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MONETARY POLICY, EXPECTATIONS AND COMMITMENT

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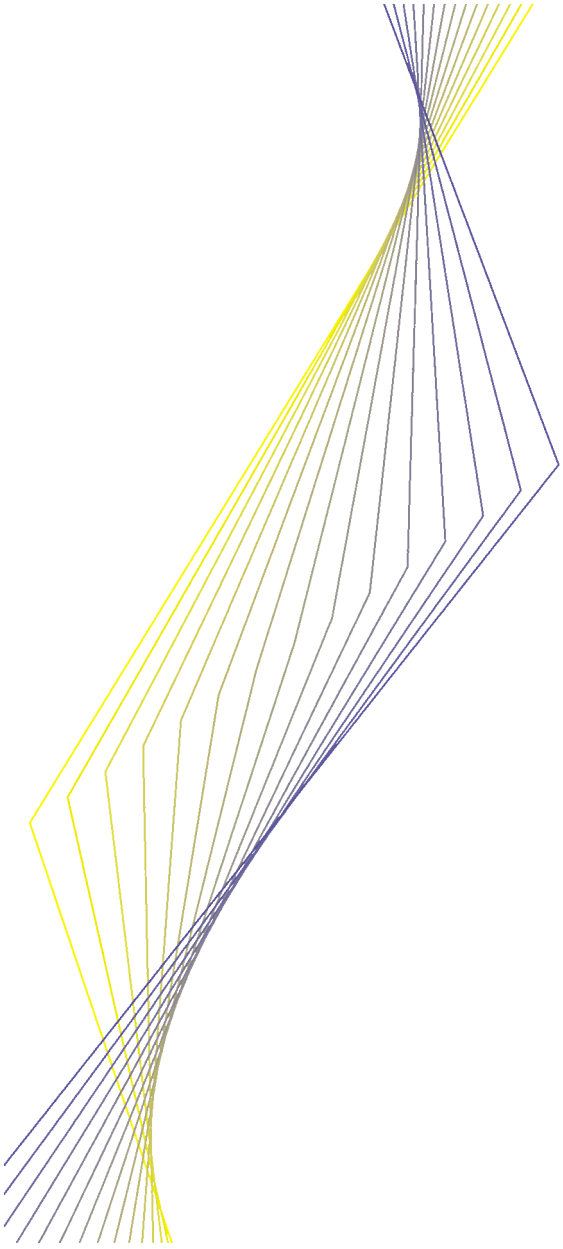

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Abstract

Full commitment in monetary policy leads to equilibria that are superior to those from optimal discretionary policies. Different types of reactions functions to implement and instrument rules to approximate full commitment have been proposed in the literature. We assess optimal reaction functions and instrument rules, in terms of whether they lead to an RE equilibrium that is both locally determinate and stable under adaptive learning by private agents. The reaction function that appropriately depends explicitly on private expectations performs best on both counts.

Key words: Commitment, interest rate setting, adaptive learning, stability, determinacy.

JEL classification: E52, E31, D84.

Non-Technical Summary

Many recent models of monetary policy emphasize the importance of forward looking aspects of the economy, in which expectations of private agents significantly influence the economic outcome. If expectations about the future are part of the equilibrating mechanisms in the economy it is well known that standard intertemporal optimization of economic policy by the government is in general subject to the problem of time inconsistency. Lack of time consistency means that a policy maker has incentives to deviate, in later periods, from the optimal plan obtained in the first period. In contrast, discretionary policies are obtained through policy optimization separately in each period and are time consistent, but typically the resulting sequence of discretionary policy decisions will not lead to the overall intertemporal optimum of the economy. The losses from discretionary policies can be quantitatively significant, and this has provided the impetus for finding ways to achieve the optimum or at least to improve the outcome.

The first papers on commitment vs discretion in monetary policy focused attention on the so-called inflation bias that arises from overambitious government objectives with respect to aggregate output. In most recent work, it is assumed that the policy maker does not have overambitious goals. Nevertheless, the issue of commitment vs. discretion still prevails, since discretion leads to what is called a "stabilization bias" and there are gains to commitment. Implementation of the commitment solution can be based on the timeless perspective, *which* is a rule based policy that respects the optimality conditions from the intertemporal optimization under commitment, except for the current decision-making period. Recent work has shown that the gains from committing to this policy, relative to the discretionary policies, can be significant.

Almost all of the recent literature on monetary policy, including all of the references above, has been conducted under the hypothesis of rational expectations (RE). However, this may not be an innocuous assumption as was explicitly shown by Bullard and Mitra (2001) and Evans and Honkapohja (2001). In the analysis of economic policy the assumption of RE should not

be taken for granted, since expectations can be out of equilibrium, at least for a period of time, as a result of exogenous events such as structural shifts in the economy. Economic policies should be designed to avoid instabilities that can arise from expectational errors and the corrective behavior of economic agents in the face of such errors.

For monetary policy Evans and Honkapohja (2001) show that certain standard forms of optimal discretionary interest rate setting by the central bank can lead to instability as economic agents unsuccessfully try to correct their forecast functions over time, with the result that the economy may not converge to the desired rational expectations equilibrium (REE). They also propose a new way for implementing optimal discretionary policy that always leads to stability under learning. Bullard and Mitra (2001) consider the stability of equilibria when monetary policy is conducted using some variant of the Taylor interest rate rule. Bullard and Mitra argue that monetary policy making should take into account the learnability constraints, which imply constraints on the parameters of policy behavior. A related concern addressed by various authors is that it is desirable for policy rules to yield determinacy, i.e. locally unique REE, to ensure that there are no nearby suboptimal REE.

The research on adaptive learning and monetary policy has so far considered the performance of discretionary optimal policies or ad hoc interest rate rules. A partial exception is Evans and Honkapohja (2001), but they restrict attention to limited forms of commitment. Learnability of optimal policy with full commitment has not been analyzed thus far. In this paper we study whether optimal monetary policy by the central bank is conducive to long run convergence of private expectations to the optimal REE.

On intuitive grounds one might think that commitment favors stability under learning by leading to more forecastable dynamics of the economy than when policy is re-optimized every period. We will argue that while this can indeed be the case, stability depends critically on the way the monetary policy with full commitment is implemented. Certain standard forms of central bank reaction functions or instrument rules that approximate the policy target do not (or do not always) provide stability under learning. However, there is another implementation, depending explicitly on private expectations, that performs well in this respect. We propose a specific implementation of optimal policy that always leads to both determinacy and stability under learning.

1 Introduction

Many recent models of monetary policy emphasize the importance of forward looking aspects of the economy, in which expectations of private agents significantly influence the economic outcome. If expectations about the future are part of the equilibrating mechanisms in the economy it is well known that standard intertemporal optimization of economic policy by the government is in general subject to the problem of time inconsistency, as was first pointed out by (Kydland and Prescott 1977). Lack of time consistency means that a policy maker has incentives to deviate, in later periods, from the optimal plan obtained in the first period. In contrast, discretionary policies are obtained through policy optimization separately in each period and are time consistent, but typically the resulting sequence of discretionary policy decisions will not lead to the overall intertemporal optimum of the economy. The losses from discretionary policies can be quantitatively significant, and this has provided the impetus for finding ways to achieve the optimum or at least to improve the outcome. The discussion is often framed as the question of rules vs. discretion in policy making.

The tensions between the non-optimality arising from discretionary policy relative to full optimality and the time inconsistency of fully optimal policies have also been considered in the context of monetary policy. The first papers focused attention on the so-called inflation bias that arises from overambitious government objectives with respect to aggregate output, see the contributions by (Barro and Gordon 1983a) and (Barro and Gordon 1983b) and the subsequent literature. In most recent work, the problem of inflation bias has received less attention, as it is often assumed that the policy maker does not have overambitious goals. Nevertheless, the issue of commitment vs. discretion still prevails, since discretion leads to what is called a "stabilization bias" and there are gains to commitment, see (Woodford 1999a), (Woodford 1999b), (Svensson and Woodford 1999) and (McCallum and Nelson 2000) among others.[1]

It has been suggested by (Woodford 1999a) and (Woodford 1999b) that, to implement the commitment solution, monetary policy making ought be based on the *timeless perspective*. This concept is a rule based policy that is obtained by respecting the optimality conditions from the full intertempo-

[1] See (Clarida, Gali, and Gertler 1999) for a recent survey on this literature on monetary policy.

ral optimization under commitment, except for the current decision-making period. In other words, such a rule stipulates that the policy maker follows "the pattern of behavior to which it would have wished to commit itself at a date far in the past" (p.293 in (Woodford 1999a)). Recent work has shown that the gains from committing to this policy, relative to the discretionary policies, can be significant, see (McCallum and Nelson 2000). In this paper we will adopt the timeless perspective formulation and refer to the corresponding optimal monetary policy with commitment as the "full commitment solution."

