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Modelling euro banknote quality in circulation

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Contents

Abstract

The quality of banknotes in the cash cycles of countries in the Eurosystem varies, despite all of these countries using identical euro banknotes. While it is known that this is dependent on national characteristics, such as public use and the involvement of the central bank in cash processing operations, the influence of all relevant parameters has not yet been established. This paper presents two computer-based models for the simulation of banknote cash cycles. The first model simulates a cash cycle using a theoretical approach based on key figures and models banknote fitness as a one-dimensional profile of fitness levels. The model identifies: (i) the frequency with which banknotes are returned to the central bank; (ii) the fitness threshold used in automated note processing at the central bank; and (iii) the note lifetime as the main drivers of banknote quality in circulation as well as central bank cash cycle costs. Production variations in new banknotes, the fitness threshold applied by commercial cash handlers and the accuracy of the fitness sensors used in the sorting process have been found to have a lower but non-trivial impact. The second model simulates banknotes in circulation as single entities and is oriented towards modelling country-specific cash cycles using available single-note data. The model is constructed using data collected by monitoring banknotes in circulation over the duration of a “circulation trial” carried out in three euro area countries. We compare the predicted quality results of the second data-based model against actual cash cycle data collected outside the circulation trial, discuss the reasons for the deviations found and conclude with considerations for an optimal theoretical national cash cycle.

Keywords: banknotes, banknote circulation, banknote quality, banknote lifetime, circulation modelling

JEL classification: C46, C63, E42, E58

Non-technical summary

In 2016, euro area national central banks (NCBs) checked 32.3 billion euro banknotes for quality and authenticity using their high-speed machines. Commercial cash handlers (CHs) processed a similar number. CHs disburse banknotes of good quality to their customers and eventually return all poor-quality ones (unfit banknotes) and surplus stock to the NCBs. The NCBs destroy all unfit banknotes after a final authenticity and quality check. In 2016, NCBs replaced 5.4 billion unfit banknotes (around 27% of the banknotes in circulation) with new ones in order to maintain stable quality in circulation. All these steps are part of the cash cycle.

Banknotes become unfit from soiling and by receiving defects such as tears, folded corners and so forth. To uphold the quality of banknotes in the cash cycle, the Eurosystem has defined minimum note fitness standards for NCBs and a second set of slightly lower standards for CHs. When comparing national note quality (expressed as a percentage of banknotes found in circulation that are unfit in accordance with NCB standards) with the results of an opinion poll on the public perception of banknote quality, there is a certain quantity of banknotes that are unfit by NCB standards but that the public appears to accept without any negative opinion on banknote quality.

The involvement of NCBs and CHs, as well as the general use of cash by the public, varies between euro area countries. Data collected by the Eurosystem show significant differences between the countries' cash cycles in relation to per-capita note destruction and note quality in circulation. These differences often do not follow the intuitive assumption that high note destruction results in clean circulation of banknotes – an indication that other effects play a determinant role for a cash cycle.

We therefore developed two computer-based models in order to better understand these differences in euro area cash cycles and to provide a tool for NCBs to better understand and optimise their cash cycles. The first model simulates a cash cycle using a theoretical approach based on key figures, such as banknote lifetime due to soiling, sorting volumes, banknotes in circulation and fit/unfit thresholds applied in automated note processing. It also includes other factors, such as the accuracy of the fitness sensors used, the likelihood of a note suffering a defect and the volume of note inflows into and outflows from a national cash cycle from/to other euro area countries as well as euro banknotes returning from outside the euro area. The model simulates fitness as a one-dimensional profile of fitness levels. By studying scenarios based on two theoretical cash cycles, we found that the frequency of note return to the central bank, the fitness threshold used in automated note processing at the central bank and the note lifetime are the main drivers of banknote quality in circulation and cash cycle costs. We also found that the volume of note inflows into and outflows from a national cash cycle has a substantial impact on circulation quality and an NCB's ability to control banknote quality. While such flows are difficult to determine, we found that they need to be included in the model for results to match actual banknote quality data.

The second model simulates banknotes in circulation as single entities and is oriented towards modelling country-specific cash cycles using available per-banknote data. The model is constructed using data collected by monitoring banknotes in circulation over the duration of a “circulation trial” carried out in Austria, Ireland and the Netherlands. We compared the predicted quality results of the second data-based model with the quality of an actual note sample from circulation. The deviations between our second model and real-life data are attributed to: (i) inaccuracies in translating the fitness values measured by the NCB sorting machines to the homogeneous fitness levels used in the model; (ii) the unavailability of data on note inflows and outflows; and (iii) the model being based on data collected at the NCBs and not directly from circulation.

We conclude by providing suggestions on how future circulation trials can be improved to receive data which allow more accurate modelling of national cash cycles. We use the results of the scenario study for our first model to provide suggestions on how national cash cycles can be improved so as to reduce replacement costs for new banknotes and the costs to NCBs for processing banknotes.

On the policy impact, we acknowledge the national specificities of the different cash cycles and conclude that our models need to be adjusted and filled with accurate national data. They can then be used to derive quantitatively correct national models, which NCBs can use to optimise their national cash cycles.

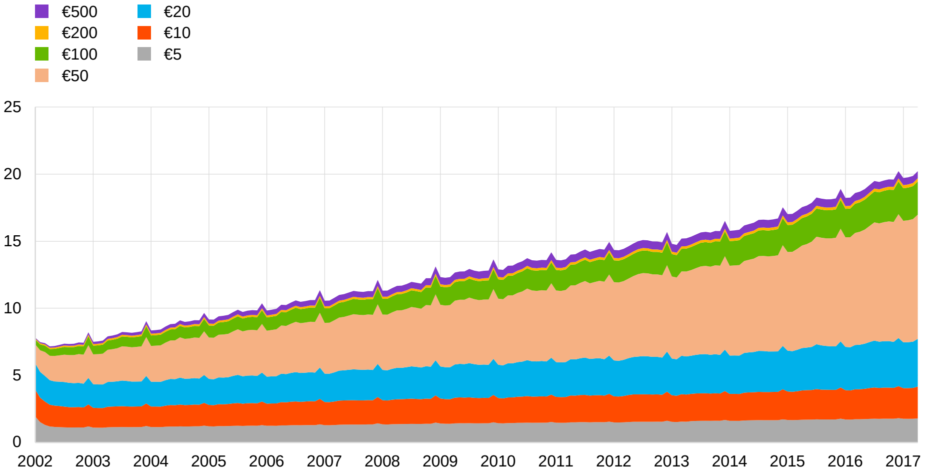
1 Introduction

At the end of December 2016, 20.2 billion euro banknotes with a nominal value of €1.12 trillion were in circulation. Compared with end-2015, banknote circulation had increased by 7.0% in volume terms and by 3.9% in value terms. These figures are well in line with the average annual increases over the last five years of 7.8% in volume and 6.1% in value (see Chart 1). The euro banknote circulation increases if NCBs issue banknotes; by contrast, it decreases if NCBs receive banknote lodgements – usually poor-quality banknotes or surplus stock – from commercial cash handlers (CHs)[1].

