, except for the structure of the exogenous shocks. The Households experience a common, persistent but mean reverting, shock to their (dis)utility of supplying labor. Such a shock is estimated in a full information setting by Smets and Wouters (2003), and in our simple setting this shock is the only economywide disturbance. In addition to the common preference shock, firms experience idiosyncratic shocks to their wage bargaining skills that are uncorrelated across firms and time. This is meant to capture, in a stylized way, the empirical finding that a significant part of the variation in average wages at the firm level seem to be random.[7] Firms cannot observe wages paid at other firms, and thus have to form a hierarchy of average wage expectations. In what follows, lower case letters denote the log deviations from steady state values of the corresponding capital letter. The representative households maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \Xi_t \frac{{N_t}^{1+\varphi}}{1+\varphi} \right) \tag{6.1}$$

where N_t is the aggregate labor supply in period t and β is the discount rate. C_t is the the usual CES consumption aggregator

$$C_t = \left(\int_0^1 C_t\left(j\right)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{6.2}$$

[7] Martins (2003).

and Ξ_t is a shock to the disutility of providing labor services which in logs follow the AR(1) process

$$\xi_t = \rho\xi_{t-1} + \nu_t \tag{6.3}$$

$$\nu_t \sim N(0, \sigma_v^2) \tag{6.4}$$

Firm j produces the differentiated good $Y_t(j)$, using a linear technology with labor as the sole input.

$$Y_t(j) = N_t(j) \tag{6.5}$$

The absence of a storage technology and imposing market clearing implies that aggregate consumption will equal aggregate production

$$Y_t = C_t \tag{6.6}$$

where the standard CES aggregator was used again. The Euler equation of the representative household then implies the IS-equation

$$y_t = E_t\left[y_{t+1}\right] - \frac{1}{\gamma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) \tag{6.7}$$

where i_t is the nominal interest rate that follows the Taylor rule

$$i_t = \phi_\pi \pi_t + \phi_y y_t. \tag{6.8}$$

The marginal cost of firm j will be the real wage paid at firm j, which is determined by the intratemporal labor supply decision of the households

$$w_t - p_t - \gamma c_t - \varphi n_t - \xi_t = 0 \tag{6.9}$$

and a firm specific wage bargaining shock $\varepsilon_t(j)$. The bargaining shock introduces an idiosyncratic component to firms' marginal cost and firm $j's$ marginal cost will be

$$mc_t(j) = \gamma c_t + \varphi n_t + \xi_t + \varepsilon_t(j) \tag{6.10}$$

or substituting in (6.5) and (6.6)

$$mc_t(j) = (\gamma + \varphi)\, y_t + \xi_t + \varepsilon_t(j) \tag{6.11}$$

Firm j's marginal cost is thus determined by aggregate output y_t, the preference shock ξ_t and the idiosyncratic bargaining shock $\varepsilon_t(j)$.

The timing of the model is the following. Fist, the preference shock ξ_t is realized. Then, firms and households bargain over wages, where real wages are contracted in the form

$$w_t(j) - p_t = (\gamma + \varphi)\, y_t + \omega_t(j) \tag{6.12}$$

where $\omega_t(j) = \xi_t + \varepsilon_t(j)$. Firms cannot by direct observation distinguish between the shock to preferences and the firm specific bargaining shock, but only observe the sum of the two, $\omega_t(j)$, and the component dependent on output, $(\gamma + \varphi)\, y_t$. The latter can be interpreted as a contract specifying higher hourly wages for (aggregate) overtime. Firms set prices before production takes place, and due to the overtime premium firms do not know their own marginal cost with certainty when prices are chosen, but have to form an expectation of what the aggregate output level will be. As before, they will also need to form higher order expectations of current marginal cost and current and future price levels. When prices are set, households choose labor supply and consumption simultaneously with the determination of the interest rate. It is natural to assume that households know the preference shock with certainty, and we will further assume that there is no information sharing between households and firms. Firm j's information set is thus defined by

$$I_t(j) = \left\{\omega_t(j), p_{s-1}, y_{s-1}, \varrho, \beta, \theta, \gamma, \varphi, \sigma_\varepsilon^2, \sigma_v^2 \mid s \leq t\right\}. \tag{6.13}$$

The state of the economy is firms ' hierarchy of expectations of the preference shock ξ_t. Similar to the previous section, we conjecture a solution in the form

$$\pi_t = \widetilde{\mathbf{c}} \begin{bmatrix} \xi_t^{(0)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} \tag{6.14}$$

$$y_t = \widetilde{\mathbf{d}} \begin{bmatrix} \xi_t^{(0)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} \tag{6.15}$$

$$\begin{bmatrix} \xi_t^{(0)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} = \widetilde{M} \begin{bmatrix} \xi_{t-1}^{(0)} \\ \vdots \\ \xi_{t-1}^{(\overline{k})} \end{bmatrix} + \widetilde{\mathbf{n}} v_t \tag{6.16}$$

$\widetilde{\mathbf{c}}$ is now a function of $\widetilde{\mathbf{d}}$, since firms set prices based on their beginning-of-period estimate of their own marginal cost

$$E_t(j) mc_t(j) = (\gamma + \varphi) \widetilde{\mathbf{d}}_{1:\overline{k}} \begin{bmatrix} \xi_t^{(1)}(j) \\ \vdots \\ \xi_t^{(\overline{k})}(j) \end{bmatrix} + \omega_t(j) \tag{6.17}$$

Output is chosen by the perfectly informed households who form rational expectations (in the standard sense) of future inflation and interest rates. The preference shock and firms' hierarchy of expectations are treated as $\overline{k}+1$ predetermined state variables and y_t as a forward looking jump variable. This introduces an additional step in the solution algorithm, and is described in detail in the Appendix.

6.1. **Simulating the Model.** Once we have the model in the form of (6.14), (6.15) and (6.16) we can compare the impulse responses of inflation in general equilibrium with the impulse responses from the previous section where marginal costs were entirely exogenous. The following parameterization was used $\{\rho, \theta, \gamma, \varphi, \beta\} = \{.9, .9, 4, 5, .995\}$. Figure 5 plots the impulse response of inflation to a unit shock to preferences.

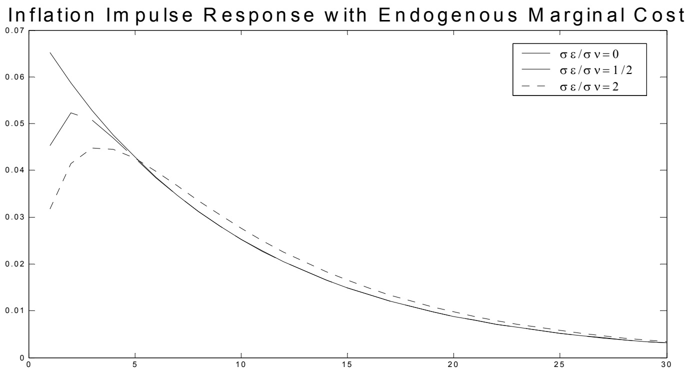


Figure 5

Figure 4 shows that the responses of inflation to a preference shock in the general equilibrium model is similar to the responses to the exogenous marginal cost shock from the previous section. The main difference is that the impulse responses with non-zero variance ratios between the idiosyncratic component and the economy wide component lie closer to the perfect information zero ratio curve. This is due to the additional information now available to firms through the observation of lagged output, which is partly determined by the perfectly informed households. This makes firms hierarchy of expectations converge quicker to the true value of the shock.