Almost all of the recent literature on monetary policy, including all of the references above, has been conducted under the hypothesis of rational expectations (RE). However, this may not be an innocuous assumption as was explicitly shown by (Bullard and Mitra 2001b) and (Evans and Honkapohja 2001a). In the analysis of economic policy the assumption of RE should not be taken for granted, since expectations can be out of equilibrium, at least for a period of time, as a result of exogenous events such as structural shifts in the economy. Economic policies should be designed to avoid instabilities that can arise from expectational errors and the corrective behavior of economic agents in the face of such errors.

The possibility of temporary errors in forecasting, and the consequent correction mechanisms, have been widely studied in recent research using the adaptive learning approach.[2] For monetary policy (Evans and Honkapohja 2001a) show that certain standard forms of optimal discretionary interest rate setting by the central bank can lead to instability as economic agents unsuccessfully try to correct their forecast functions over time, with the result that the economy may not converge to the desired rational expectations equilibrium (REE). They also propose a new way for implementing optimal discretionary policy that always leads to stability under learning. (Bullard and Mitra 2001b) consider the stability of equilibria when monetary policy is conducted using some variant of the Taylor interest rate rule. Bullard and Mitra argue that monetary policy making should take into account the learnability constraints, which imply constraints on the parameters of policy behavior.[3] A related concern addressed by various authors is that it is desir-

[2] (Evans and Honkapohja 2001b) provides an extensive treatise on the analysis of adaptive learning and its implications in macroeconomics. (Evans and Honkapohja 1999), (Evans and Honkapohja 1995), (Marimon 1997), (Sargent 1993) and (Sargent 1999) provide surveys of the field.

[3] Other papers on monetary policy using the learning approach include (Bullard and

able for policy rules to yield determinacy, i.e. locally unique REE, to ensure that there are no nearby suboptimal REE.

The research on adaptive learning and monetary policy has so far considered the performance of discretionary optimal policies or ad hoc interest rate rules. A partial exception is (Evans and Honkapohja 2001a). However, they restrict attention to limited forms of commitment for which the REE takes the same form (but with different parameter values) as the optimal discretionary equilibrium. Learnability of optimal policy with full commitment has not been analyzed thus far. In this paper our goal is to study whether optimal monetary policy by the central bank is conducive to long run convergence of private expectations to the optimal REE.

On intuitive grounds one might think that commitment favors stability under learning by leading to more forecastable dynamics of the economy than when policy is re-optimized every period. We will argue that while this can indeed be the case, stability depends critically on the way the monetary policy with full commitment is implemented. Certain standard forms of central bank reaction functions or instrument rules that approximate the policy target do not (or do not always) provide stability under learning. However, there is another implementation, depending explicitly on private expectations, that performs well in this respect. We propose a specific implementation of optimal policy that always leads to both determinacy and stability under learning.

2 The Model

We use a linearized model that is very commonly employed in the literature, see (Clarida, Gali, and Gertler 1999) for this particular formulation. The original nonlinear framework is based on a representative consumer, a continuum of firms producing differentiated goods under monopolistic competition and subject to constraints on the frequency of price changes, as originally suggested by (Calvo 1983).[4]

Mitra 2001a), (Mitra 2001), (Honkapohja and Mitra 2001), (Honkapohja and Mitra 2002) and (Carlstrom and Fuerst 2001). A predecessor to this work is (Howitt 1992), though he did not use the New Keynesian framework.

[4]See e.g. (Woodford 1996) for the nonlinear model and its linearization.

The behavior of the private sector is described by two equations

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t, \tag{1}$$

which is the "IS" curve derived from the Euler equation for consumer optimization, and

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \tag{2}$$

which is the price setting rule for the monopolistically competitive firms. Here x_t and π_t denote the output gap and inflation for period t, respectively. i_t is the nominal interest rate, expressed as the deviation from the steady state real interest rate. The determination of i_t will be discussed below. $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$ denote the private sector expectations of the output gap and inflation next period. Since our focus is on learning behavior, these expectations need not be rational (E_t without $*$ denotes RE). The parameters φ and λ are positive and β is the discount factor of the firms so that $0 < \beta < 1$.

The shocks g_t and u_t are assumed to be observable and follow

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix}, \tag{3}$$

where

$$F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix},$$

$0 < |\mu| < 1$, $0 < |\rho| < 1$ and $\tilde{g}_t \sim iid(0, \sigma_g^2)$, $\tilde{u}_t \sim iid(0, \sigma_u^2)$ are independent white noise. g_t represents shocks to government purchases and as well as to potential output. u_t represents any cost push shocks to marginal costs other than those entering through x_t. For simplicity, we assume throughout the paper that μ and ρ are known (if not, they could be estimated).

Assume RE for the moment. Monetary policy is derived from minimization of a quadratic loss function

$$E_t \sum_{s=0}^{\infty} \beta^s (\pi_{t+s}^2 + \alpha x_{t+s}^2). \tag{4}$$

This type of optimal policy is often called "flexible inflation targeting" in the current literature, see e.g. (Svensson 1999) and (Svensson 2001). α is the

relative weight on the output target and strict inflation targeting would be the case $\alpha = 0$. Note that, first, the policy maker is assumed to have the same discount factor as the private sector and, second, the target value of the output gap is set at zero implying that the classical problem of inflation bias does not arise.[5] Thus the target for output is set at the flexible price output level. For brevity, the inflation target is set at zero (introducing non-zero targets would not change the conclusions of our analysis regarding determinacy and stability under learning). As is well known, the quadratic loss function (4) can be viewed as an approximation of the utility function of the representative consumer.[6]

The full intertemporal optimum, usually called the commitment solution, is obtained by maximizing (4) subject to (2) for all periods $t, t+1, t+2, \ldots$ The first order conditions are given by[7]

$$2\alpha x_{t+s} + \lambda \omega_{t+s} = 0 \text{ for } s = 0, 1, 2, \ldots \tag{5}$$

$$2\pi_{t+s} + \omega_{t+s-1} - \omega_{t+s} = 0 \text{ for } s = 1, 2, \ldots \tag{6}$$

and

$$2\pi_t - \omega_t = 0. \tag{7}$$

Here ω_{t+s}, $s = 0, 1, 2, ...$, denote Lagrange multipliers associated with the constraints (2) for each time period.

The time inconsistency of the commitment solution is evident from (7), since this places a requirement that is specific to the current period and is different from the corresponding requirement (6) for later periods. The current decision places a constraint for the future periods that is non-optimal when the future periods actually arrive. A planned re-optimization for such a period would lead to violation of (6), so that the optimal plan would not be continued.

[5] See e.g. (Clarida, Gali, and Gertler 1999) for a discussion of the inflationary bias in the context of this kind of model.

[6] It should be noted that, like much of the literature on monetary policy, we do not explicitly introduce the budget constraint of the government to the analysis. This is justified by assuming that fiscal policy is set "passively" in the sense of (Leeper 1991) and ensures that the intertemporal budget constraint of the government is satisfied.

[7] See (Woodford 1999b), section 3.1 for the derivation of the first order conditions using Lagrange multipliers in a very similar setup.

As noted in the Introduction, the timeless perspective resolution to the problem of the time inconsistency of optimal policy is that the policy maker should respect the optimality conditions above, except for the current period when the optimization is done. In our context this amounts to using (5) and (6) also for the current period (and neglecting (7)). This yields the *full commitment optimality condition*[8]

$$\lambda\pi_t = -\alpha(x_t - x_{t-1}). \tag{8}$$

We remark that (8) is sometimes called a "specific targeting rule" in the literature.

We next compute the REE of interest. It can be shown that the dynamic system in x_t and π_t defined by (2) and (8) has a unique nonexplosive solution. This solution can be expressed as a linear function of the state variables x_{t-1} and u_t and is known as the "minimal state variable" (MSV) solution (see (McCallum 1983)). To obtain it explicitly we use the method of undetermined coefficients, expressing the REE in the form[9]

$$\begin{aligned} x_t &= b_x x_{t-1} + c_x u_t, & (9)\\ \pi_t &= b_\pi x_{t-1} + c_\pi u_t. & (10) \end{aligned}$$

Under RE we have $E_t\pi_{t+1} = b_\pi(b_x x_{t-1} + c_x u_t) + c_\pi \rho u_t$, so that substituting into (2) and (8) yields

$$\begin{aligned} \lambda\pi_t &= -\alpha[(b_x - 1)x_{t-1} + c_x u_t],\\ \pi_t &= \lambda(b_x x_{t-1} + c_x u_t) + \beta[b_\pi(b_x x_{t-1} + c_x u_t) + c_\pi \rho u_t] + u_t. \end{aligned}$$

This implies the equations

$$\begin{aligned} b_\pi &= -\frac{\alpha}{\lambda}(b_x - 1),\\ c_\pi &= -\frac{\alpha}{\lambda}c_x, & (11)\\ b_\pi &= \lambda b_x + \beta b_\pi b_x,\\ c_\pi &= \lambda c_x + \beta(b_\pi c_x + c_\pi \rho) + 1 \end{aligned}$$

[8] (Clarida, Gali, and Gertler 1999), p.1681 and (Woodford 1999a), appendix also derive this optimality condition.