Chart 1

Cumulative number of euro banknotes in circulation

(y-axis: number of euro banknotes in billions)



Source: Eurosystem Currency Information System 2.
Note: The volumes for the €5 to €50 banknotes are the sum of the first series and Europa series banknotes.

The Eurosystem has a duty to ensure public confidence in euro banknotes by maintaining adequate quality in circulation. Poor-quality banknotes are likely to be rejected by vending machines and also make it difficult for the public and retailers to spot counterfeits. Two factors are mainly responsible for maintaining quality. The first is providing durable banknotes: To this end, the lifespan of the Europa series €5 and €10 banknotes has been extended by applying an additional protective varnish layer. The second is the involvement of NCBs in the cash cycle, replacing soiled and defective banknotes detected during machine processing. However, banknote quality in circulation also depends on various other factors. For example, if only few ATMs dispense €5 banknotes, these will stay in circulation longer to make up for their limited availability as change. Retailers will retain them for use rather than return

[1] CHs are the institutions and economic agents referred to in Article 6(1) of Regulation (EC) No 1338/2001, later amended by Regulation (EC) No 44/2009 ("*Credit institutions, and, within the limits of their payment activity, other payment service providers, and any other institutions engaged in the processing and distribution to the public of notes and coins* […]"). For the purposes of this paper, all parties other than NCBs that process banknotes for recirculation are considered CHs.

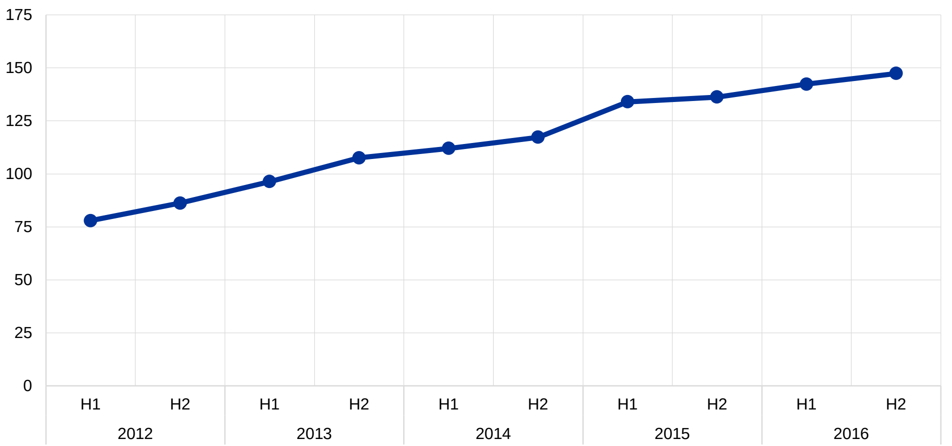
them to the NCB, which is therefore unable to remove any soiled banknotes from circulation.

Since 2011, CHs have been able to disburse used banknotes (i.e. recirculate them) as long as they observe specific rules set out in ECB Decision ECB/2010/14 on the authenticity and fitness checking and recirculation of euro banknotes (the "Recirculation Framework" (European Central Bank, 2010))[2]. More specifically, any recirculated euro banknotes must have been processed on banknote sorting machines that have been tested by the Eurosystem and are listed on the ECB's website (European Central Bank, 2017c). This test encompasses the detection of all topical counterfeits and the accurate automatic separation of fit from unfit banknotes. In addition, CHs are obliged to report half-yearly, alongside other information, the number and type of machines in use together with the volume of banknotes processed, recirculated and sorted as unfit. The Recirculation Framework was swiftly adopted by CHs. Since the initial reporting of machines used in accordance with the Recirculation Framework, the number of compliant banknote handling machines in operation has almost doubled (from around 78,000 in 2012[3] to more than 147,000 by the end of 2016, as seen in Chart 2). Recirculating fit banknotes rather than lodging them with an NCB helps commercial cash handlers to reduce transport and handling costs substantially but also reduces an NCBs ability to steer banknote quality.

Chart 2

Number of banknote handling machines in operation complying with the ECB Recirculation Framework

(y-axis: number of BHMs in operation (in 1,000))



Source: Eurosystem Central Registry System.

In 2016, the number of banknotes processed by cash handlers (35.7 billion) surpassed the NCBs' sorting volumes (32.3 billion) for the first time. This constitutes, on a euro area level, a shift in the operational involvement in the cash cycle from NCBs to CHs (see Chart 3). Of the total number of banknotes processed by the

2 In some euro area countries, the recirculation of banknotes was not permitted before that time; in other countries recirculation was carried out based on bilateral agreements between CHs and the NCB.

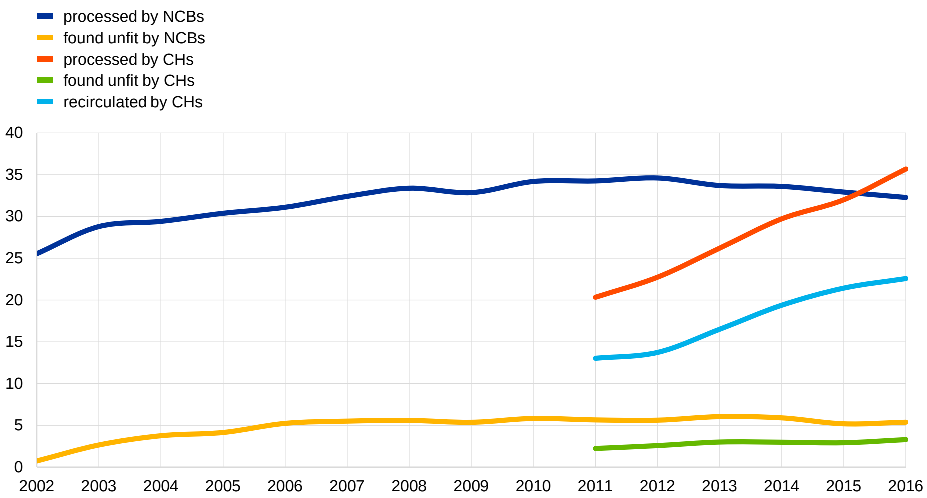
3 The Recirculation Decision entered into force on 1 January 2011 with a one-year transitional period for statistical reporting.

latter, about two-thirds (22.6 billion) were found fit and returned directly to circulation (recirculated); the remainder were returned to the NCBs. Only 2.3 billion of these returned banknotes were unfit, meaning that they did not comply with the minimum quality standard stipulated in the Recirculation Framework for CHs; the remainder were fit surplus banknotes.