Not surprisingly, output responds negatively to a shock to the disutility of supplying labor. This is illustrated in Figure 6, where the responses of output to a unit shock to preferences is illustrated for the usual values of the idiosyncratic bargaining shock variance.

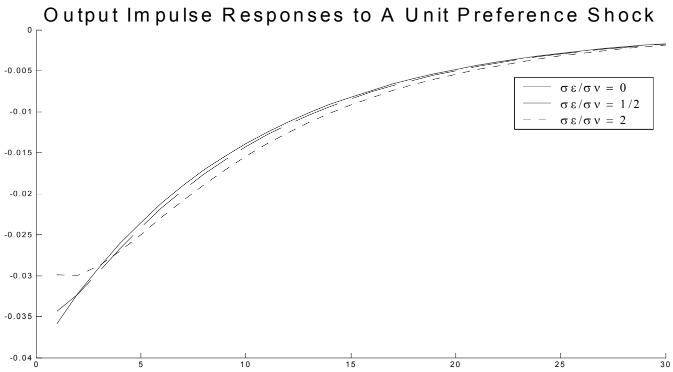


Figure 6

Households, having fully informed rational expectations, correctly anticipate the hump shaped responses of inflation under non-zero bargaining shocks. This reduces households' expected real interest rates in the Euler equation (6.7) and makes output respond less negatively to the preference shock in the first couple of periods after impact.

6.2. **The Model and U.S. and Euro Area Inflation Dynamics.** In the preceding sections we have presented qualitative evidence in the form of hump shaped impulse responses on how imperfect common knowledge can introduce inflation inertia. In this section we ask the question whether our model can account for the observed inflation inertia in U.S. and Euro area data, with quantitatively realistic amounts of information imperfections. We will pursue this question by generating data from our simple general equilibrium model and estimate the Hybrid New Keynesian Phillips Curve (1.1) by GMM. If our model is the true data generating process, then the Hybrid New Keynesian Phillips Curve is of course misspecified. The experiment we perform here is thus to check whether the misspecified econometric model (1.1) applied to our theoretical model would produce results similar to those obtained when the Hybrid New Keynesian Phillips Curve is estimated on actual data. Gali, Gertler and Lopez-Salido (2003b) provide a range of estimates for the U.S. and the Euro area, using slightly different choices of instruments and formulations of the orthogonality condition. The estimates of the backward looking parameter γ_b ranges from 0.035 to 0.27 for the Euro area and from 0.32 to 0.36 for the U.S. A robust feature across methodologies is that the estimated inflation inertia is lower in the Euro area than in the U.S.

The table below displays estimates of the Hybrid New Keynesian Phillips Curve (1.1) using simulated data from our theoretical model for different ratios of the variance of the bargaining shock and the innovation in the preference shock process.[8]

$\sigma_\varepsilon^2/\sigma_v^2$	γ_b	γ_f	λ
0.1	0.24	0.76	0.015
0.5	0.32	0.68	0.014
2	0.43	0.57	0.009

The simulated data was transformed to "quarterly" frequencies by taking three period averages and the orthogonality condition

$$E_t\left[\pi_t - (1-\gamma_b)\pi_{t+1} - \gamma_b\pi_{t-1} - \lambda mc_t\right] = 0$$

was then estimated by GMM using lagged marginal costs and inflation rates as instruments. To approximately replicate the inertia in U.S. inflation it is enough with an innovation variance ratio of 1/2. One should note that this implies a significantly smaller ratio of the bargaining shock over the actual marginal cost (and thus average real wage) variance, which is a function of the output gap variance

$$\sigma_{mc}^2 = (\gamma + \varphi)^2 \sigma_{\widehat{y}}^2$$

where the output gap $\widehat{y}_t$ is defined as the difference between actual output y_t and the level of output that would prevail under flexible prices $\overline{y}_t$

$$\begin{aligned}\widehat{y}_t &= y_t - \overline{y}_t \\ \overline{y}_t &= -\frac{\xi_t}{\gamma+\varphi}\end{aligned}$$

The variance of the output gap in turn is given by

$$\sigma_{\widehat{y}}^2 = \left[\widetilde{\mathbf{d}} + \frac{1}{\gamma+\varphi}\mathbf{e}_1\right]\Sigma_{\xi\xi}\left[\widetilde{\mathbf{d}} + \frac{1}{\gamma+\varphi}\mathbf{e}_1\right]'$$

where $\mathbf{e}_1 = \left[\begin{array}{cc} 1 & \mathbf{0}_{1x\overline{k}} \end{array}\right]$ and $\Sigma_{\xi\xi}$ is the variance of the preference shock hierarchy given by the solution to the discrete Lyaponov equation

$$\Sigma_{\xi\xi} = \widetilde{M}\Sigma_{\xi\xi}\widetilde{M}' + \widetilde{\mathbf{n}}\widetilde{\mathbf{n}}'$$

Plugging in $\sigma_\varepsilon^2/\sigma_v^2 = 1/\ 2$ in the benchmark specification yields variance ratio of $\sigma_\varepsilon^2/\sigma_{mc}^2 = 0.21$. This implies that the variance of idiosyncratic bargaining shock only have to be about 1/5 of the average real wage variance to generate the observed U.S. inertia.

7. Conclusions

In this paper we have argued that when firms have idiosyncratic components in their marginal cost, they cannot compute the current price level perfectly before they choose their own optimal price. Instead firms have to form an estimate of the price level using the information contained in their own marginal cost and observations of past inflation. This structure, coupled with the Calvo mechanism of price adjustment, results in a Phillips curve with a role for higher order expectations of marginal cost and future inflation. Even though the pricing decision is entirely forward looking, lagged inflation will still have an impact on current inflation since lagged inflation contains information on three quantities relevant for the optimal price of the firm: (i) The current average marginal cost, (ii) the current average marginal cost expectation hierarchy and (iii) the hierarchy of expected future prices. We show that this 'information effect' can explain the positive coefficient on lagged inflation in estimates of the so called Hybrid New-Keynesian Phillips Curve.

[8]300 "monthly" observations was transformed into 100 "quarterly" observations. The estimates displayed in the table are averages over 20 independent samples.

We solve the model by imposing that all firms have rational expectations given their information set and that the structure of the economy is common knowledge. These two assumptions, together with a structural model that implies that the impact of higher order expectations are decreasing as the order of expectation increases, allow us to derive a solution algorithm of arbitrary accuracy. The complexity of the solution of the model presented here is naturally a drawback. Future research should focus on deriving reduced forms of the model and specifying under what changes in the environment they would be robust predictors of pricing behavior.