[9] The derivation follows closely (McCallum and Nelson 2000). Note that the MSV solution excludes sunspots and bubbles by construction.

that determine the presently unknown coefficients in (9) and (10).

The first and third equations in (11) lead to a quadratic equation in b_x

$$\beta b_x^2 - \gamma b_x + 1 = 0,$$

where $\gamma = 1 + \beta + \lambda^2/\alpha$. Given a solution for b_x, the solution for b_π is obtainable from $b_\pi = (\alpha/\lambda)(1 - b_x)$. Finally, the solutions for c_π and c_x are obtained from the second and fourth equation in (11), which are linear, given the solution for b_π. The quadratic in b_x has two solutions, but the solution of interest is[10]

$$\bar{b}_x = (2\beta)^{-1}[\gamma - (\gamma^2 - 4\beta)^{1/2}]. \tag{12}$$

This delivers a stationary REE for all values of structural parameters, since $0 < \bar{b}_x < 1$, and corresponds to the policy optimum. We will therefore refer to the REE, given by (9)-(10), (11) and (12), as the *optimal REE*.

3 Optimal Interest Rate Setting

Thus far we have formulated the concept of optimal monetary policy under RE and reviewed the derivation of the REE of interest using the existing literature. This derivation did not rely on the aggregate demand curve (1), which depends on the interest rate and which can be used to determine the interest rate that implements the desired optimal equilibrium. Computation of the appropriate interest rate will lead to a functional relationship that will be called a *reaction function*, since the reaction functions aims to set interest rates so that the optimality condition (8) will be exactly met. Interest rate rules that respond to endogenous and exogenous variables, but do not respect (8), are instead called *instrument rules* and we will analyze some instrument rules below in Section 4. We note that the terminology used by different authors appears to vary in the literature.[11]

[10] The other root for b_x is always larger than one and therefore generates explosive time paths.

[11] This largely agrees with the terminology of (Svensson and Woodford 1999) and (Svensson 2001). They call the optimality condition (8) a "specific targeting rule" and the setting of the interest rate instrument, with (8) satisfied, a "reaction function" of the policy maker.

As has become apparent from the earlier literature (see the references below), interest rate setting in the form of a reaction function can be implemented in different ways depending on what is assumed to be known in the policy optimization. We now consider three cases, following the analysis in (Evans and Honkapohja 2001a) for discretionary policy. For each form of the reaction function we will test its performance in two ways.

First, we will determine if the resulting REE is determinate. This means that it is the unique REE under the reaction function. If a solution is indeterminate there exist other stationary RE solutions nearby and, as is well known, these can include a dependence on extraneous variables or "sunspots." Second, we determine whether the REE corresponding to the reaction function implementing optimal policy is stable under adaptive learning by private agents. Here we formally analyze whether the RE solution is E-stable, since E-stability is known to determine whether the solution is locally stable if private update their forecasts using least squares or related learning schemes. Our aim is to look for reaction functions for the interest rate that are both determinate and stable under learning.

3.1 The Fundamentals Based Reaction Function

In the REE constructed above the inflation and output gap forecasts satisfy (9) and (10) with the parameter values $\bar{b}_\pi$, $\bar{c}_\pi$, $\bar{b}_x$ and $\bar{c}_x$, where $\bar{b}_x$ is given in (12) and

$$\begin{aligned} \bar{b}_\pi &= (\alpha/\lambda)(1-\bar{b}_x), \\ \bar{c}_x &= -[\lambda + \beta\bar{b}_\pi + (1-\beta\rho)(\alpha/\lambda)]^{-1}, \\ \bar{c}_\pi &= -(\alpha/\lambda)\bar{c}_x. \end{aligned}$$

RE are thus given by

$$\begin{aligned} E_t\pi_{t+1} &= \bar{b}_\pi\bar{b}_x x_{t-1} + (\bar{b}_\pi\bar{c}_x + \bar{c}_\pi\rho)u_t, \\ E_t x_{t+1} &= \bar{b}_x^2 x_{t-1} + (\bar{b}_x + \rho)\bar{c}_x u_t. \end{aligned}$$

Inserting these into (1) and solving for the interest rate yields

$$i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t, \tag{13}$$

with

$$\begin{aligned}\psi_x &= \bar{b}_x[\varphi^{-1}(\bar{b}_x - 1) + \bar{b}_\pi], &(14)\\ \psi_g &= \varphi^{-1}, &(15)\\ \psi_u &= [\bar{b}_\pi + \varphi^{-1}(\bar{b}_x + \rho - 1)]\bar{c}_x + \bar{c}_\pi\rho. &(16)\end{aligned}$$

We refer to (13) as the *fundamentals based reaction function*, since its derivation is based solely on the model (1) and (2), the optimality condition (8) and the assumption that the economy is in a stationary REE.

We emphasize that the derivation of this interest rate rule presupposes RE on the part of both the private agents and the policy maker. In the REE specified it indeed implements the optimal policy, as is evident from the way (13) was derived. (13) states that interest rates are set so as to respond to lagged output gap and observable exogenous shocks. The dependence on lagged output gap reflects the commitment aspect of the optimal policy.[12] We note that interest rate setting according to (13) is quite similar to the "reaction functions" in equation (2.30) in (Svensson and Woodford 1999) and (3.5) in (Svensson 2001). Their models differ from the model in this paper, but the setting of interest rates according to lagged output and observable exogenous variables is the key common feature for their setups and (13).[13]

We are now ready to analyze the model with interest rate setting according to (13) for determinacy and stability under learning. For this purpose, combining (1), (2) and (13), we write the reduced form of the model in terms of general (possibly non-rational) expectations as

$$\begin{aligned}\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} &= \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + &(17)\\ &\quad \begin{pmatrix} -\varphi\psi_x & 0 \\ -\lambda\varphi\psi_x & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & -\varphi\psi_u \\ 0 & 1 - \lambda\varphi\psi_u \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix}.\end{aligned}$$

[12] The corresponding interest rate function under discretion does not depend on x_{t-1}, see (Evans and Honkapohja 2001a). In fact, the coefficient ψ_u with $\bar{b}_x = \bar{b}_\pi = 0$, is identical to the discretionary case.

[13] Our model does not include the unobservable judgement variables that are introduced in (Svensson 2001) to capture further model uncertainties.

3.1.1 Does the Fundamentals Based Reaction Function Yield Determinacy?

To analyze determinacy, we apply well-known methodology, see e.g. the Appendix of Chapter 10 of (Evans and Honkapohja 2001b). Key technical details are given in Appendix A.2. The basic steps are to rewrite the model in first-order form and to compare the number of non-predetermined variables with the number of eigenvalues of the forward-looking matrix that lie outside the unit circle. When these numbers are equal the model is determinate and has a unique nonexplosive solution. Intuitively, each root outside the unit circle provides a side condition that ties down one nonpredetermined variable. If there are fewer eigenvalues outside the unit circle than non-predetermined variables then the model is indeterminate and there exist multiple nonexplosive solutions. In particular, in the indeterminate case there exist multiple stationary solutions that depend on sunspot variables. In contrast to the optimal REE these other REE will not satisfy (8), the necessary conditions for an optimum.[14]

The conditions for determinacy are given in Appendix A.3. Whether the determinacy condition holds depends on the structural parameters of the model, and we have

Proposition 1 *Under the fundamentals based reaction function there are parameter regions in which the model is determinate and other parameter regions in which it is indeterminate.*

As an illustration we consider the calibrated parameter values suggested in (Woodford 1999b).

Calibrated Example. $\beta = 0.99$, $\varphi = (0.157)^{-1}$ and $\lambda = 0.024$.

Straightforward numerical calculations show that for small values of α the steady state is indeterminate, while for larger values of α the model is determinate. (With the calibrated parameter values the borderline is approximately $\alpha = 0.16$.) Determinacy thus arises only for some values of α. In particular, sufficient flexibility in inflation targeting is needed to ensure determinacy of equilibrium under the reaction function (13). Similar results are found if other parameters are varied. For example, for $\beta = 0.99, \lambda = 0.024$

[14] Other stationary REE that satisfy (2) cannot satisfy (8) because, as previously noted, the system (2) and (8) has a unique stationary RE solution.

and $\alpha = 0.2$, large values of φ lead to determinacy but small values of φ generate indeterminacy.