Chart 3

Number of banknotes processed/recirculated by CHs and NCBs per annum

(y-axis: number of euro banknotes in billions)



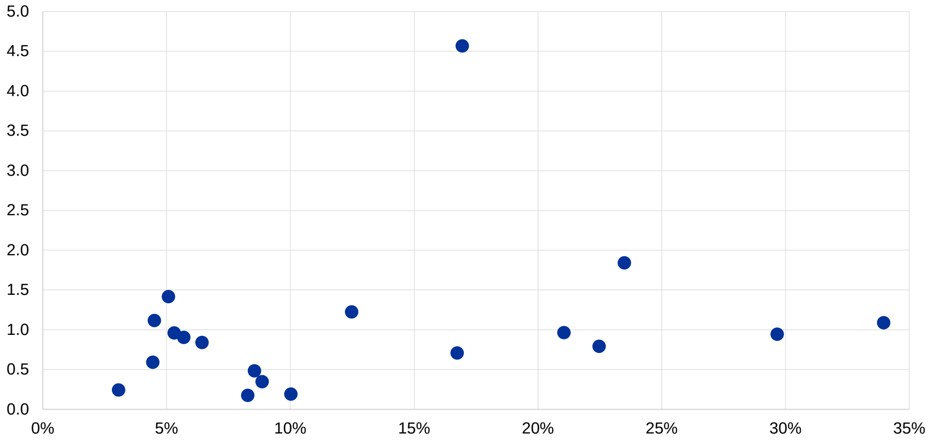
Source: Eurosystem Currency Information System 2.

The intuitive assumption regarding the quality of euro banknotes in circulation in different euro area countries is that higher destruction of poor-quality banknotes – and their replacement with new banknotes – results in improved quality. When comparing a country's quality in circulation with per-capita banknote destruction, however, such behaviour is not in fact visible (see Chart 4). The chart shows a heterogeneous picture for the Europa series €5 banknote, with most countries having a fairly low destruction rate of about one banknote per person per annum, but it contains some significant outliers. Some countries have lower quality in circulation despite a high destruction rate; others have very high quality in circulation, although they destroy less than half as many banknotes as the euro area average. This is an indication that differences in the national cash cycles play a significant role; however, these specific national influences are not yet sufficiently understood.

Chart 4

Percentage of unfit banknotes found in a representative sample of a country's circulation vs. banknotes destroyed per inhabitant per annum for the Europa series €5 banknotes

(x-axis: banknotes found to be unfit as a percentage of notes in circulation; y-axis: note destruction per capita from May 2014 to April 2015)



Sources: 2015 Eurosystem banknote quality survey, Eurosystem Currency Information System 2, Eurostat population figures (for larger countries 2015 Eurostat population figures were used; for smaller countries a correction (increase in population) for tourism and migration has been introduced using national statistical data sources).
Notes: One data point per euro area country. See Section 2.4 for an explanation of how a representative sample of a country's circulation has been collected. The sample for the Eurosystem banknote quality survey from which the unfit in circulation (x-axis) was derived, was collected from March to May 2015. Banknote destruction per person and year (y-axis) is calculated using the sum of banknotes destroyed for the preceding 12 months (i.e. from 1 May 2014 to 30 April 2015).

Developing a model of the circulation has been an ongoing research topic, and other authors have developed circulation models (Lacker & Wolman, 1997) and applied them to solve policy issues (Janicki, Main, Waddle & Wolman, 2007). Earlier publications have also applied models to study the impact of banknote durability on total costs when using polymer instead of cotton paper for banknotes (Menzies, 2004), (Van Hove, 2015), (Bouhdaoui, Bounie & Van Hove, 2013) or have focused on forecasting banknote demand when economic variables (such as interest rates or GDP growth) change (Cabrero, Camba-Mendez, Hirsch & Nieto, 2002). However, studies have so far not covered the impact of cash cycle parameters relevant for analysing national euro cash cycles. Such parameters include in particular the impact of recirculation by commercial cash handlers, hoarding of banknotes as a store of value, in- and outflows of banknotes from/to other (euro area) countries and the modelling of inaccuracies in fitness classification of banknotes by the sorting machines used by NCBs or CHs (see Section 2.2 for further explanations).

We developed a computer-based model incorporating all main parameters that are known to affect a cash cycle and that can be applied on a national and an aggregate euro area level. Such a model helps to better understand the different euro area countries' cash cycles and the factors that influence note consumption and note quality. The model is published together with this publication and can also be applied to non-euro area cash cycles. Apart from the Eurosystem, the simulation of local cash cycles is of relevance for other (large) countries in which different regions present a similarly heterogeneous picture. This model, hereafter referred to as Model A, is used to study the behaviour of well-defined cash cycles. In addition, a second model, Model B, has been developed, which is focused on simulating country-specific cash cycles using per-banknote data. Data from an external circulation trial[4], which was conducted by the three NCBs of Austria, Ireland and the Netherlands from mid-2014 to end-2016, were used to calibrate Model B.

Section 2 of our paper explains the main stakeholders in a cash cycle and the key parameters that influence banknote fitness. These include the differences in sorting standards, the mechanisms behind banknote ageing and issues relating to the automated classification of fitness by comparison with human perception. In Section 3 we present the basic algorithms of the two models and the results we derived from using them to simulate either theoretical or real-life cash cycles (Section 4). Section 5 provides a summary and discusses potential policy changes, concluding with an outlook for future work.

4 An exercise where one or more NCBs issue within a very short period a statistically relevant number of banknotes, which are then monitored, typically by serial number reading in NCB cash centres.