We also propose a new approach to modeling imperfect common knowledge. Instead of working with noise and signals, we let the agents observe real, but partly idiosyncratic quantities perfectly, as well as lagged aggregate variables. The variance of the idiosyncratic component determines how precise these quantities are as indicators of economywide averages. This approach allows us to compare variances of the model with variances in the data and ask whether information imperfections are likely to be large enough in reality to be important for the dynamics of the model. It is hard to argue that these observations are the only information available to agents in reality. However, a *necessary* condition for limited information availability (or limited capacity to process information) based explanations of economic phenomena to be plausible, is that quantities that are immediately and costlessly observable to agents are not too informative. In the specific case considered here, we found that the firm level idiosyncratic wage variances necessary to replicate U.S. inflation dynamics is about 1/5 of the overall variance of real wages. Quantitative information on the magnitude of unexplained firm level variations in real wages are hard to come by, but there are some studies where this information can be extracted as a by-product. Martins (2003) investigates the competitiveness of the Portuguese garment industry labor market using yearly data, and finds that between 30 and 40 percent of the firm average wage variations cannot be explained by neither labor market conditions, changes in the skills of workers, production techniques or (time dependent) firm level fixed effects. Though not necessarily representative, it is not obvious if this is likely to be lower or higher than in other industries and countries and a variance ratio of 1/5 is not conspicuously unrealistic. Our model also suggests an explanation for the observed higher inflation inertia in the U.S. relative to the Euro area. European wage bargaining is often centralized, while in the U.S. a larger fraction of wages are set at the firm level.[9] There is thus likely to be more firm level variations in wages in the U.S. than in Europe, which in our model would lead to more inertia, and could thus explain the observed differences. The relation between the degree of wage centralization and inflation inertia, as well as actual magnitudes of random variations in firm level wages will be investigated in future work.

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Appendix A. The Optimal Reset Price of a Firm

Firm j resetting its price in period t maximizes

$$E_t(j)\sum_{i=0}^{\infty}(\theta\beta)^i\left[\frac{P_t(j)}{P_{t+i}}Y_{t+i}(j)-MC_{t+i}(j)Y_{t+i}(j)\right] \tag{A.1}$$

subject to the demand constraint

$$Y_{t+i}(j)=\frac{P_t(j)}{P_{t+i}}Y_{t+i} \tag{A.2}$$

where

$$Y_t=\left(\int_0^1 Y_t\left(j\right)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.3}$$

and

$$P_t=\left(\int_0^1 P_t\left(j\right)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}}. \tag{A.4}$$

Substituting (A.2) into (A.1) and taking derivatives w.r.t $P_t(j)$ gives the first order condition

$$E_t(j)\sum_{i=0}^{\infty}(\theta\beta)^iY_{t+i}\left[\frac{1-\epsilon}{P_{t+i}}\left[\frac{P_t^*(j)}{P_{t+i}}\right]^{-\epsilon}-MC_{t+i}(j)\frac{\epsilon}{P_{t+i}}\left[\frac{P_t^*(j)}{P_{t+i}}\right]^{-\epsilon-1}\right]=0 \tag{A.5}$$

Rearranging and simplifying yields

$$P_t^*(j)E_t(j)\left[\sum_{i=0}^{\infty}(\theta\beta)^iY_{t+i}P_{t+i}^{\epsilon-1}\right]=\left(1+\mu\right)E_t(j)\left[\sum_{i=0}^{\infty}(\theta\beta)^iMC_{t+i}(j)P_{t+i}Y_{t+i}P_{t+i}^{\epsilon-1}\right] \tag{A.6}$$

where

$$(1+\mu)=\frac{\epsilon}{\epsilon-1}.$$

Log linearize

$$\left[\sum_{i=0}^{\infty}(\theta\beta)^i\right](p_t^*(j)-p_t)+\sum_{i=0}^{\infty}(\theta\beta)^i\left[y_{t+i}+(\epsilon-1)p_{t+i}\right] \tag{A.7}$$

$$=\sum_{i=0}^{\infty}(\theta\beta)^i\left[p_{t+i}+mc_{t+i}+y_{t+i}+(\epsilon-1)p_{t+i}\right]$$

and simplify

$$p_t^*(j)=\left(1-\beta\theta\right)E_t(j)\sum_{i=0}^{\infty}\left(\beta\theta\right)^i\left(p_{t+i}+mc_{t+i}(j)\right) \tag{A.8}$$

Appendix B. Deriving a Forward Looking Phillips Curve with Imperfect Common Knowledge

Let the price level follow

$$p_t=\theta p_{t-1}+(1-\theta)p_t^* \tag{B.1}$$

where p_t^* is the average price chosen by firms resetting their price in period t. The optimal price of firm j is a discounted sum of firm j's current and future nominal marginal costs given by

$$p_t^*(j)=\left(1-\beta\theta\right)E_t(j)\sum_{i=0}^{\infty}\left(\beta\theta\right)^i\left(p_{t+i}+mc_{t+i}(j)\right) \tag{B.2}$$

Rewrite as

$$p_t^*(j) = (1-\beta\theta)\, E_t(j)\, (p_t + mc_t(j)) + E_t(j)\beta\theta p_{t+1}^*(j) \tag{B.3}$$

To set the optimal price, firm j need to form an estimate of the price level. Substitute (B.1) into(B.3) to get

$$p_t^*(j) = (1-\beta\theta)\, E_t(j)\, ([\theta p_{t-1} + (1-\theta)p_t^*] + mc_t(j)) + E_t(j)\beta\theta p_{t+1}^*(j) \tag{B.4}$$

where the average reset price p_t^* is

$$p_t^* = (1-\beta\theta)\, \overline{E}_t\, (p_t + mc_t) + \overline{E}_t\beta\theta p_{t+1}^* \tag{B.5}$$

Repeated substitution of (B.5) and (B.1) into (B.4) yields

$$\begin{aligned} p_t^*(j) \quad = \quad & (1-\beta\theta)\, E_t(j)((\theta p_{t-1} + \\ & (1-\theta)\,(1-\beta\theta)\, \overline{E}_t(\theta p_{t-1} + \\ & (1-\theta)\,(1-\beta\theta)\, \overline{E}_t\, (\theta p_{t-1} + (1-\theta)p_t^* + mc_t) \\ & + \overline{E}_t\beta\theta p_{t+1}^* + mc_t) + \overline{E}_t\beta\theta p_{t+1}^*) \\ & + mc_t(j)) + E_t(j)\beta\theta p_{t+1}^*(j) \end{aligned} \tag{B.6}$$

Continued substitution and averaging across firms yields

$$\begin{aligned} p_t^* \quad = \quad & (1-\beta\theta)\sum_{k=0}^{\infty}((1-\beta\theta)(1-\theta))^k\, mc_t^{(k)} + \\ & + \frac{(1-\beta\theta)\theta}{1-((1-\theta)(1-\beta\theta))}p_{t-1} + \theta\beta\sum_{k=0}^{\infty}((1-\beta\theta)(1-\theta))^k\, p_{t+1|t}^{*(k+1)} \end{aligned} \tag{B.7}$$

Substitute (B.7) into (B.1) to get

$$\begin{aligned} p_t \quad = \quad & (1-\theta)(1-\beta\theta)\sum_{k=0}^{\infty}((1-\beta\theta)(1-\theta))^k\, mc_t^{(k)} + \\ & + \left(\theta + \frac{(1-\theta)(1-\beta\theta)\theta}{1-((1-\theta)(1-\beta\theta))}\right)p_{t-1} + (1-\theta)\,\theta\beta\sum_{k=0}^{\infty}((1-\beta\theta)(1-\theta))^k\, p_{t+1}^{*(k)} \end{aligned} \tag{B.8}$$