3.1.2 Instability Under Learning with the Fundamentals Based Reaction Function

Derivation of the interest rate reaction function (13) presupposed that economic agents in the model have RE. However, suppose instead that private agents may have possibly non-rational expectations, which they try to correct through adaptive learning. We assume that the policy maker does take this private agent learning into account, and continues to set policy according to (13). We are thus analyzing whether, when interest rates are set according to (13), the optimal REE is robust to transient errors in forecasting by private agents. (The formulation will be analogous in later sections when some other reaction function or instrument rule for interest rate setting is considered.)

In this analysis we employ the standard methodology of adaptive learning in macroeconomics, see (Evans and Honkapohja 2001b) that provides an extensive treatise on the subject. We now explain briefly the formulation and definition of stability of an REE under adaptive learning. For convenience, Appendix A.1 provides the general underpinnings of adaptive learning and the derivation of the stability conditions for the settings needed in this paper.

The basic idea in adaptive learning is the assumption that at any period t the private agents have a perceived law of motion (PLM) that they use to make forecasts. The PLM takes the form

$$y_t = a_t + b_t y_{t-1} + c_t v_t, \tag{18}$$

where the vector notation

$$y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}, v_t = \begin{pmatrix} g_t \\ u_t \end{pmatrix}$$

has been used for brevity. Note that for the reduced form (17) the optimal REE can be written as

$$y_t = \bar{a} + \bar{b} y_{t-1} + \bar{c} v_t,$$

where $\bar{a} = 0$ and where the second column of $\bar{b}$ is zero. Thus the PLM (18) has the same form as this REE, but in general the parameters (a_t, b_t, c_t) do not correspond to the REE values $(0, \bar{b}, \bar{c})$.

Given the PLM and the current value of v_t, the forecast functions of the private agents are

$$E_t^* y_{t+1} = a_t + b_t(a_t + b_t y_{t-1} + c_t v_t) + c_t F v_t,$$

where (a_t, b_t, c_t) are the parameter values of the forecasts functions that agents have estimated on the basis of past data up to and including period $t-1$.[15] These forecasts are used in decisions for period t, which yields the temporary equilibrium, also called the actual law of motion (ALM), for $y_t = (x_t, \pi_t)$ with the given PLM. The temporary equilibrium or ALM provides a new data point and agents are then assumed to re-estimate the parameters (a_t, b_t, c_t) with data through period t and use the updated forecast functions for period $t+1$ decisions. This in turn yields the temporary equilibrium for period $t+1$ and the learning dynamics proceeds further with the same steps. The REE $(0, \bar{b}, \bar{c})$ is said to be *stable under learning* if the sequence (a_t, b_t, c_t) converges to $(0, \bar{b}, \bar{c})$ over time.

Appendix A.1 gives the technical details on how to obtain the stability conditions for convergence to an REE. The central idea is to obtain a mapping T from the PLM parameters (a, b, c) to the implied ALM parameters, $T(a, b, c)$. The REE corresponds to a fixed point of this map and one can define a stability condition, known as E-stability, in terms of a differential equation describing partial adjustment of the PLM parameters towards the ALM parameters. E-stability turns out to provide the conditions for stability of an REE under least squares and related learning rules.

Earlier work by (Evans and Honkapohja 2001a) showed that discretionary policy, using interest rate setting based on fundamentals, leads to instability because learning by private agents fails to lead the economy to the REE corresponding to the optimal policy without commitment. It would seem possible that the full commitment policy implemented with (13) would perform better than discretion in this respect, because of the feedback of the output gap on interest rates. However, we have:

Proposition 2 *The fundamentals based reaction function leads to instability under learning for all structural parameter values.*

The proof is given in the Appendix A.3.

[15] In line with most of the literature we are assuming that current exogenous variables, and lagged but not current endogenous variables, are in the information set when forecasts are made.

The source of the instability lies in the interaction between the IS curve (1) and the price setting curve (2). The simplest intuition is obtained from considering a PLM (a, b, c) in which all of the parameters are held fixed at the optimal REE values, except for a_π, the intercept term in the PLM for inflation. In this case the mapping from PLM to ALM becomes one-dimensional and takes the form

$$T_{a_\pi}(a_\pi) = constant + (\beta + \lambda\varphi)a_\pi.$$

Since β is close to one and $\beta, \lambda, \varphi > 0$, for most parameter values we have $\beta+\lambda\varphi > 1$. A value of $a_\pi > 0$ will therefore tend to be adjusted upward, away from the equilibrium value. Intuitively, $a_\pi > 0$ corresponds to an exogenous positive shock to inflation expectations. This directly increases inflation by β times the shock. In addition via (1) the inflation expectations shock lowers the real interest rate, increasing output by φ times the shock, and through (2) this raises inflation indirectly by $\lambda\varphi$ times the shock. If $\beta + \lambda\varphi > 1$ then revisions to expected inflation in the direction of actual inflation will lead to a cumulative movement away from equilibrium and we have instability.

Under least-squares learning the dynamics are, of course, much more complicated and in particular all of the parameters (a, b, c) adjust to forecast errors. The proof of Proposition 2 shows that under the fundamentals based interest rate policy, the system is always locally unstable, even in the case $\beta + \lambda\varphi < 1$.

In summary, under private agent learning, the policy maker's ability to commit to optimal policies is not sufficient to stabilize the economy, if the policy reaction function is based on observable exogenous shocks and the lagged output gap in the way suggested by the standard theory for optimal policy. We now show how the instability problem can be overcome if private expectations are observable and interest rate policy conditions appropriately on their values.

3.2 An Expectations Based Reaction Function

The computation deriving the fundamentals based reaction function in Section 3.1 relied heavily on the assumption that the economy is in the optimal REE. We now obtain a different reaction function for interest rate setting, under optimal monetary policy, which does not make direct use of the RE assumption. Instead, recognizing the possibility that private agents may have

non-rational expectations during the learning transition, the policy rule is obtained by combining the optimality condition, the price setting equation and the IS curve, for given private expectations.[16] This leads to a monetary policy in which interest rates depend on observed private expectations as well as on fundamentals. We call this rule the *expectations based reaction function*.

Formally, combine the price-setting equation (2) and the optimality condition (8), treating private expectations as given. This leads to

$$x_t = \frac{\lambda}{\alpha + \lambda^2}\left[\frac{\alpha}{\lambda}x_{t-1} - \beta E_t^* \pi_{t+1} - u_t\right].$$

Next, substitute this expression in the IS curve (1) and solve for i_t. This yields the expectations based reaction function for interest rate setting:

$$i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t, \tag{19}$$

where

$$\begin{aligned}
\delta_L &= \frac{-\alpha}{\varphi(\alpha + \lambda^2)}, \\
\delta_\pi &= 1 + \frac{\lambda\beta}{\varphi(\alpha + \lambda^2)}, \\
\delta_x &= \varphi^{-1}, \\
\delta_g &= \varphi^{-1}, \\
\delta_u &= \frac{\lambda}{\varphi(\alpha + \lambda^2)}.
\end{aligned}$$

Looking at the rule (19) it can be seen that its coefficients stipulate a relatively large response to expected inflation ($\delta_\pi > 1$) and that effects coming from expected output gap and aggregate demand shock are fully neutralized ($\delta_x = \delta_g = \varphi^{-1}$). The positive coefficients on private expectations are crucial for ensuring stability of the REE and the sizes of the coefficients are chosen so that the economy is led to the optimal REE.

[16] This general approach was suggested and studied in (Evans and Honkapohja 2001a) in the context of discretionary policy.

The reduced form of the economy under (19) is

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 0 & -\frac{\lambda\beta}{\alpha+\lambda^2} \\ 0 & \frac{\alpha\beta}{\alpha+\lambda^2} \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + \begin{pmatrix} \frac{\alpha}{\alpha+\lambda^2} & 0 \\ \frac{\alpha\lambda}{\alpha+\lambda^2} & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} -\frac{\lambda}{\alpha+\lambda^2} \\ \frac{\alpha}{\alpha+\lambda^2} \end{pmatrix} u_t. \tag{20}$$

We now consider both determinacy and the stability under learning for the expectations based reaction function (19).

The methodology of Appendix A.2 can be applied to the reduced form (20). In Appendix A.3 we obtain the following result.