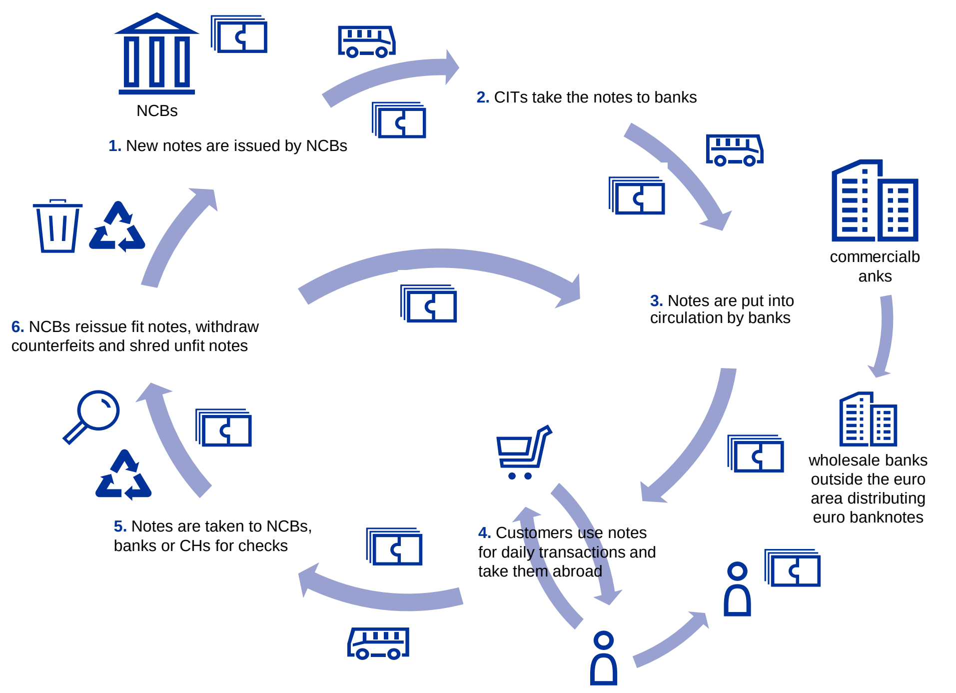
2 The banknote lifecycle

2.1 Overview of the stakeholders

The lifecycle of a banknote involves several interacting processes and stakeholders. A schematic overview of a typical euro area cash cycle[5] is shown in Figure 1 below. Based on an annual banknote demand forecast, the final consolidated volume of euro banknotes (European Central Bank, 2017a) are produced by accredited printing works and delivered to the NCBs. NCBs issue the banknotes to CHs, usually cash-in-transit companies (CITs), who deliver them to commercial banks, to retailers or directly to ATMs. The public receive banknotes primarily via ATMs or as change from retailers, and to a lesser extent over the counter at banks. Retailers' excess banknotes are deposited at a bank branch or picked up by CITs. They are then either returned directly to the NCB for quality (fitness) and authenticity checks or put back into circulation (recirculated) by a CH. Depending on the denomination of the note and the country, a banknote may be processed and returned to the cash cycle multiple times before becoming unfit and sorted out for destruction.

Figure 1

The banknote lifecycle



5 In this paper we use the term cash cycle synonymously for banknote cash cycle. Coins are not considered in our models.

In addition, euro banknotes are exported to and imported from countries outside the euro area, not only by specialised CHs (banknote wholesale banks supplying bureaux de change, for example), but also by the public, be it for tourism or commercial purposes. The cumulative net exports to non-euro area countries via bulk shipments of euro banknotes by banknote wholesale banks had reached €172.8 billion by end-2016. While this constitutes about 15% of the total value of euro banknotes in circulation, this figure is considered too low for the total euro banknotes held outside the euro area, given that euro banknotes leave and re-enter the euro area through several other channels. The ECB (European Central Bank, 2017b) provides an improved estimate stating that “at the end of 2016 residents outside the euro area held approximately €341 billion in euro banknotes” together with a lower bound estimate of €274 billion and an upper bound estimate slightly above €405 billion. Other authors (Bartzsch, Rösl & Seitz, 2011a) (Bartzsch, Rösl & Seitz, 2011b) estimate that more than 50% in value of the euro banknotes issued in Germany circulates outside the euro area. No estimations of the denominational breakdown have so far been established.

Such inflows and outflows of banknotes from and to outside the euro area, but also between different countries of the euro area, substantially affect the national cash cycles. Some euro area countries experience negative net issuance of certain denominations. This happens if an NCB receives more banknotes than it issues. The reason for such negative net issuance lies in denominations flowing into the country from abroad, either from another euro area country or from regions outside the euro area. Such banknote migration is caused primarily by tourism and cross-border commuting. In order to balance such “natural” flows, the Eurosystem regularly carries out large-volume cross-border transports of euro banknotes between its Member States. These ensure that countries with positive net issuance can meet their banknote demand at any time.

While the general cash cycle holds true for all NCBs, the processing shares and roles of NCBs and CHs vary greatly from country to country. This is due to national specificities. While in 2016 the aggregated ratio of CH to NCB processing was close to 1.1 (see Chart 3), national figures range from 0 (i.e. no recirculation by CHs at all) to CHs processing more than five times the NCB sorting volume.

2.2 Banknote fitness as judged by humans and machines

During their lifespan, banknotes deteriorate and decrease in quality, i.e. banknote fitness declines. The fitness of a banknote is defined by its soil level and whether it bears any defects. Previous research into banknote ageing identified soiling as one of the main reasons for a circulating banknote to become unfit (Balke, 2011), (Kyrychok, Shevchuk, Nesterenko & Kyrychok, 2014) over time. Soil consists primarily of human sebum (a waxy substance produced by skin glands) that is transferred onto the banknote by handling, as well as dirt particles (Balke, 2011). Apart from soiling, the second unfit category comprises defects such as stains, graffiti markings, tape, dog-ears and tears. While soiling is typically a gradual process, a banknote usually becomes defective at a defined moment in time (e.g.

when it is torn or stained), and the underlying process is therefore binary. The final overall fitness of a banknote is a result of soiling and defects, with the contribution of each two varying between national cash cycles.

The quality of banknotes in circulation is assessed by the public, and – as already stated – the Eurosystem has a duty to ensure public confidence by providing good-quality banknotes. The difficulty is, however, that banknote quality is judged at NCB level not by humans, but by automated high-speed processing machines. Ensuring that the machine's judgement correlates well with human perception of the fitness of a banknote is therefore of prime concern for an NCB. Fitness measurement is usually performed by banknote processing (sorting) machines that process up to 33 banknotes per second, capture an image of the banknote, apply various algorithms to the image and finally decide whether the note is fit or unfit for circulation (Kropnick, 2012). While the machines' assessment of mechanical defects is closely related to the human eye's perception and mainly follows the same scales, this is not the case to the same extent for soil assessment. Here, the machine-specific algorithms (which are proprietary and not usually disclosed by the manufacturer) deliver arbitrary fitness/soil values that do not correlate to any standard fitness scale and need to be aligned to human perception of soil. Soil assessment by sorting machines is influenced by different factors, most significantly:

- imperfections in the banknote transport and camera system;
- dust coming from the processed banknotes (such as residues of paper fibres or ink) affecting the image quality;
- the gloss on new banknotes, which has been shown to significantly affect the soil assessment;
- despite strict quality control during the production of euro banknotes, production variations resulting in slight differences in new banknote production batches.

In order to ensure that NCBs apply soil standards that match human perception, the ECB has created a standardised batch ("soil fitness test deck")[6] of euro banknotes from circulation for use in evaluating sorting machines. The test deck contains banknotes of all fitness levels in terms of soil. Based on the visual assessment by Eurosystem experts, a "true" fitness value has been allocated to each banknote in the test deck based on its soil intensity. Naturally, the judgement of any automated fitness sensor will not exactly match this soil-based fitness value derived by human expert judgement, so there will be cases of misclassifications of the quality of the banknotes. This means that either fit banknotes are incorrectly destroyed prematurely ("false-unfit banknotes") or unfit banknotes are judged fit and reissued ("false-fit banknotes"). Eurosystem research applying the test deck has confirmed that different high-speed sorting machines have substantially different classification accuracy (assessed based on false-fit/false-unfit rates). This is attributed primarily to

6 This test deck is only used for NCB sorting machines. The testing of CH machines is discussed in the section below.

the different technologies and algorithms used for soil classification (Buitelaar, 2008). Evaluating sorting machines based on their false-fit and false-unfit rates is only a simplistic approach, and the Eurosystem is currently developing alternative methods based on overall correlation of the machine fitness values with human perception.[7]

2.3 Eurosystem standards for measuring banknote fitness

The Eurosystem has defined minimum thresholds at which an NCB has to classify a banknote as unfit for circulation ("Eurosystem threshold[8]"), in line with common practice among central banks (Federal Reserve Bank Services – Currency Technology Office, 2008) (Central Bank of Russia). These thresholds include limits for soiling and for any of the defect categories. The Eurosystem threshold for banknotes is very much "on the fit side", and such banknotes are readily accepted by the public. All NCBs need to adhere to these minimum requirements, and only a small percentage of the banknotes that they reissue are permitted not to fulfil these criteria. This tolerance margin, which is set at 8% of the banknotes reissued, is due to the uncertainties of the note classification by sorting machines, as described in the previous section. An NCB can apply a stricter sorting policy by adjusting its sorting fitness threshold (the "NCB threshold"") to counteract low quality of banknotes in the national cash cycle, which could be due, for example, to a low frequency of banknote return to the NCB.