First, note that

$$\begin{aligned} \left(\theta + \frac{(1-\theta)(1-\beta\theta)\theta}{1-((1-\theta)(1-\beta\theta))}\right) \quad & = \quad \frac{\theta\,(1-((1-\theta)(1-\beta\theta))) + \theta(1-\theta)(1-\beta\theta)}{1-((1-\theta)(1-\beta\theta))} \\ & = \quad \frac{1}{1+\beta-\theta\beta} \end{aligned} \tag{B.9}$$

then add $\frac{1}{1+\beta-\theta\beta}p_t$ and subtract $\frac{1}{1+\beta-\theta\beta}p_{t-1}$ and p_t to/from both sides to get

$$\begin{aligned} \frac{1}{1+\beta-\theta\beta}(p_t - p_{t-1}) \quad = \quad & (1-\theta)(1-\beta\theta)\sum_{k=1}^{\infty}((1-\beta\theta)(1-\theta))^{k-1}\, mc_t^{(k)} + \\ & + \left(\frac{1}{1+\beta-\theta\beta} - 1\right)p_t \\ & + (1-\theta)\,\theta\beta\sum_{k=0}^{\infty}((1-\beta\theta)(1-\theta))^k\, p_{t+1|t}^{*(k+1)} \end{aligned} \tag{B.10}$$

Divide through by $\frac{1}{1+\beta-\theta\beta}$

$$\begin{aligned}(p_t - p_{t-1}) &= (1-\theta)(1-\beta\theta)\sum_{k=0}^{\infty}(1+\beta-\theta\beta)\left((1-\beta\theta)(1-\theta)\right)^k mc_t^{(k)} \\ &\quad + (1-1-\beta+\beta\theta)\,p_t \qquad\qquad \text{(B.11)} \\ &\quad + (1-\theta)\,\theta\beta\sum_{k=0}^{\infty}(1+\beta-\theta\beta)\left((1-\beta\theta)(1-\theta)\right)^k p_{t+1|t}^{*(k+1)}\end{aligned}$$

simplify and add and subtract

$$\beta\,(\theta-1)\,(1-(1-\beta\theta)(1-\theta))\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k p_t^{(k+1)}$$

and use that $\theta\left((1+\beta-\theta\beta)\right) = 1-(1-\beta\theta)(1-\theta)$ to get

$$\begin{aligned}\pi_t &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\,(1-(1-\beta\theta)(1-\theta))\sum_{k=1}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^{k-1} mc_t^{(k)} \\ &\quad + (\theta-1)\,\beta p_t \qquad\qquad \text{(B.12)} \\ &\quad +\beta\,(1-\theta)\,(1-(1-\beta\theta)(1-\theta))\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k p_{t+1|t}^{*(k+1)} \\ &\quad +\beta\,(\theta-1)\,(1-(1-\beta\theta)(1-\theta))\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k p_t^{(k+1)} \\ &\quad -\beta\,(\theta-1)\,(1-(1-\beta\theta)(1-\theta))\,E_t\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k p_t^{(k+1)}\end{aligned}$$

Inflation can now be rewritten as a function of higher order expectations of current marginal cost and inflation plus an error term that is a sum of the discrepancies of the higher order beliefs of the pricelevel and the actual pricelevel

$$\begin{aligned}\pi_t &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\,(1-(1-\beta\theta)(1-\theta))\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k mc_t^{(k)} \\ &\quad +\beta\,(1-(1-\beta\theta)(1-\theta))\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k \pi_{t+1|t}^{(k+1)} \qquad\qquad \text{(B.13)} \\ &\quad + (\theta-1)\,\beta\left[p_t - (1-(1-\beta\theta)(1-\theta))\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k p_t^{(k+1)}\right]\end{aligned}$$

where we used

$$\theta p_t + (1-\theta)E_t p_{t+1}^* - p_t = (\theta-1)\,p_t + (1-\theta)E_t p_{t+1}^* = E_t\pi_{t+1}$$

This is equation (2.7) in Section 2 of the main text.

To write the model in the form of (C.1) use that

$$p_t = p_{t-1} + \pi_t. \qquad\qquad \text{(B.14)}$$

Substitute (B.14) into (B.13) to get

$$\begin{aligned}\pi_t &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(1-(1-\beta\theta)(1-\theta)\right)\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k mc_t^{(k)} \\ &\quad +\beta\left(1-(1-\beta\theta)(1-\theta)\right)\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k \pi_{t+1|t}^{(k+1)} \\ &\quad +(\theta-1)\beta[(\theta p_{t-1}+\theta\pi_t)- \\ &\quad \left(1-(1-\beta\theta)(1-\theta)\right)\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k\left(p_{t-1}^{(k+1)}+\pi_t^{(k+1)}\right)].\end{aligned} \tag{B.15}$$

Collect all terms with actual period t inflation on the left hand side and use that the lagged pricelevel is common knowledge

$$\begin{aligned}&(1-(\theta-1)\beta)\pi_t = \\ &\frac{(1-\theta)(1-\beta\theta)}{\theta}\left(1-(1-\beta\theta)(1-\theta)\right)\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k mc_t^{(k)} \\ &+\beta\left(1-(1-\beta\theta)(1-\theta)\right)\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k\pi_{t+1|t}^{(k+1)} \\ &-(\theta-1)\beta\left[\left(1-(1-\beta\theta)(1-\theta)\right)\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k\pi_t^{(k+1)}\right].\end{aligned} \tag{B.16}$$

Divide both sides with $(1-(\theta-1)\beta)$

$$\begin{aligned}\pi_t &= \frac{(1-\theta)(1-\beta\theta)}{(1-(\theta-1)\beta)\theta}\left(1-(1-\beta\theta)(1-\theta)\right)\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k mc_t^{(k)} \\ &\quad +\frac{\beta\left(1-(1-\beta\theta)(1-\theta)\right)}{(1-(\theta-1)\beta)}\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k\pi_{t+1|t}^{(k+1)} \\ &\quad -\frac{(\theta-1)\beta}{(1-(\theta-1)\beta)}\left[\left(1-(1-\beta\theta)(1-\theta)\right)\sum_{k=0}^{\infty}\left((1-\beta\theta)(1-\theta)\right)^k\pi_t^{(k+1)}\right]\end{aligned} \tag{B.17}$$

The Phillips curve (B.17) is now in the required form for the solution algorithm of Appendix C.

Appendix C. Solution with Exogenous Marginal Cost

C.1. **Step 1: The Impact of Expectations on Inflation.** In the first part of the solution we will use the two consistency restrictions to eliminate expectation hierarchies of the endogenous inflation rate from the Phillips curve (2.7). First, use that

$$p_t = p_{t-1} + \pi_t$$

to rewrite the Phillips curve (2.7) in the following form

$$\pi_t = \mathbf{f}\begin{bmatrix} mc_t^{(0)} \\ \vdots \\ mc_t^{(\overline{k})}\end{bmatrix} + \mathbf{g}\begin{bmatrix}\pi_{t+1|t}^{(1)} \\ \vdots \\ \pi_{t+1|t}^{(\overline{k})}\end{bmatrix} + \mathbf{h}\begin{bmatrix}\pi_{t|t}^{(1)} \\ \vdots \\ \pi_{t|t}^{(\overline{k})}\end{bmatrix} \tag{C.1}$$

where $\mathbf{f}, \mathbf{g}$ and $\mathbf{h}$ are row vectors of appropriate dimensions. Denote the elements of the row vectors as $\begin{bmatrix} f_1 & f_2 & \cdots & f_{\overline{k}} \end{bmatrix}$ etc., where the respective elements are given by

$$f_k = \frac{(1-\theta)(1-\beta\theta)}{(1-(\theta-1)\beta)\theta}(1-(1-\beta\theta)(1-\theta))[(1-\beta\theta)(1-\theta)]^{k-1} \tag{C.2}$$

$$g_k = \frac{\beta(1-(1-\beta\theta)(1-\theta))}{(1-(\theta-1)\beta)}[(1-\beta\theta)(1-\theta)]^{k-1} \tag{C.3}$$

$$h_k = -\frac{(\theta-1)\beta}{1-(\theta-1)\beta}(1-(1-\beta\theta)(1-\theta))[(1-\beta\theta)(1-\theta)]^{k-1} \tag{C.4}$$