Proposition 3 *Under the expectations based reaction function (19) the REE is determinate for all structural parameter values..*

It is clearly a desirable property of our proposed monetary policy rule that it does not permit the existence of other suboptimal REE. However, as we have seen in the case of the fundamentals based reaction function, having a determinate REE does not always ensure that it is attainable under learning. To analyze stability under learning we can again use the general matrix framework in Appendix A.1. As in the preceding section we endow private agents with the PLM, compute the corresponding forecast function and substitute them into (20). This yields the temporary equilibrium or ALM and it is possible to study whether learning converges to the REE under the expectations based reaction function (19).

The next proposition shows that our interest rate rule performs well (see Appendix A.3 for the formal proof).

Proposition 4 *Under the expectations based reaction function (19), the optimal REE is stable under learning for all structural parameter values.*

Partial intuition for this result can be seen from the reduced form (20). An increase in inflation expectations now leads to an increase in actual inflation that is smaller than the change in expectations since $\alpha\beta/(\alpha+\lambda^2) < 1$. This dampened effect arises from the interest-rate reaction to changes in $E_t^*\pi_{t+1}$ and is a crucial element of the stability result. Of course, stability under learning requires convergence over time of all the PLM parameters, and Proposition 4 thus provides a remarkably strong result: Under the interest rate setting rule (19), learning is stable and the economy is guided

specifically to the optimal REE, and this result holds for all possible values of the structural parameters.

Our analysis has shown that the reaction function (19) is a robust method for implementing optimal monetary policy with commitment, passing both of the performance tests we discussed earlier. Because the optimal REE is determinate under the expectations based reaction function, there are no nearby sunspot equilibria that are consistent with the policy. Because it is stable under learning, the reaction function is robust to expectational errors by private agents. These positive results complement the analysis of (Evans and Honkapohja 2001a) for the corresponding implementation of optimal discretionary policy and show that a full policy optimum is obtainable with a well designed interest rate rule.

We remark that we have chosen our recommended rule carefully to ensure both determinacy and stability under learning for all parameter values. Alternative ways for interest rate setting have appeared in the literature, which can be interpreted as expectations based reaction functions, but which do not meet our tests. For example, the interest rate reaction function

$$i_t = (1 - \frac{\lambda}{\alpha\varphi})E_t\pi_{t+1} + \varphi^{-1}g_t. \tag{21}$$

is suggested in (Clarida, Gali, and Gertler 1999), Section 4.2.2. Replacing $E_t\pi_{t+1}$ with $E_t^*\pi_{t+1}$ leads to a policy reaction function based in part on observed expectations. This policy rule is consistent with the optimal policy under commitment under the RE assumption. However, as Clarida, Gali and Gertler note, this reaction function can lead to indeterminacy. Furthermore, it can be shown that if $\beta + \lambda^2/\alpha > 1$ the optimal REE is not stable under learning.

3.2.1 Discussion

The key to our stability results is that monetary authorities raise interest rates, *ceteris paribus*, in response to increases in inflation and output forecasts by private agents, and lower interest rates in response to decreases in private expectations. Given the fundamentals u_t, g_t and x_{t-1}, overly optimistic or pessimistic forecasts by private agents have the potential to destabilize the economy under least squares learning. Our expectations based policy is designed to offset this tendency and to guide the economy to the optimal REE.

Several points should be made concerning our results. First, although we have demonstrated our results in the context of least squares learning, the stability results will obtain under a number of alternative schemes. They would, for example, hold for related learning schemes such as stochastic gradient learning, with convergence to the optimal REE if our expectations based rule is followed. In fact, the stability results for the expectations based reaction function hold even for some forecast rules that do not converge to RE. For example, suppose private agents forecast both output and prices using the simple adaptive expectations rules:

$$\begin{aligned} x^e_{t+1} &= \gamma x_{t-1} + (1-\gamma)x^e_t, \\ \pi^e_{t+1} &= \gamma \pi_{t-1} + (1-\gamma)\pi^e_t, \end{aligned}$$

where $0 < \gamma < 1$. This forecast rule has a venerable history, but it is not rational in the current model. Nonetheless it can be shown that under our expectations based rule the economy is stable for all $0 < \gamma < 1$, though it will not, of course, converge to the optimal REE.

Second, since our recommended policy reaction function depends explicitly on private expectations, it is desirable to have high quality observations or estimates of private forecasts. However, our stability results extend to the case in which the reaction function depends on private expectations observed with a white noise measurement error. In this case there is convergence to an REE that deviates from full optimality by an amount depending on the measurement error variance. Furthermore, even if contemporaneous observations of expectations are not available, it may nonetheless be possible to either fully implement or approximate our policy, provided suitable auxiliary assumptions are made about the expectation formation process. Most obviously, if it is known that agents make forecasts based on a PLM of the form (18), with coefficients estimated using least squares, then policy-makers can construct accurate proxies for private expectations. To do so, policy-makers would proxy $E^*_t \pi_{t+1}$ and $E^*_t x_{t+1}$ by linear functions of $y'_{t-1} = (x_{t-1}, \pi_{t-1})$ and $v'_t = (g_t, u_t)$ with estimated coefficients, following the same procedure used by private agents. If, in addition, private expectations are observed with a lag, then the assumed behavior of private expectations can be tested and validated statistically.

Third, our discussion has implicitly assumed that the coefficients of the structural model (1) and (2) are known. This, however, is stronger than necessary. In our analysis of discretionary policy, in (Evans and Honkapohja

2001a), we showed that an expectations based policy could be implemented using estimated structural parameters and that the REE was stable under simultaneous learning by private agents and policy makers. An analogous argument should be applicable in the case of optimal policy with commitment.

4 Approximating Optimal Policy

(McCallum and Nelson 2000) have recently suggested that, in place of interest rate setting by a reaction function satisfying the optimality condition (8), there are well performing instrument rules that can approximate this condition. These instrument rules specify that the interest rate is moved towards a specified target value in response to deviations from the full commitment optimality condition (8).[17] In this section we analyze the performance of such rules for determinacy and stability under learning.

4.1 The Approximate Targeting Instrument Rule

To begin, we consider instrument rules of the form[18]

$$i_t = \pi_t + \theta[\pi_t + (\alpha/\lambda)(x_t - x_{t-1})]. \tag{22}$$

From now on we will call this rule the *approximate targeting instrument rule*, or simply the *approximate targeting rule.* This terminology describes better the underpinnings (22) than the general term "instrument rule", which is used in (McCallum and Nelson 2000).[19] Substituting (22) into the model

[17] See (McCallum 1999) for a general discussion of this approach.

[18] (McCallum and Nelson 2000) include a constant real interest target but this does not affect our results. They also suggest adding a lagged nominal interest rate to the rule, but this term is dropped in their numerical results. For simplicity, we also ignore such an inertial term. (Bullard and Mitra 2001a) analyze policies with interest rate inertia using the learning approach.

[19] (22) is a particular type of instrument rule in the terminology of (Svensson and Woodford 1999) and (Svensson 2001). The widely studied Taylor rules constitute another example of instrument rules.

(1)-(2) leads to the reduced form

$$\begin{aligned} & \begin{pmatrix} 1+\alpha\varphi\theta\lambda^{-1} & \varphi(1+\theta) \\ -\lambda & 1 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} \\ = & \begin{pmatrix} 1 & \varphi \\ 0 & \beta \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + \\ & \begin{pmatrix} \alpha\varphi\theta\lambda^{-1} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} g_t \\ u_t \end{pmatrix}. \end{aligned} \tag{23}$$

To assess determinacy we rewrite (23) in first-order form and compute the roots of the forward-looking matrix. Details are given in Appendix A.3. The eigenvalues do not lend themselves to clear theoretical results and thus we have studied them numerically. For the calibrated example $\beta = 0.99$, $\varphi = (0.157)^{-1}$ and $\lambda = 0.024$ in Section 3.1.1 we find numerically that the steady state seems to be determinate for all values of α and θ. Thus, the approximate targeting instrument rule performs well in this respect.

Next, we consider learning stability of the REE under the instrument rule (22). We apply the general methodology of Appendix A.1, see the explanations in Appendix A.3. Again, theoretical results cannot be obtained, so that numerical analysis must be used. Using the calibration given in Section 3.1.1 and setting $\rho = \mu = 1$ we have found stability under learning for all values of α and θ. We conclude that the approximate targeting rule (22) appears to lead to both determinacy and stability under learning.

4.2 Variants of the Approximate Targeting Rule

As pointed out by (McCallum and Nelson 2000), a difficulty with the approximate targeting rule (22) is that it presupposes that the policy maker can observe current output gap and inflation when setting the interest rate. This is not a satisfactory assumption and (McCallum and Nelson 2000) propose some alternative formulations. We explore some of these in this section.