The minimum fitness standards for CHs ("CH threshold") are defined by the Recirculation Framework and are lower than those for NCBs, meaning that banknotes of lower quality than the NCB standard are considered fit by CHs. This is to ensure that, even including the measuring tolerances, the banknotes reissued by an NCB are also fit for the CHs and can be recirculated before reaching the end of their lives.

To ensure that the machines used by CHs fulfil the minimum standards defined in the Recirculation Decision, the mandatory testing of any machine used by CHs includes a fitness detection test. This test is conducted using a standardised set of genuine euro banknotes with soiling and defects that are similar to those encountered in circulation. The test is passed if not more than 5% of the unfit euro banknotes are classified as fit by the respective machine. An NCB may, after informing the ECB, lay down stricter standards for CHs for one or more denominations of euro banknotes if this is justified, for example due to a deterioration in the quality of the euro banknotes in circulation (European Central Bank, 2010).

Figure 2 below shows banknote ageing over its lifespan, together with the inaccuracies in the production and processing steps.

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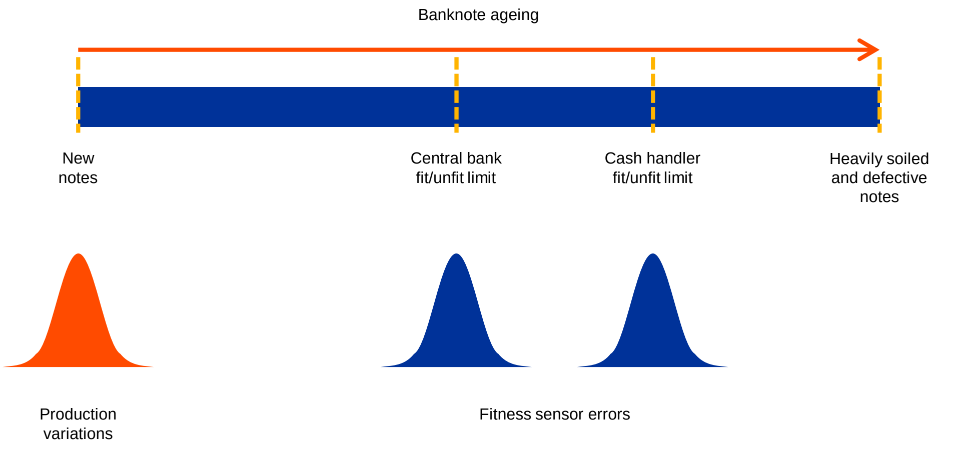
[7] Furthermore the current alternative methods which are under development within the Eurosystem have shown that the accuracy of a sensor to assess soiling is dependent on the denomination and also not consistent along a normalised fitness range.

[8] When presenting percentages of unfit banknotes in circulation, this paper – if not otherwise specified – refers to banknotes unfit according to the Eurosystem threshold.

Figure 2
Overview of the variations and errors in a banknote's life



2.4 How the Eurosystem measures quality in circulation

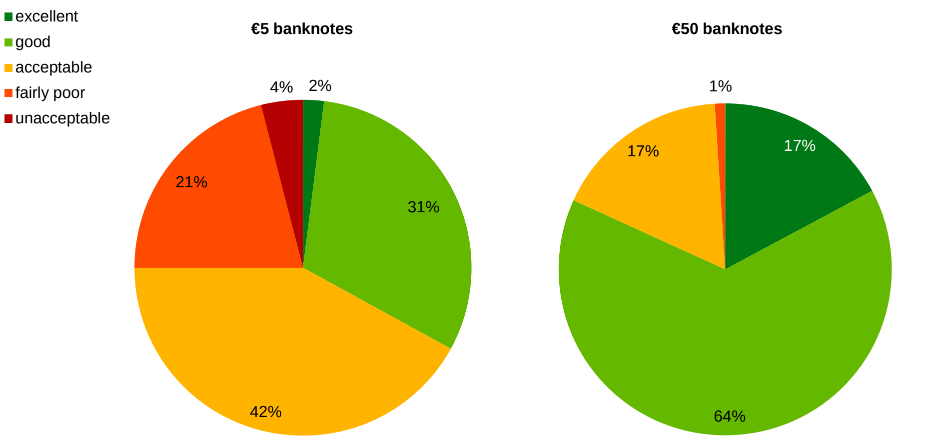
Different methods can be used to determine the actual banknote quality in circulation, all of which rely on some kind of sampling regime. Each year the Eurosystem collects a representative sample of the transactional denominations (€5-€100) and determines the share of unfit banknotes in this sample according to the Eurosystem threshold. Here, samples of the national circulation of each euro area country are processed on the high-speed sorting machines of two NCBs, and the average share of unfit notes – be that due to soil or defects – in each sample is determined. The results of this annual quality survey (QS) are an important input for the NCBs, enabling them to observe if the banknote quality in circulation is adequate and to adjust their sorting policies.

In addition to the QS, the Eurosystem carries out a public opinion poll every two years to gain an insight into public perception of note quality. Since 2012, an online poll has also been available (Online survey on the quality of euro banknotes). Both polls focus on the quality of the €5 and the €50 banknotes. The €5 note usually has lower quality because it remains in circulation as change, returning to NCBs less often. Taking the euro area average, the quality of the €5 note is considered good, with 75% of participants ranking quality as acceptable or higher (see Chart 5). Almost all respondents (99%) consider the €50 note to be of at least acceptable quality. Comparing the national results of the quality survey with the national responses in the online poll reveals a good correlation between the percentage of unfit banknotes found in a country's sample and public opinion of the €5 note (see Chart 6). However, this pattern is not observed for the €50 note; as this is generally of good quality, there are no data in the unfit/negative responses area (top right) of the graph.