One should not that the elements of (C.2), (C.3) and (C.4) decreases as k increases since $0 < (1-\beta\theta)(1-\theta) < 1$. Use the rationality assumptions to move the hierarchy of contemporaneous inflation expectations in (C.1) to the left hand side

$$\begin{bmatrix} 1 & -h_1 & \cdots & -h_{\overline{k}-1} \\ 0 & 1 & -h_1 & \vdots \\ 0 & 0 & 1 & -h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t^{(0)} \\ \vdots \\ \vdots \\ \pi_t^{(\overline{k})} \end{bmatrix} = \tag{C.5}$$

$$\begin{bmatrix} f_1 & f_2 & \cdots & f_{\overline{k}} \\ 0 & f_1 & f_2 & \vdots \\ 0 & 0 & f_1 & f_2 \\ 0 & 0 & 0 & f_1 \end{bmatrix} \begin{bmatrix} mc_t^{(0)} \\ \vdots \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} + \begin{bmatrix} g_1 & g_2 & \cdots & g_{\overline{k}} \\ 0 & g_1 & g_2 & \vdots \\ 0 & 0 & g_1 & g_2 \\ 0 & 0 & 0 & g_1 \end{bmatrix} \begin{bmatrix} \pi_{t+1|t}^{(1)} \\ \vdots \\ \vdots \\ \pi_{t+1|t}^{(\overline{k}+1)} \end{bmatrix}$$

Denote the left hand side coefficient matrix H. Pre-multiply with the inverse of H to get

$$\begin{bmatrix} \pi_t^{(0)} \\ \vdots \\ \vdots \\ \pi_t^{(\overline{k})} \end{bmatrix} = H^{-1}F \begin{bmatrix} mc_t^{(0)} \\ \vdots \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} + H^{-1}G \begin{bmatrix} \pi_{t+1|t}^{(1)} \\ \vdots \\ \vdots \\ \pi_{t+1|t}^{(\overline{k}+1)} \end{bmatrix}. \tag{C.6}$$

Inflation can now be written as a function of only the hierarchies of expectations of current marginal cost and future inflation

$$\pi_t = \mathbf{a} \begin{bmatrix} mc_t^{(0)} \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} + \mathbf{b} \begin{bmatrix} \pi_{t+1|t}^{(1)} \\ \vdots \\ \pi_{t+1|t}^{(\overline{k})} \end{bmatrix} \tag{C.7}$$

where $\mathbf{a}$ and $\mathbf{b}$ are the first rows of $H^{-1}F$ and $H^{-1}G$ respectively.

By imposing the consistency assumption, any order of expected inflation can be written as

$$\pi_{t+s|t}^{(k)} = \mathbf{a}_{1:\overline{k}-k}(I - b_1 M_{1:\overline{k}-k})^{-1} M_{1:\overline{k}-k}^{s} \begin{bmatrix} mc_t^{(k)} \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} + \sum_{i=0}^{\infty} b_1^i \mathbf{b}_{1:\overline{k}-k} \begin{bmatrix} \pi_{s+i|s}^{(k+1)} \\ \vdots \\ \pi_{s+i|s}^{(\overline{k})} \end{bmatrix} \tag{C.8}$$

where

$$M^s_{1:\overline{k}-k} = \begin{bmatrix} m_{1,1} & \cdots & m_{1,(\overline{k}-k)} \\ \vdots & \ddots & \vdots \\ m_{(\overline{k}-k),1} & \cdots & m_{(\overline{k}-k),(\overline{k}-k)} \end{bmatrix}^s \tag{C.9}$$

$$\mathbf{a}_{1:\overline{k}-k} = \begin{bmatrix} a_1 & \cdots & a_{\overline{k}-k} \end{bmatrix}, \quad \mathbf{b}_{1:\overline{k}-k} = \begin{bmatrix} b_1 & \cdots & b_{\overline{k}-k} \end{bmatrix} \tag{C.10}$$

Repeated forward substitution of (C.8) into (C.7) will yield a convergent solution for inflation as a function of the hierarchy of current marginal cost expectations as long as the elements of $\mathbf{b}$ and the largest eigenvalue of M are smaller than one in absolute value. Restricting the solution to only include expectations up to the $\overline{k}^{th}$ order allow us to write inflation as a function of a finite number of convergent weighted sums of current and expected future hierarchies of marginal cost expectations. This solution method is conceptually similar to solving a complete information rational expectations model by forward substitution, where we treat the marginal cost expectation hierarchy as the fundamental.

C.1.1. *Illustrating the method with $\overline{k}$=2.* We can illustrate the methodology by setting $\overline{k} = 2$. While not sufficient to achieve an accurate solution to the model, it does serve well to illustrate the methodology. Start by rewriting (C.7) with $\overline{k} = 2$

$$\pi_t = \mathbf{a}_{1:3} \begin{bmatrix} mc_t^{(0)} \\ mc_t^{(1)} \\ mc_t^{(2)} \end{bmatrix} + \mathbf{b}_{1:2} \begin{bmatrix} \pi_{t+1|t}^{(1)} \\ \pi_{t+1|t}^{(2)} \end{bmatrix} \tag{C.11}$$

Use the first consistency restriction, i.e. rationality of the first order expectation, and the conjectured law of motion (4.2) to eliminate $\pi_{t+1|t}^{(1)}$

$$\begin{aligned} \pi_t &= \mathbf{a}_{1:3} \begin{bmatrix} mc_t^{(0)} \\ mc_t^{(1)} \\ mc_t^{(2)} \end{bmatrix} + b_1 \mathbf{a}_{1:2} M_{1:2} \begin{bmatrix} mc_t^{(1)} \\ mc_t^{(2)} \end{bmatrix} \\ &\quad + b_2 \left[\pi_{t+1|t}^{(2)}\right] + b_1 \mathbf{b}_{1:2} \begin{bmatrix} \pi_{t+2|t}^{(1)} \\ \pi_{t+2|t}^{(2)} \end{bmatrix}. \end{aligned} \tag{C.12}$$

Continued forward substitution eliminates all first order expectation of inflation

$$\begin{aligned} \pi_t &= \mathbf{a}_{1:3} \begin{bmatrix} mc_t^{(0)} \\ mc_t^{(1)} \\ mc_t^{(2)} \end{bmatrix} + \mathbf{a}_{1:2} (I - b_1 M_{1:2})^{-1} b_1 M_{1:2} \begin{bmatrix} mc_t^{(1)} \\ mc_t^{(2)} \end{bmatrix} \\ &\quad + \sum_{i=0}^{\infty} b_1^i b_2 \left[\pi_{t+i+1|t}^{(2)}\right]. \end{aligned} \tag{C.13}$$