Several possibilities appear natural. One possibility is to replace actual values of x_t and π_t by their forecasts, i.e. set the interest rate according to

$$i_t = E_t^* \pi_t + \theta[E_t^* \pi_t + (\alpha/\lambda)(E_t^* x_t - x_{t-1})].$$

However, (McCallum and Nelson 2000) find that this rule performs very poorly under REE. Another possibility would be to use a lagged version of

(22), but intuitively such a rule would lead to instability. We will thus discard these two variants.

(McCallum and Nelson 2000) find that making the approximate targeting rules forward looking leads to better performance under RE. In this case the policy maker adjusts the current interest rate in response to the discrepancy from the optimality condition (8) anticipated for the next period. This suggests interest rate setting according to

$$i_t = \tilde{E}_t \pi_{t+1} + \theta[\tilde{E}_t \pi_{t+1} + (\alpha/\lambda)(\tilde{E}_t x_{t+1} - \tilde{E}_t x_t)]. \tag{24}$$

Here $\tilde{E}_t(.)$ denotes the expectations of the policy maker and this can be interpreted in several ways.

One interpretation comes from working out the implied REE, i.e. setting $\tilde{E}_t \pi_{t+1} = E_t \pi_{t+1}$ etc. and substituting these expressions into (24). This would lead to an instrument rule of the form $i_t = \xi_0 + \xi_g g_t + \xi_u u_t$, which is known to lead to both indeterminacy and instability under learning, see (Evans and Honkapohja 2001a).

Another interpretation is to assume that the expectations of the policy maker are formed like those of private agents, i.e. $\tilde{E}_t \pi_{t+1} = E_t^* \pi_{t+1}$ etc. and that these are updated by least squares learning.[20] This leads to an expectations based rule, which is different from the well-performing reaction function (19) considered above.

We next analyze determinacy and learnability of the economy with the instrument rule (24) using the latter interpretation for expectations. Substituting (24), with $\tilde{E}_t(.) = E_t^*(.)$, into (1) yields the reduced form

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} -\alpha\varphi\theta\lambda^{-1} & -\varphi\theta \\ -\alpha\varphi\theta & \beta - \varphi\theta\lambda \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + \begin{pmatrix} \alpha\varphi\theta\lambda^{-1} & 0 \\ \alpha\varphi\theta & 0 \end{pmatrix} \begin{pmatrix} E_t^* x_t \\ E_t^* \pi_t \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix}. \tag{25}$$

Details of the conditions for determinacy and learning stability are given in Appendix A.3.

Determinacy depends on the values of the parameters. We adopt the calibrated values in Section 3.1.1 and examine the issue numerically. Determinacy obtains for sufficiently small values of the reaction parameter θ, but

[20] A further possibility would be to assume that the policy maker uses its own forecasts in (24). See (Honkapohja and Mitra 2002) for such an analysis for optimal discretionary policies and Taylor rules.

larger values of θ lead to indeterminacy. The boundary between determinacy and indeterminacy depends on the model parameters and, in particular, on the degree of flexibility α in inflation targeting. Indeterminacy obtains for smaller values of θ with more flexible inflation targeting as illustrated in the following table for which the calibration in Section 3.1.1 was used. (In the table $\theta > \hat{\theta}$ leads to indeterminacy.)

Table 1. Approximate critical θ value for indeterminacy

α	0.05	0.1	0.15
$\hat{\theta}$	0.038	0.019	0.013

Thus we have indeterminacy with the forward looking approximate targeting rules except for small values of the reaction parameter. This is problematic since, under RE, rules with a small value of θ imply that deviations from optimality lead to only small corrections towards meeting the optimality condition. We remark that (McCallum and Nelson 2000) consider large values of the reaction parameter in their quantitative analyses.

Numerical analysis indicates learning stability obtains for sufficiently small values of the reaction parameter θ, but larger values of θ will destabilize the economy under forward looking approximate targeting rules. To illustrate this we again revert to the calibration of Section 3.1.1 and we also set $\rho = \mu = 0.35$. It turns out that for sufficiently small values of θ the REE is stable under learning. However, instability obtains already for slightly larger values. The following table illustrates the dependence of the critical value for θ on the degree of flexibility α in inflation targeting. More flexible inflation targeting leads to instability with smaller values of the reaction parameter. (In the table $\theta > \bar{\theta}$ leads to instability.)

Table 2. Approximate critical θ value for learning instability

α	0.05	0.1	0.15
$\bar{\theta}$	0.118	0.055	0.039

The conclusion regarding learning stability of forward looking approximate targeting rules (24) resembles what we obtained with respect to determinacy for these rules. These rules lead to stability under learning only with sufficiently small values of θ, but such values imply long-lasting deviations from optimality. Large values of θ imply instability. In addition, less strict inflation targeting leads more easily to instability.

5 Concluding Remarks

This paper has analyzed determinacy and stability under learning for alternative interest rate reaction functions or instrument rules that aim to implement optimal monetary policy under commitment. Determinacy is desirable because it implies that there do not exist other (nonoptimal) REE near the solution of interest. Stability under learning is desirable because it indicates that if private agents follow least squares learning they will converge over time to the optimal REE.

Our analysis leads to the conclusion that the two desiderata are met by a policy that sets interest rates according to our expectations based reaction function. In this monetary policy reaction function, interest rates respond to private expectations as well as to fundamentals, i.e., exogenous shocks and the lagged output gap. This interest rate reaction function performs well as it unambiguously delivers both determinacy and stability under learning for the economy, with the economy converging over time to the optimal REE.

Attempting to implement of optimal policy through the fundamentals based formulation does not perform well. Such a policy will in some cases yield determinacy, but it always leads to instability under learning. The dependence on lagged output gap implied by commitment is not sufficient to achieve convergence under learning when interest rate setting is carried out using the reaction function based solely on fundamentals.

We also considered a class of approximate targeting instrument rules in which interest rates respond to the deviation from the optimality condition. Such rules may or may not be satisfactory, depending on the information available at the time interest rates are set. If contemporaneous observations of inflation and output gap are available to the policy maker, our numerical results indicate that these instrument rules do deliver both determinacy and stability under learning. However, it is arguably more realistic to assume that this contemporaneous data is not available, and for this reason a forward looking approximation of the target has been suggested as the basis for interest rate setting. Both indeterminacy and instability problems can arise for such formulations unless the reaction parameter is set at a sufficiently low value. However, for low reaction parameters the resulting REE is likely to deviate substantially from the optimal policy.

A Appendices

A.1 Stability Under Learning, General Methodology

The starting point for models of adaptive learning is agents have much less information than is presumed under RE. Instead of RE, private agents optimize using subjective (possibly non-rational) probability distributions over future variables. Their optimal behavior is still characterized by Euler equations involving the subjective expectations. The Euler equations for the current period are taken to be the behavioral rule giving current decisions as functions of the expected state next period. These Euler equations are then supplemented by forecasting rules for the state variables next period. Given the forecasts, the agents make decisions for the current period according to the Euler equations. This kind of behavior is boundedly rational but, in our view, reasonable. The decisions of the agents lead to a temporary equilibrium for the current period, which provides a new data point for the agents. Given this new data, the forecast functions are updated using standard adaptive learning rules such as least squares. The question of interest is whether this kind of (adaptive) learning behavior converges over time to REE of interest. (Note that the REE is a fixed point of the adjustment of forecast functions.)

When agents adjust their forecast functions over time, the dynamics of the economy is mathematically a stochastic recursive algorithm, which is a special type of nonlinear time varying stochastic system. The conditions for convergence of such dynamics are formally obtained from the local stability conditions of an associated ordinary differential equation. The latter conditions are in turn given by what are called *expectational or E-stability conditions*. (Evans and Honkapohja 2001b) provides an extensive treatise on the analysis of adaptive learning and its implications in macroeconomics (see also the other references in footnote 2). In this paper we will simply exploit this connection between convergence of learning dynamics and E-stability and carry out the analysis directly in terms of the E-stability conditions.

We derive E-stability conditions for a general matrix model

$$y_t = ME_t^* y_{t+1} + Ny_{t-1} + Pv_t. \tag{26}$$

Clearly, the model of monetary policy with different ways for setting the interest rate lead to particular cases of (26) by setting

$$y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}, v_t = \begin{pmatrix} g_t \\ u_t \end{pmatrix}.$$

For (26) an MSV REE takes the form

$$y_t = \bar{a} + \bar{b}y_{t-1} + \bar{c}v_t.$$

The description of learning proceeds as outlined in Section 3.1.2. To define E-stability we drop the time subscript from the parameters of the PLM but otherwise proceed in the same way to determine the temporary equilibrium or ALM for the given PLM. We consider PLMs of the form

$$y_t = a + by_{t-1} + cv_t. \tag{27}$$

E-stability conditions can be obtained using the methods of Chapter 10 of (Evans and Honkapohja 2001b). As above, we get

$$E_t^* y_{t+1} = (I + b)a + b^2 y_{t-1} + (bc + cF)v_t.$$

Inserting these expectations into (26) yields the ALM

$$y_t = M(I + b)a + (Mb^2 + N)y_{t-1} + (Mbc + NcF + P)v_t. \tag{28}$$

This equation defines the crucial mapping from PLM to ALM

$$T(a, b, c) = (M(I + b)a, Mb^2 + N, Mbc + NcF + P).$$

An MSV REE $(\bar{a}, \bar{b}, \bar{c})$ is a fixed point of this map.