Chart 5

Physical condition of euro banknotes as found in the 2012 public opinion survey

(Eurosystem averages)

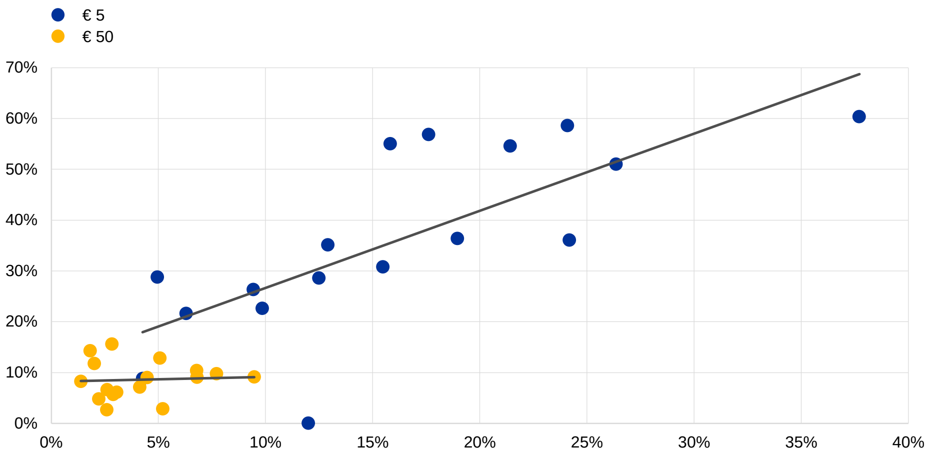


Source: 2012 ECB public opinion survey on euro banknotes.
Note: Answers to the question: "How would you generally describe the physical condition of the €5/€50 banknotes in circulation?" Results shown are for 2012, as the latest figures (2014) were affected by the introduction of the Europa series €5 note.

Chart 6

Correlation between banknotes unfit by central bank standards found in circulation and online feedback received for €5 and €50 banknotes (one data point per euro area country)

(x-axis: banknotes found to be unfit as a percentage of notes in circulation; y-axis: percentage of negative responses (fairly poor/unacceptable) in the online poll)



Sources: 2012 online survey on the quality of euro banknotes, 2012 quality survey.
Notes: One data point per euro area country. The percentage of negative responses shown on the y-axis is the sum of fairly poor/unacceptable responses in the 2012 online survey (see the note of Chart 5 for the detailed questions and answer options).

In our paper, we consistently rely on the share of unfit notes according to Eurosystem criteria as a measurement of banknote quality. However, it is acknowledged, that this does not necessarily provide the best correlation with public perception of banknote quality. The way in which the public assess banknote quality has been studied by other authors (van der Horst, Meeter, Theeuwes & van der Woude, 2011), who have found that public appreciation is in fact more complex and the fit/unfit judgement correlates with the sum of soil and different defects of a banknote.

2.5 The lifespan of a banknote

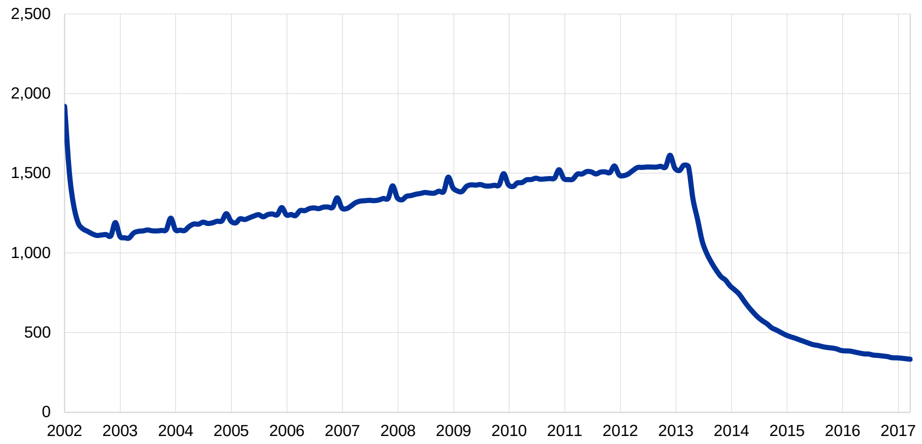
All euro banknotes within a denomination have the same substrate and print specifications; however, the lifespan of a note from its first issuance to destruction at an NCB depends not only on its physical durability but also on national cash cycle characteristics. The way in which banknotes are treated by the public (e.g. whether banknotes are stored in wallets or in trouser pockets) and also environmental factors such as humidity play a significant role in the time it takes for a banknote to become unfit. The frequency of banknote return to either CHs or NCBs then has an impact on how quickly unfit banknotes can be removed from circulation.

The lifespan of a banknote is commonly defined as the total number of banknotes in circulation divided by the banknotes destroyed per annum. However, this formula does not take into account banknotes that do not actually circulate because they are used as a store of value, have been lost or have migrated outside of a national cash cycle or even the euro area. As an example, Chart 7 shows that, despite the issuance of the Europa series €5 banknote already back in May 2013, together with the destruction of all €5 banknotes of the first series upon return to the NCBs, 342 million first series €5 banknotes had not yet been returned to NCBs as of end-2016 and should therefore not be considered part of active circulation.

Chart 7

First series €5 banknotes in circulation

(y-axis: number of banknotes in millions)



Source: Eurosystem Currency Information System 2.
Notes: The peak in 2002 was due to frontloading of banknotes to commercial banks to ensure sufficient banknote supply during the euro introduction.

More accurately, the lifespan of a banknote can be determined by

$$Lifespan[years] = \frac{Notes\ in\ \boldsymbol{active}\ circulation}{Notes\ destroyed\ per\ annum}$$

However, available data do not allow accurate determination of the active circulation for each country and denomination. Even though there is no clear boundary between an active and an inactive note, we can qualitatively define the active banknotes as those that are used in transactions and are regularly sorted by an NCB or CH (ter

Huurne, Post, Duijndam, Overakker, Vis & Broeder, 2010). Here, the authors defined all banknotes returning to the NCB within 200 days as active circulation for the Dutch cash cycle. The Eurosystem therefore has to rely on estimates that take into account national data on NCB and CH processing, as well as NCB destruction volumes. Banknote flows due to cross-border commuting, tourism and CH shipments are also included in these national estimates to the extent known.

When speaking about the lifespan of a banknote, we also need to draw a distinction between the time it takes for a banknote to reach the NCB fitness threshold and the time after which a banknote is actually returned to the NCB and destroyed (Den Butter & Coenen, 1982). It is also clear that not every banknote has the same lifespan. While each banknote ages independently, the overall population follows a certain distribution. This will be discussed in the next section and has also been studied previously (Den Butter & Coenen, 1982), (Koeze, 1979), (Martin & Meuer, 2001).

3 Models – definition and application

So far, models published on banknote circulation (Lacker & Wolman, 1997), (Menzies, 2004), (Van Hove, 2015) have not covered all the cash cycle specificities mentioned above, such as sorting inaccuracies, the different mechanisms behind defects and soil, or the impact of production variations and inflows and outflows. We have therefore tried to incorporate these into our two models, which are presented in this section.