Note that, by ignoring terms of higher than second order and by invoking the second restriction that firms do not believe that other firms use a different model, the sum of second order inflation expectations on the second line of (C.13) can be written as a function of only second order expectations of marginal cost

$$\sum_{i=0}^{\infty} b_1^i b_2 \left[\pi_{t+i+1|t}^{(2)}\right] = \mathbf{a}_{1:1} b_2 \sum_{i=0}^{\infty} b_1^i M_{1:1}^i (I - b_1 M_{1:1})^{-1} M_{1:1} \left[mc_t^{(2)}\right] \tag{C.14}$$

$$= \mathbf{a}_{1:1} b_2 (I - b_1 M_{1:1})^{-1} (I - b_1 M_{1:1})^{-1} M_{1:1} \left[mc_t^{(2)}\right] \tag{C.15}$$

We have thus found an approximate, though with $\overline{k} = 2$ not very accurate, solution of inflation as a function of only the hierarchy of expectations of current marginal cost

$$\pi_t = \mathbf{a}_{1:3}\begin{bmatrix} mc_t^{(0)} \\ mc_t^{(1)} \\ mc_t^{(2)} \end{bmatrix} + \mathbf{a}_{1:2}(I - b_1 M_{1:2})^{-1} b_1 M_{1:2}\begin{bmatrix} mc_t^{(1)} \\ mc_t^{(2)} \end{bmatrix} \quad (C.16)$$
$$+\mathbf{a}_{1:1} b_2 (I - b_1 M_{1:1})^{-1}(I - b_1 M_{1:1})^{-1} M_{1:1}\left[mc_t^{(2)}\right].$$

To find the vector $\mathbf{c}$ in the conjectured solution (4.1), we add the subvectors that each involve the hierarchy of marginal cost expectations from order 0 to 2, from 1 to 2, and 2 in the following way

$$\mathbf{c} = \mathbf{c}_0 + \begin{bmatrix} 0 & \mathbf{c}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \mathbf{c}_2 \end{bmatrix} \quad (C.17)$$

$$\pi_t = \mathbf{c}\begin{bmatrix} mc_t^{(0)} \\ mc_t^{(1)} \\ mc_t^{(2)} \end{bmatrix} \quad (C.18)$$

where

$$\mathbf{c}_0 = \mathbf{a}_{1:3} \quad (C.19)$$
$$\mathbf{c}_1 = \mathbf{a}_{1:2}(I - b_1 M_{1:2})^{-1} b_1 M_{1:2} \quad (C.20)$$
$$\mathbf{c}_2 = \mathbf{a}_{1:1} b_2 (I - b_1 M_{1:1})^{-1}(I - b_1 M_{1:1})^{-1} M_{1:1} \quad (C.21)$$

C.1.2. *The general case.* For the general case the same strategy is followed to find $\mathbf{c}_0, \mathbf{c}_1, ...\mathbf{c}_k, ..., \mathbf{c}_{\overline{k}}$ in the expression

$$\pi_t = \mathbf{c}_0\begin{bmatrix} mc_t^{(0)} \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} + \mathbf{c}_1\begin{bmatrix} mc_t^{(1)} \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} + ... + \mathbf{c}_k\begin{bmatrix} mc_t^{(k)} \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} + ... + \mathbf{c}_{\overline{k}}\left[mc_t^{(\overline{k})}\right] \quad (C.22)$$

$\mathbf{c}$ is then given by

$$\mathbf{c}{=}\mathbf{c}_0 + \begin{bmatrix} 0 & \mathbf{c}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \mathbf{c}_2 \end{bmatrix} + ... + \begin{bmatrix} 0 & \cdots & 0 & \mathbf{c}_{\overline{k}} \end{bmatrix} \quad (C.23)$$

The formulas for the individual subvectors $\mathbf{c}_0, \mathbf{c}_1, ..., \mathbf{c}_k, ..., \mathbf{c}_{\overline{k}}$ get increasingly complex as k increases and the general formula for $k \geq 2$ is

$$\mathbf{c}_k = \sum \alpha \underbrace{b_q b_r ....b_s}_{h=\#\ of\ b's} \mathbf{a}_{1:\overline{k}-k}\left(I - b_1 M_{1:\overline{k}-5}\right)^{-1}\left(I - b_1 M_{1:\overline{k}-5}\right)^{-1} M_{1:\overline{k}-k}^{h} \quad (C.24)$$

where we sum over

$$\forall \ q, r, ...s : (q-1) + (r-1) + ... + (s-1) = k-1$$

and α is equal to the number of unique orderings of $b_q, b_r, ..., b_s$.

The structure of (C.24) can be understood in the following way. We have organized the subvectors of $\mathbf{c}$ such that $\mathbf{c}_k$ captures the impact of expectations from order k to $\overline{k}$. $\mathbf{c}_k$ thus do not capture any effects of lower than the k^{th}order. The number α as the number of 'paths' including the $q^{th}, r^{th}, ..., s^{th}$ orders of expectation, in h 'steps' that will 'loose' k orders (i.e. from zero to $k-1$) of marginal cost expectations. For an example, second order expectations of third order inflation expectations of inflation will not be a function of marginal cost expectations of lower or equal order than four, i.e. $(2-1)+(3-1)+1$. The same is true for the reverse 'path' of expectations, i.e. third order expectations of second order inflation expectations. Once we have calculated $\mathbf{c}$ for the desired order of expectations we can continue to the second step of the solution algorithm.

C.2. **Step 2: Finding the Implied Law of Motion of Expectations.** In this section we will derive the law of motion for the hierarchy of marginal cost expectations, taking its impact on inflation as given. We will use a state representation similar in spirit to the one in Woodford (2002). However, due to the forward looking nature of inflation, we will not be able to reduce the state to a 2×1 vector. Instead, actual average marginal cost and the hierarchy of average expectations of average marginal cost up to the $\overline{k}^{th}$ order will be treated as a hidden state, that firms will estimate using the Kalman filter. Since the state to be estimated includes the hierarchy of estimates, the Kalman filter determines both the estimates *and* and the law of motion of the state that is being estimated. Denote the state X_t and define it as

$$X_t \equiv \begin{bmatrix} mc_t^{(0)} \\ \vdots \\ mc_t^{(\overline{k}-1)} \\ mc_{t-1}^{(0)} \\ \vdots \\ mc_{t-1}^{(\overline{k}-1)} \end{bmatrix} \tag{C.25}$$

Use the conjectured law of motion of the the marginal cost hierarchy (4.2) to find the implied law of motion for the state X_t

$$X_t = WX_{t-1} + \begin{bmatrix} \mathbf{n} \\ \mathbf{0} \end{bmatrix} v_t \tag{C.26}$$

$$W = \begin{bmatrix} M & \mathbf{0} \\ I & \mathbf{0} \end{bmatrix}. \tag{C.27}$$

For a given W, firm j's estimate of X_t will evolve according to the updating equation

$$X_{t|t}(j) = WX_{t-1|t-1}(j) + K\left[Z_t(j) - LWX_{t-1|t-1}(j)\right] \tag{C.28}$$

where K is the Kalman gain matrix, $Z_t(j)$ is firm j's observation vector containing its own marginal cost and the lagged aggregate pricelevel. L is a matrix that maps an expected state into an expected observation vector. They are given by

$$Z_t(j) = \begin{bmatrix} mc_t(j) \\ \pi_{t-1} \end{bmatrix} \tag{C.29}$$

$$L = \begin{bmatrix} 1 & \mathbf{0}_{1\times(\overline{k}-1)} & \mathbf{0}_{1\times(\overline{k})} \\ 0 & \mathbf{0}_{1\times(\overline{k}-1)} & \mathbf{c} \end{bmatrix} \tag{C.30}$$

$$K = PL'(LPL' + \Sigma_{\varepsilon\varepsilon})^{-1} \tag{C.31}$$

$$P = W(P - PL'(LPL' + \Sigma_{\varepsilon\varepsilon})^{-1}LP)W' + \Sigma_{uu} \tag{C.32}$$

$$\Sigma_{\varepsilon\varepsilon} = \begin{bmatrix} \sigma_{\varepsilon\varepsilon}^2 & 0 \\ 0 & 0 \end{bmatrix}. \tag{C.33}$$