E-stability of an REE $(\bar{a}, \bar{b}, \bar{c})$ is defined as local asymptotic stability under the differential equation

$$d/d\tau(a, b, c) = T(a, b, c) - (a, b, c).$$

Specifically, we are led to the matrix differential equations

$$\begin{aligned} \frac{da}{d\tau} &= M(I + b)a - a \\ \frac{db}{d\tau} &= Mb^2 + N - b \\ \frac{dc}{d\tau} &= Mbc + McF + P - c. \end{aligned} \tag{29}$$

The second equation in (29) is nonlinear and it must be linearized. Moreover, the second and third equations in (29) are matrix values that need to vectorized.

We remark that the time concept τ in (29) is notional time. Indeed, (29) is a partial adjustment formula in time τ, where the parameters are gradually adjusted in response to deviations from equilibrium. Such gradual adjustment also characterizes the asymptotic behavior of least squares and related learning rules, since in such rules parameter adjustment responds to forecast errors and, after a long history, new data points have only a small effect on parameter estimates. See Chapter 2 of (Evans and Honkapohja 2001b) for a more detailed explanation of this connection.

The stability conditions can be stated in terms of the derivative matrices

$$\begin{aligned} DT_a &= M(I + \bar{b}) && (30)\\ DT_b &= \bar{b}' \otimes M + I \otimes M\bar{b} && (31)\\ DT_c &= F' \otimes M + I \otimes M\bar{b}, && (32) \end{aligned}$$

where $\otimes$ denotes the Kronecker product.

Remark 5 *The necessary and sufficient condition for E-stability is that all eigenvalues of $DT_a - I$, $DT_b - I$ and $DT_c - I$ have negative real parts.*[21]

Finally, some cases of interest rate setting lead to frameworks without lagged endogenous variables, but with expectations of current values present in the reduced form. Consider the general matrix model

$$y_t = BE_t^* y_{t+1} + CE_t^* y_t + Kv_t \tag{33}$$

In this case, the MSV solution takes the form

$$y_t = a + hv_t, \tag{34}$$

where in the REE the coefficients satisfy $a = (B+C)a$ and $h = BhF+Ch+K$.

For E-stability we use the PLM (34) with general values for a and h. Computing expectations we get

$$\begin{aligned} E_t^* y_t &= a + hv_t,\\ E_t^* y_{t+1} &= a + hFv_t \end{aligned}$$

and using (33) gives the ALM

$$y_t = (B + C)a + (BhF + Ch + K)v_t.$$

[21] We are excluding the exceptional cases where one or more eigenvalue has zero real part.

E-stability conditions now require that the eigenvalues of the matrices

$$DT_a - I = B + C - I \quad (35)$$
$$DT_h - I = F' \otimes B + I \otimes C - I \quad (36)$$

have negative real parts.

A.2 Determinacy

The general methodology for ascertaining determinacy is given in the Appendix to Chapter 10 of (Evans and Honkapohja 2001b). For models with reduced form (26) we need only consider the special case in which the second column of N is zero. Writing $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ and $N = \begin{pmatrix} n_{11} & 0 \\ n_{21} & 0 \end{pmatrix}$, assuming rational expectations, introducing the new variable $x_t^L \equiv x_{t-1}$, and noting that for any random variable z_{t+1} we have $E_t z_{t+1} = z_{t+1} + \varepsilon_{t+1}^z$ where $E_t \varepsilon_{t+1}^z = 0$, we can rewrite (26) as

$$\begin{pmatrix} 1 & 0 & -n_{11} \\ 0 & 1 & -n_{12} \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \\ x_t^L \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{t+1} \\ \pi_{t+1} \\ x_{t+1}^L \end{pmatrix} + other,$$

where "other" includes terms that are not relevant in assessing determinacy. Assuming $n_{11} \neq 0$ this can be rewritten as

$$\begin{pmatrix} x_t \\ \pi_t \\ x_t^L \end{pmatrix} = J \begin{pmatrix} x_{t+1} \\ \pi_{t+1} \\ x_{t+1}^L \end{pmatrix} + other \quad (37)$$

where

$$J = \begin{pmatrix} 1 & 0 & -n_{11} \\ 0 & 1 & -n_{12} \\ 1 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Because this model has one predetermined variable, i.e. x_t^L, the condition for determinacy is that exactly two eigenvalues of J lie inside the unit circle and one eigenvalue outside. If one or no roots lie inside the unit circle (with the other roots outside), then the model is indeterminate.

For models with reduced form (33) the reduced form can be rewritten as

$$(I - C)y_t = By_{t+1} + other,$$

where $y_t = (x_t, \pi_t)'$. Since this model has no predetermined variable the determinacy condition is that $J = (I - C)^{-1}B$ have both roots inside the unit circle. If one or both roots lie outside the unit circle then the model is indeterminate.

A.3 Derivations

Proof of Proposition 1. Applying the methodology of Section A.2 to the reduced form (17) we obtain

$$\begin{aligned} J &= \begin{pmatrix} 1 & 0 & \varphi\psi_x \\ 0 & 1 & \lambda\varphi\psi_x \\ 1 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1 & \varphi & 0 \\ \lambda & \beta + \lambda\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & \beta & \lambda \\ (\varphi\psi_x)^{-1} & {\psi_x}^{-1} & -(\varphi\psi_x)^{-1} \end{pmatrix}. \end{aligned}$$

Since the model has two free variables, determinacy requires that exactly two eigenvalues of J are inside the unit circle. Straightforward numerical calculations for the calibrated example show that two eigenvalues of J lie outside the unit circle, and one lies inside, for small values of α, so that the steady state is indeterminate, while for larger values of α exactly one root lies outside the unit circle, and the model is determinate. We remark that continuity of eigenvalues implies that both regions contain open sets of parameters.

Proof of Proposition 2. We apply the general methodology outlined above in Appendix A.1, when the general model (26) takes the specific form (17). In this case

$$\begin{aligned} M &= \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix}, \\ N &= \begin{pmatrix} -\varphi\psi_x & 0 \\ -\lambda\varphi\psi_x & 0 \end{pmatrix} \text{ and} \\ P &= \begin{pmatrix} 0 & -\varphi\psi_u \\ 0 & 1 - \lambda\varphi\psi_u \end{pmatrix}. \end{aligned}$$

Looking at the differential equations (29) defining E-stability in this case, it can be seen that the equations for the elements of b are independent of the other variables, while the equations for a and c are dependent on b but not on each other. Because of this recursive structure, a necessary condition for stability is that $DT_a - I$, evaluated at the REE, has eigenvalues with negative real parts. This condition is equivalent to $\mathrm{tr}(DT_a - I) < 0$ and $\det(DT_a - I) > 0$.

Using the notation $b = (b_{ij})$, $j = 1, 2$ and evaluating variables at the REE, we have $b_{11} = \bar{b}_x$, $b_{21} = \bar{b}_\pi$ and $b_{12} = b_{22} = 0$. The coefficient matrix for a in (30) for the reduced form (17) has the explicit form

$$DT_a - I = \begin{pmatrix} \bar{b}_x + \varphi\bar{b}_\pi & \varphi \\ \lambda(\bar{b}_x + 1) + (\beta + \lambda\varphi)\bar{b}_\pi & (\beta + \lambda\varphi) - 1 \end{pmatrix}. \tag{38}$$

The determinant of the coefficient matrix (38) is

$$(\beta - 1)\bar{b}_x - \varphi\bar{b}_\pi - \lambda\varphi < 0$$

since the parameters λ, φ are positive, $0 < \beta < 1$ and the REE values $\bar{b}_x$ and $\bar{b}_\pi$ are positive. The result follows.