3.1 Common modelling concepts

Both models, Model A and Model B, simulate a cash cycle and, based on the input parameters, derive outputs of interest. The most important results are the NCB destruction rate, the quality in circulation (% unfit in circulation), the note replacement cost and the actual banknote lifespan in circulation. Similarly to the approach described in Lacker & Wolman (1997), the model starts with a population of banknotes which is evolved over a number of iterations until a steady state is reached. Each iteration in the model simulates the cash cycle activities of sorting and ageing in a one-week[9] period. The cash cycle's steady state is the equilibrium point where the sorting activities of CHs and the NCBs counteract the ageing of banknotes in circulation due to soiling and defects, as well as any growth/decrease and inflows/outflows in circulation volume. While the paper mostly discusses steady-state results, both models also allow the simulation of dynamic step changes to a cash cycle, a practical example of which is presented in Section 4.1.4.

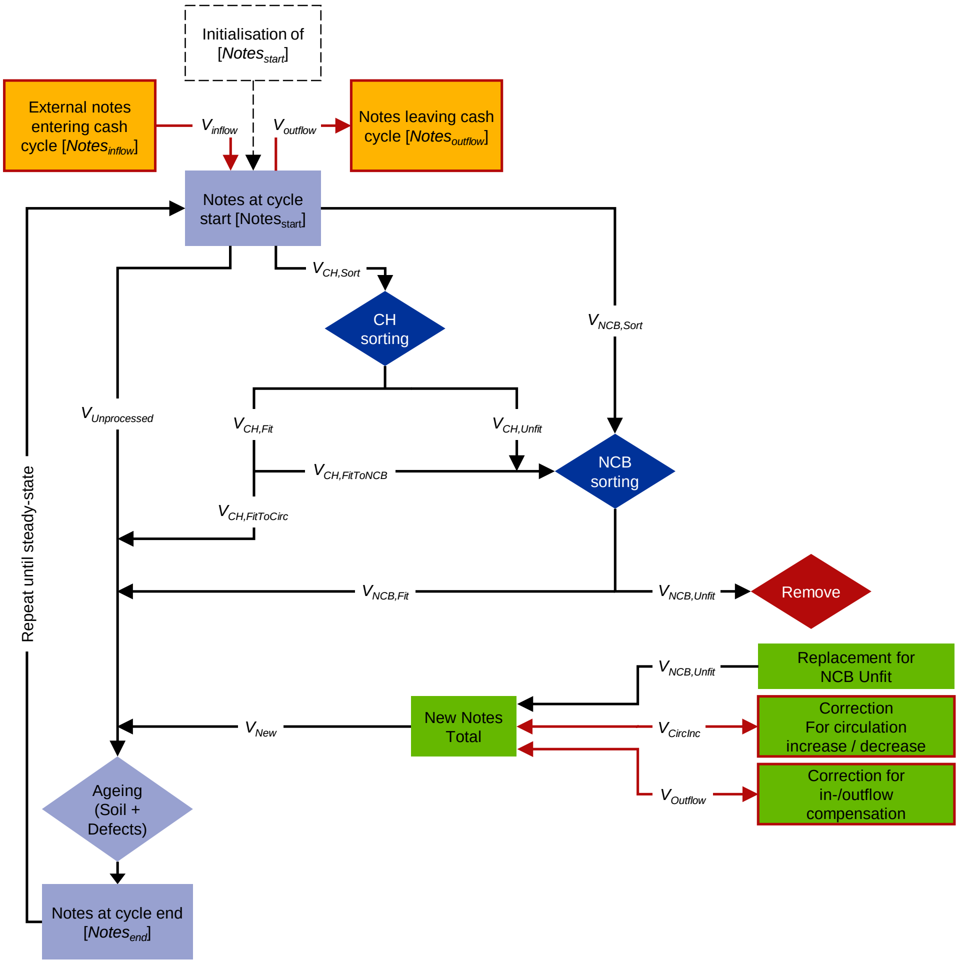
In the theoretical model (Model A), we have introduced variables for a general note volume increase due to increased demand and also inflows and outflows of banknotes. However, this was not implemented in the Model B, which is based on real-life data, due to the absence of accurate figures (the respective blocks are marked in red in the flowchart shown in Figure 3).

The cash cycle as implemented in the models is shown in Figure 3.

9 One week was chosen as a simulation time unit for all subsequent simulations, because it also matched the reporting frequency of the data used in Model B. While both models allow the cycle time to be defined, we have found that as long as the cycle time is much lower than the banknote lifetime, the model results do not show substantial variations.

Figure 3

Block diagram of the two banknote circulation models (blocks framed in red are implemented in Model A only)



In the models, banknote fitness is defined according to a fitness scale from 1 to 100, with the Eurosystem threshold set at 50. A fitness level of 1 corresponds to the cleanest new banknote, and 100 denotes any banknote that is more than twice as soiled as the Eurosystem threshold, as well as any defective banknotes. Defects do not follow a continuous scale in our model. Instead, a note becomes defective in an instantaneous event (see Section 3.2.1 for more details), resulting in the current fitness level of the note being changed to 100. This ensures that defective notes are consistently sorted to unfit by CHs and NCBs[10]. With this approach, any population of banknotes can be characterised by a fitness profile, which is the frequency of soil levels in this [1…100] population.

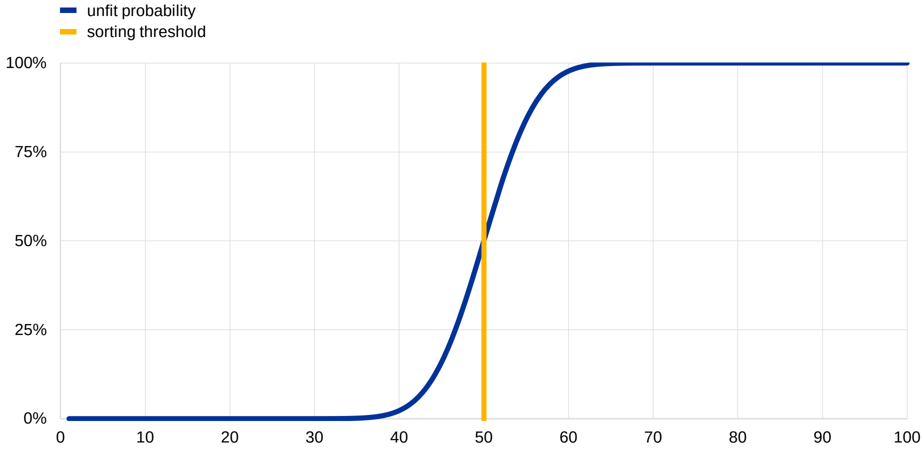
10 This is the predominant case in reality, as the camera systems usually have no problem detecting defects such as dog-ears, etc.

The inaccuracy of NCB and CH fitness sensors is modelled by a Gaussian distribution with a standard deviation defined in the 1 to 100 fitness scale (see Chart 8).