Denote the ith element of the first column of K $k_{i,1}$, then the innovation covariance matrix Σ_{uu} is given by

$$\Sigma_{uu} = \begin{bmatrix} \sigma_v^2 & k_{1,1}\sigma_v^2 & \cdots & \cdots & k_{\overline{k}-1,1}\sigma_v^2 & \mathbf{0} \\ k_{1,1}\sigma_v^2 & k_{1,1}^2\sigma_v^2 & k_{1,1}k_{2,1}\sigma_v^2 & \cdots & k_{1,1}k_{\overline{k}-1,1}\sigma_v^2 & \mathbf{0} \\ \vdots & k_{2,1}k_{1,1}\sigma_v^2 & k_{2,1}^2\sigma_v^2 & \cdots & k_{2,1}k_{\overline{k}-1,1}\sigma_v^2 & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ k_{\overline{k}-1,1}\sigma_v^2 & k_{\overline{k}-1,1}k_{1,1}\sigma_v^2 & k_{\overline{k}-1,1}k_{2,1}\sigma_v^2 & \cdots & k_{\overline{k}-1,1}^2\sigma_v^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{C.34}$$

As will be shown below, the structure of the filtering problem (C.28), the definition of the state X_t (C.25) and the conjectured law of motion (4.2) implies that in equilibrium

$$\Sigma_{uu} = \sigma_v^2 \begin{bmatrix} \mathbf{n} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{n} \\ \mathbf{0} \end{bmatrix}' \tag{C.35}$$

and $\mathbf{n}$ will thus be

$$\mathbf{n} = \begin{bmatrix} 1 \\ k_{1,1} \\ \vdots \\ k_{\overline{k}-1,1} \end{bmatrix} \tag{C.36}$$

But we are not quite there yet. First, take averages over (C.28) to get

$$X_{t|t} = [W - KLW]\, X_{t-1|t-1} + K \begin{bmatrix} mc_t \\ \pi_{t-1} \end{bmatrix}. \tag{C.37}$$

One should note that the idiosyncratic component gets 'washed out' in aggregation. Partition and use the definition of X_t

$$\begin{bmatrix} mc_t^{(1)} \\ \vdots \\ mc_t^{(\overline{k})} \\ mc_{t-1|t}^{(1)} \\ \vdots \\ mc_{t-1|t}^{(\overline{k})} \end{bmatrix} = \begin{bmatrix} W_{11} - [KLW]_{11} & W_{12} - [KLW]_{12} \\ W_{21} - [KLW]_{21} & W_{22} - [KLW]_{22} \end{bmatrix} \begin{bmatrix} mc_{t-1}^{(1)} \\ \vdots \\ mc_{t-1}^{(\overline{k})} \\ mc_{t-2|t-1}^{(1)} \\ \vdots \\ mc_{t-2|t-1}^{(\overline{k})} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \rho mc_{t-1} + v_t \\ \pi_{t-1} \end{bmatrix} \tag{C.38}$$

where $W_{12} - [KLW]_{12} = W_{22} - [KLW]_{22} = \mathbf{0}$. Substitute the solution of inflation lagged one period

$$\pi_{t-1} = \mathbf{c} \begin{bmatrix} mc_{t-1}^{(0)} \\ \vdots \\ mc_{t-1}^{(\overline{k})} \end{bmatrix} \tag{C.39}$$

and use the average actual marginal cost process (3.1) to get the desired form (4.2)

$$\begin{bmatrix} mc_t^{(0)} \\ mc_t^{(1)} \\ \vdots \\ mc_t^{(\overline{k})} \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ \rho K_{11} + K_{12} c_1 & W_{11} - [KLW]_{11} + K_{12} \mathbf{c}_{2:\overline{k}} \end{bmatrix} \begin{bmatrix} mc_{t-1}^{(0)} \\ mc_{t-1}^{(1)} \\ \vdots \\ mc_{t-1}^{(\overline{k})} \end{bmatrix} + \begin{bmatrix} 1 \\ K_{11} \end{bmatrix} v_t \tag{C.40}$$

M and $\mathbf{n}$ in the conjectured solution is thus given by

$$M = \begin{bmatrix} \rho & 0 \\ \rho K_{11} + K_{12} c_1 & W_{11} - [KLW]_{11} + K_{12} \mathbf{c}_{2:\overline{k}} \end{bmatrix} \tag{C.41}$$

$$\mathbf{n} = \begin{bmatrix} 1 \\ k_{1,1} \\ \vdots \\ k_{\overline{k}-1,1} \end{bmatrix}. \tag{C.42}$$

Solving the model implies finding a fixed point of the system described by (C.23), (C.27), (C.30), (C.31), (C.32) (C.34), (C.41) and (C.42).

Appendix D. Solution with Endogenous Marginal cost

We restrict the explanation of the solution under endogenous marginal cost to the instances where it differs from the solution under exogenous marginal costs.

Output is determined by the IS equation and the Taylor rule

$$y_t = E_t y_{t+1} - \frac{1}{\gamma} (i_t - E_t \pi_{t+1}) \tag{D.1}$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t \tag{D.2}$$

and the actual marginal cost of firm j is given by

$$mc_t(j) = (\gamma + \varphi) y_t + \xi_t + \varepsilon_t(j) \tag{D.3}$$

Conjecture that output and inflation are linear functions of the hierarchy of preference shock expectations

$$\pi_t = \widetilde{\mathbf{c}} \begin{bmatrix} \xi_t^{(0)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} \tag{D.4}$$

$$y_t = \widetilde{\mathbf{d}} \begin{bmatrix} \xi_t^{(0)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} \tag{D.5}$$

$$\begin{bmatrix} \xi_t^{(0)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} = \widetilde{M} \begin{bmatrix} \xi_{t-1}^{(0)} \\ \vdots \\ \xi_{t-1}^{(\overline{k})} \end{bmatrix} + \widetilde{\mathbf{n}} v_t \tag{D.6}$$

D.1. **Finding $\widetilde{\mathbf{c}}$.** Replace the actual average marginal cost of firms in the Phillips curve (C.1) with the average of firms' beginning-of-period estimate of their own marginal cost

$$E_t(j)mc_t(j) = (\gamma+\varphi)\,\widetilde{\mathbf{d}}_{1:\overline{k}} \begin{bmatrix} \xi_t^{(1)}(j) \\ \vdots \\ \xi_t^{(\overline{k})}(j) \end{bmatrix} + \omega_t(j) \tag{D.7}$$