Proof of Proposition 3. Applying the methodology of Section A.2 to the reduced form (20) we obtain

$$J = \begin{pmatrix} 1 & 0 & \frac{-\alpha}{\alpha+\lambda^2} \\ 0 & 1 & \frac{-\alpha\lambda}{\alpha+\lambda^2} \\ 1 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 & -\frac{\lambda\beta}{\alpha+\lambda^2} & 0 \\ 0 & \frac{\alpha\beta}{\alpha+\lambda^2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \beta & \lambda \\ 0 & \frac{\beta\lambda}{\alpha} & \frac{\alpha+\lambda^2}{\alpha} \end{pmatrix}.$$

The roots of J are 0 and $(2\alpha)^{-1}\left(\alpha + \alpha\beta + \lambda^2 \pm \sqrt{(\alpha + \alpha\beta + \lambda^2)^2 - 4\alpha^2\beta}\right)$. It can be verified that the nonzero roots are real and positive, with one root less than one and the other root larger than one. Since exactly two roots are inside the unit circle, the result follows.

Proof of Proposition 4. We note that in this case the matrices in the E-stability conditions can be written as

$$DT_b - I = \begin{pmatrix} \frac{-\beta\lambda\bar{b}_\pi}{\alpha+\lambda^2} - 1 & \frac{-\beta\lambda\bar{b}_x}{\alpha+\lambda^2} & 0 & 0 \\ \frac{\alpha\beta\bar{b}_\pi}{\alpha+\lambda^2} & \frac{\alpha\beta\bar{b}_x}{\alpha+\lambda^2} - 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{39}$$

$$DT_c - I = \begin{pmatrix} \frac{-\lambda\beta\bar{b}_\pi}{\alpha+\lambda^2} - 1 & -\frac{\lambda\beta\rho}{\alpha+\lambda^2} & 0 & 0 \\ \frac{\alpha\beta\bar{b}_\pi}{\alpha+\lambda^2} & \frac{\alpha\beta\rho}{\alpha+\lambda^2} - 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{40}$$

and

$$DT_a - I = \begin{pmatrix} \frac{-\beta\lambda\bar{b}_\pi}{\alpha+\lambda^2} - 1 & \frac{-\beta\lambda}{\alpha+\lambda^2} \\ \frac{\alpha\beta\bar{b}_\pi}{\alpha+\lambda^2} & \frac{\alpha\beta}{\alpha+\lambda^2} - 1 \end{pmatrix}. \tag{41}$$

Looking at the coefficient matrix (39), it has two eigenvalues equal to -1 while the remaining two eigenvalues are those of the 2×2 matrix in the top left corner of $DT_b - I$. The trace of this 2×2 matrix is given by

$$\frac{-\beta\lambda\bar{b}_\pi}{\alpha + \lambda^2} + \frac{\alpha\beta\bar{b}_x}{\alpha + \lambda^2} - 2,$$

which is negative since the only positive term is less than one. Its determinant is equal to

$$\frac{\beta\lambda\bar{b}_\pi}{\alpha + \lambda^2} - \frac{\alpha\beta\bar{b}_x}{\alpha + \lambda^2} + 1,$$

which is positive as the only negative term is less than one absolute value (since $\beta < 1$ and $0 < \bar{b}_x < 1$). Thus the matrix (39) is stable (i.e. all of its eigenvalues have negative real parts).

Next, consider the matrices (40) and (41). The matrix (40) has two eigenvalues equal to -1 and the remaining two are those of the 2×2 matrix in the top left corner. The trace of this 2×2 matrix is

$$\frac{-\lambda\beta\bar{b}_\pi}{\alpha + \lambda^2} + \frac{\alpha\beta\rho}{\alpha + \lambda^2} - 2.$$

The only positive term (if $\rho > 0$) is less than one and so the trace is always negative. (If $\rho < 0$, all terms are negative.) Its determinant is

$$\frac{\beta\lambda\bar{b}_\pi}{\alpha + \lambda^2} - \frac{\alpha\beta\rho}{\alpha + \lambda^2} + 1$$

and the only (possibly) negative term is less than one and so the determinant is positive. Thus $DT_c - I$ is a stable matrix. Finally, we note that the top

left 2×2 matrix with $\rho = 1$ is identical to the matrix (41), so that the latter is also a stable matrix.

Details on Approximate Targeting Instrument Rules:

(1) Determinacy of the approximate targeting rule (22). Using a method analogous to Section A.2 we rewrite (23) as

$$\begin{aligned} & \begin{pmatrix} 1+\alpha\varphi\theta\lambda^{-1} & \varphi(1+\theta) & -\alpha\varphi\theta\lambda^{-1} \\ -\lambda & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \\ x_t^L \end{pmatrix} \\ = & \begin{pmatrix} 1 & \varphi & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{t+1} \\ \pi_{t+1} \\ x_{t+1}^L \end{pmatrix} + other. \end{aligned}$$

This leads to an equation of the form (37) with

$$\begin{aligned} J &= \begin{pmatrix} 1+\alpha\varphi\theta\lambda^{-1} & \varphi(1+\theta) & -\alpha\varphi\theta\lambda^{-1} \\ -\lambda & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1 & \varphi & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & \beta & \lambda \\ -\frac{\lambda}{\alpha\varphi\theta} & \frac{(-1+\beta+\beta\theta)\lambda}{\alpha\theta} & \frac{\lambda+\alpha\varphi\theta+(1+\theta)\lambda^2\varphi}{\alpha\varphi\theta} \end{pmatrix}. \end{aligned}$$

Determinacy requires exactly two roots of J inside the unit circle.

(2) E-stability of the approximate targeting rule (22). The matrices in (23) have the form

$$\begin{aligned} M &= \begin{pmatrix} 1+\alpha\varphi\theta\lambda^{-1} & \varphi(1+\theta) \\ -\lambda & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & \varphi \\ 0 & \beta \end{pmatrix}, \\ N &= \begin{pmatrix} 1+\alpha\varphi\theta\lambda^{-1} & \varphi(1+\theta) \\ -\lambda & 1 \end{pmatrix}^{-1} \begin{pmatrix} \alpha\varphi\theta\lambda^{-1} & 0 \\ 0 & 0 \end{pmatrix} \text{ and} \\ P &= \begin{pmatrix} 1+\alpha\varphi\theta\lambda^{-1} & \varphi(1+\theta) \\ -\lambda & 1 \end{pmatrix}^{-1}. \end{aligned}$$

It is not possible to derive theoretical results from the E-stability conditions in Remark 5 and so we evaluate the conditions numerically. For convenience

we set $\rho = \mu = 1$ so that $F = I$. In this case the conditions for DT_a and DT_c are identical. Otherwise, the numerical example in Section 3.1.1 is used.

(3) Determinacy of the forward looking approximate targeting rule (24). Write (25) in the form

$$\begin{pmatrix} 1-\alpha\varphi\theta\lambda^{-1} & 0 \\ -\alpha\varphi\theta & 1 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} -\alpha\varphi\theta\lambda^{-1} & -\varphi\theta \\ -\alpha\varphi\theta & \beta-\varphi\theta\lambda \end{pmatrix} \begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} + other.$$

Determinacy requires that the eigenvalues of

$$J = \begin{pmatrix} 1-\alpha\varphi\theta\lambda^{-1} & 0 \\ -\alpha\varphi\theta & 1 \end{pmatrix}^{-1} \begin{pmatrix} -\alpha\varphi\theta\lambda^{-1} & -\varphi\theta \\ -\alpha\varphi\theta & \beta-\varphi\theta\lambda \end{pmatrix}$$

lie inside the unit circle.

(4) E-stability of the forward looking approximate targeting rule (24). The reduced form (25) is of the form (33). From Section A.1 the required conditions are that matrices (35) and (36) have negative real parts. First consider

$$B + C = \begin{pmatrix} 0 & -\varphi\theta \\ 0 & \beta-\varphi\theta\lambda \end{pmatrix}.$$

One of the eigenvalues of $B + C$ is zero and the other one is $\beta - \varphi\theta\lambda$, which is always less than one, so the first stability term is met.

It can be shown that the 4×4 matrix $DT_h - I$ in (36) is block diagonal with the upper left hand block given by

$$DT_{UL} - I = \begin{pmatrix} \frac{\alpha\theta\varphi(1-\mu)}{\lambda} - 1 & -\alpha\theta\varphi \\ \alpha\theta\varphi(1-\mu) & \mu(\beta-\theta\lambda\varphi) - 1 \end{pmatrix}$$

and the lower right hand block formally the same except that ρ replaces μ. We thus only need to analyze $DT_{UL} - I$. We have

$$Trace(DT_{UL} - I) = \frac{\alpha\theta\varphi(1-\mu)}{\lambda} + \mu(\beta - \theta\lambda\varphi) - 2$$

and

$$Det(DT_{UL} - I)) = \frac{\theta\lambda^2\mu\varphi + (1-\beta\mu)[\lambda - \alpha\theta\varphi(1-\mu)]}{\lambda},$$

which show that the trace and determinant can have either sign, depending on parameter values. One must thus revert to numerical analysis, as reported in the main text.

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