Taking averages

$$\int E_t(j)mc_t(j)dj = (\gamma+\varphi)\,\widetilde{\mathbf{d}}_{1:\overline{k}} \begin{bmatrix} \xi_t^{(1)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} + \xi_t \tag{D.8}$$

since

$$\int \omega_t(j) = \xi_t \tag{D.9}$$

With some abuse of notation, denote the average of firms' beginning-of-period estimate

$$\int E_t(j)mc_t(j)dj \equiv \widetilde{mc}_t^{(0)} \tag{D.10}$$

The hierarchy of beginning-of-period estimates can then be written as a function of the preference shock expectation hierarchy

$$\begin{bmatrix} \widetilde{mc}_t^{(0)} \\ \widetilde{mc}_t^{(1)} \\ \vdots \\ \widetilde{mc}_t^{(\overline{k})} \end{bmatrix} = \begin{bmatrix} \xi_t^{(0)} \\ \xi_t^{(1)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} + (\gamma+\varphi) \begin{bmatrix} 0 & d_1 & \cdots & d_{\overline{k}} \\ 0 & 0 & d_1 & \vdots \\ 0 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_t^{(0)} \\ \xi_t^{(1)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} \tag{D.11}$$

$$\begin{bmatrix} \widetilde{mc}_t^{(0)} \\ \widetilde{mc}_t^{(1)} \\ \vdots \\ \widetilde{mc}_t^{(\overline{k})} \end{bmatrix} = \widetilde{D} \begin{bmatrix} \xi_t^{(0)} \\ \xi_t^{(1)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} \tag{D.12}$$

$$\widetilde{D} = \begin{bmatrix} 1 & (\gamma+\varphi)\,d_1 & \cdots & (\gamma+\varphi)\,d_{\overline{k}} \\ 0 & 1 & (\gamma+\varphi)\,d_1 & \vdots \\ 0 & 0 & 1 & (\gamma+\varphi)\,d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{D.13}$$

which implies that

$$E_t \begin{bmatrix} \widetilde{mc}_{t+s}^{(0)} \\ \widetilde{mc}_{t+s}^{(1)} \\ \vdots \\ \widetilde{mc}_{t+s}^{(\overline{k})} \end{bmatrix} = \begin{bmatrix} 1 & (\gamma+\varphi)\,d_1 & \cdots & (\gamma+\varphi)\,d_{\overline{k}} \\ 0 & 1 & (\gamma+\varphi)\,d_1 & \vdots \\ 0 & 0 & 1 & (\gamma+\varphi)\,d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \widetilde{M}^s \begin{bmatrix} \xi_t^{(0)} \\ \xi_t^{(1)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} \tag{D.14}$$

For a given $\widetilde{\mathbf{d}}$ and $\widetilde{M}$, the subvectors $\widetilde{\mathbf{c}}_k$ of $\widetilde{\mathbf{c}}$

$$\widetilde{\mathbf{c}} = \widetilde{\mathbf{c}}_0 + \begin{bmatrix} 0 & \widetilde{\mathbf{c}}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \widetilde{\mathbf{c}}_2 \end{bmatrix} + ... + \begin{bmatrix} 0 & \cdots & 0 & \widetilde{\mathbf{c}}_{\overline{k}} \end{bmatrix} \tag{D.15}$$

can be calculated as

$$\widetilde{\mathbf{c}}_k = \sum \alpha \underbrace{b_q b_r....b_s}_{h=\#\ of\ b's} \mathbf{a}_{1:\overline{k}-k} \widetilde{D}_{1:\overline{k}-k} \left(I - b_1 \widetilde{M}_{1:\overline{k}-5}\right)^{-1} \left(I - b_1 \widetilde{M}_{1:\overline{k}-5}\right)^{-1} \widetilde{M}^h_{1:\overline{k}-k} \tag{D.16}$$

D.2. **Finding $\widetilde{\mathbf{d}}$.** The system

$$y_t = E_t y_{t+1} - \frac{1}{\gamma}(i_t - E_t \pi_{t+1}) \tag{D.17}$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t \tag{D.18}$$

$$\begin{bmatrix} \xi_t^{(0)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix} = \widetilde{M} \begin{bmatrix} \xi_{t-1}^{(0)} \\ \vdots \\ \xi_{t-1}^{(\overline{k})} \end{bmatrix} + \widetilde{\mathbf{n}} v_t \tag{D.19}$$

can be rewritten as

$$\begin{bmatrix} 1 & \left[-\frac{\phi_\pi}{\gamma}\widetilde{\mathbf{c}} - \frac{\phi_y}{\gamma}\widetilde{\mathbf{d}} + \widetilde{\mathbf{c}}\widetilde{M}\right]_{1,1} & \cdots & \left[-\frac{\phi_\pi}{\gamma}\widetilde{\mathbf{c}} - \frac{\phi_y}{\gamma}\widetilde{\mathbf{d}} + \widetilde{\mathbf{c}}\widetilde{M}\right]_{1,\overline{k}} \\ 0 & 1 & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ 0 & 0 & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} E_t y_{t+1} \\ \xi_t^{(0)} \\ \vdots \\ \xi_t^{(\overline{k})} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \mathbf{0} & 0 \\ 0 & \widetilde{m}_{1,1} & \cdots & \widetilde{m}_{1,\overline{k}+1} \\ \mathbf{0} & \vdots & \ddots & \vdots \\ 0 & \widetilde{m}_{\overline{k}+1,1} & \cdots & \widetilde{m}_{\overline{k}+1,\overline{k}+1} \end{bmatrix} \begin{bmatrix} y_t \\ \xi_{t-1}^{(0)} \\ \vdots \\ \xi_{t-1}^{(\overline{k})} \end{bmatrix} + \begin{bmatrix} 0 \\ \widetilde{\mathbf{n}} \end{bmatrix} v_t \tag{D.20}$$

For given $\widetilde{\mathbf{c}}$, $\widetilde{\mathbf{d}}$, and $\widetilde{M}$ a 'new' $\widetilde{\mathbf{d}}$ can be found by standard full information linear rational expectations solution methods if the largest eigenvalue of $\widetilde{M}$ lies within the unit circle. We can then use a Shur decomposition as in Blanchard and Kahn (1980), to get the forward looking jump variable y_t as a function of the zero to $\overline{k}$ orders of predetermined marginal cost expectations.

D.3. **Finding $\widetilde{M}$.** $\widetilde{M}$ can be found by similar methods as in the previous section. The differences will be the vector of observables $Z_t(j)$ and the matrix mapping an expected state into an expected observation , L, which are now given by

$$Z_t(j) = \begin{bmatrix} mc_t(j) \\ \pi_{t-1} \\ y_{t-1} \end{bmatrix} \tag{D.21}$$

$$L = \begin{bmatrix} 1 & \mathbf{0}_{1\times(\overline{k}-1)} & \mathbf{0}_{1\times(\overline{k})} \\ 0 & \mathbf{0}_{1\times(\overline{k}-1)} & \mathbf{c} \\ 0 & \mathbf{0}_{1\times(\overline{k}-1)} & \mathbf{d} \end{bmatrix} \tag{D.22}$$

$\widetilde{M}$ will then be given by

$$\widetilde{M} = \begin{bmatrix} \rho & 0 \\ \rho K_{11} + K_{12}c_1 + K_{13}d_1 & W_{11} - [KLW]_{11} + K_{12}\mathbf{c}_{2:\overline{k}} + K_{12}\mathbf{d}_{2:\overline{k}} \end{bmatrix} \tag{D.23}$$

Solving the model implies finding a fixed point of the system described by (D.15), (D.20), (D.22) and (D.23).

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