Chart 8

Example probability of a banknote sorted to unfit with a sensor having a standard deviation of five fitness levels operating at the Eurosystem threshold (50)

(x-axis: banknote fitness level; y-axis: percentage probability of a note sorted to unfit)



The model is initialised with a starting note population. This starting fitness profile can be chosen to resemble the fitness of banknotes in circulation or the fitness of a batch of new banknotes. For steady-state calculations, this starting population has an impact on the final model results only in the sense of the number of iterations it takes for the model to reach steady state, depending on the difference between $Notes_{Start}$ and the final steady-state profile. The following steps are then carried out in each iteration:

1. Part of the note population ($V_{CH,Sort}$) is subjected to a recirculation sorting step.

2. A further share of banknotes are then sorted by the NCB. These banknotes are a mix of the banknotes from circulation ($V_{NCB,Sort}$), the unfit notes returned by CHs ($V_{CH,Unfit}$) and the excess fit recirculation sent back by the CHs to the NCB ($V_{CH,FitToNCB}$). The banknotes processed by the NCB are sorted into fit and unfit, with all unfit banknotes being removed from circulation.

3. The volume of new banknotes added in each cycle (V_{New}) corresponds to the number of banknotes sorted to unfit by the NCB ($V_{NCB,Unfit}$) plus – in Model A – an additional correction for the general growth/decrease ($V_{CircInc}$) and compensation for any inflows or outflows ($V_{Outflow}$). The models allow any profile to be specified for V_{New}. Typically, a Gaussian distribution reflecting normal production variations is used.

4. All banknotes in circulation (i.e. banknotes not processed in the cycle, $V_{Unprocessed}$, banknotes sorted as fit by CHs, $V_{CH,Fit}$ and the NCB, $V_{NCB,Fit}$ and any new banknotes, V_{New}) are aged. Ageing is modelled as a two-step process

and entails applying algorithms to simulate how banknotes gradually become soiled and suffer defects.

5. Lastly, $Notes_{End}$ is compared with $Notes_{Start}$. If the correlation between the two profiles is higher than the specified correlation coefficient, the steady-state condition has been reached and the final results are displayed to the user. Otherwise $Notes_{Start}$ is set to $Notes_{End}$ and steps 1-5 are repeated.

As in previous models (Lacker & Wolman, 1997), the NCBs' stocks of banknotes do not play an explicit role in our model. The implicit assumption here is that the stock of used currency in inventory (of any given quality) is constant over time and any changes are to cover seasonal patterns in banknote demand without substantially affecting note circulation quality.[11]

From a design perspective, the main difference between the two models is that Model A simulates the development of a population fitness profile defined as a series of fitness levels from 1 to 100 and their respective frequency in the total population. Model B simulates single banknotes, which can carry multiple attributes, and processes these banknotes according to probabilities derived from real-life cash cycle data. Model A changes the banknote population by developing the fitness profile and as such implicitly assumes that soil is the only independent driving variable in banknote ageing. This has so far also been found by other authors (Koeze, 1979), (Den Butter & Coenen, 1982), but it limits further development should the influence of other factors need to be studied.

3.2 Model A: Theoretical model-specific concepts

3.2.1 Ageing of a banknote

The ageing of a banknote is simulated in two steps: soiling and defects. The average soiling per cycle is determined by the "theoretical note lifetime" and applied to each note via a definable distribution function. The "theoretical note lifetime" is the time it takes for a new banknote to become unfit, i.e. to go from fitness level 1 to 50. It is an input parameter for the model and depends on the banknote durability, but also on environmental factors, such as how intensively banknotes are used by the public in the simulated cash cycle.

In an earlier publication, it was found that for the Canadian $2 cash cycle, the hazard function of banknotes becoming unfit resembles a log-normal distribution (Gillieson, 1977). For the Dutch cash cycle, the lifetime distribution of a note at the time of the study could also be modelled with a gamma distribution (Koeze, 1979). Other publications later found (Den Butter & Coenen, 1982) that the probability of a banknote becoming soiled can be modelled by a gamma, an exponential or a Weibull distribution. In the most recent circulation trial carried out in the Eurosystem,

[11] As can be seen in Chart 1.

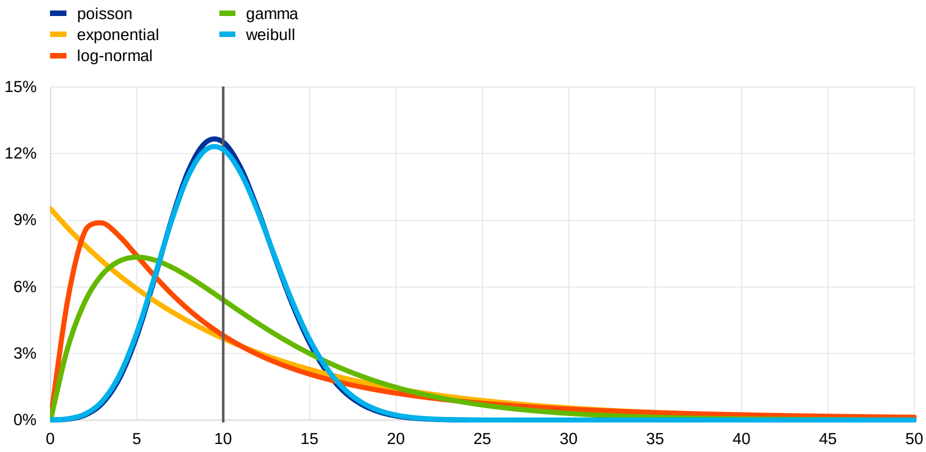
the gamma and Weibull distributions were found to best describe the survival probability of banknotes (other options included log-normal and log-logistic distributions).

In our two models the different ageing of the note population can be simulated via different distribution functions (gamma, Poisson, exponential, Weibull, logarithmic but also others), such that within each model cycle the note profile ages (i.e. is shifted to higher fitness values) by an average number of soil levels according to the specific ageing profile, as shown in Chart 9 below (example). We assume that banknotes in all fitness levels age by an identical number of levels in each calculation cycle, irrespective of the current fitness level.

Chart 9

Various ageing distribution functions, each having an average ageing of ten fitness levels per model iteration

(x-axis: increase in fitness levels, y-axis: frequency)



In the next step, defects are simulated by each banknote being assigned a probability (expressed as a percentage per year) of suffering a defect (i.e. being moved from its current fitness level to 100). The defect likelihood is applied to the banknote distribution according to a selectable profile related to the note's fitness level. This approach enables modelling that, for example, banknotes with a higher soil value have a higher likelihood of becoming defective. In our studies, we found that the closest correlation to real-life quality data can be achieved by applying, in each iteration, a defect probability that increases linearly with the degradation of (increase in) the fitness level. This is based purely on an iterative approach, to optimise the model to fit real-life data, and is not yet supported by any studies on the behaviour of defects in relation to banknotes' fitness level. There have been indications that the behaviour of defects is also substantially affected by varnishing of banknotes or – in more general terms – the banknote substrate.

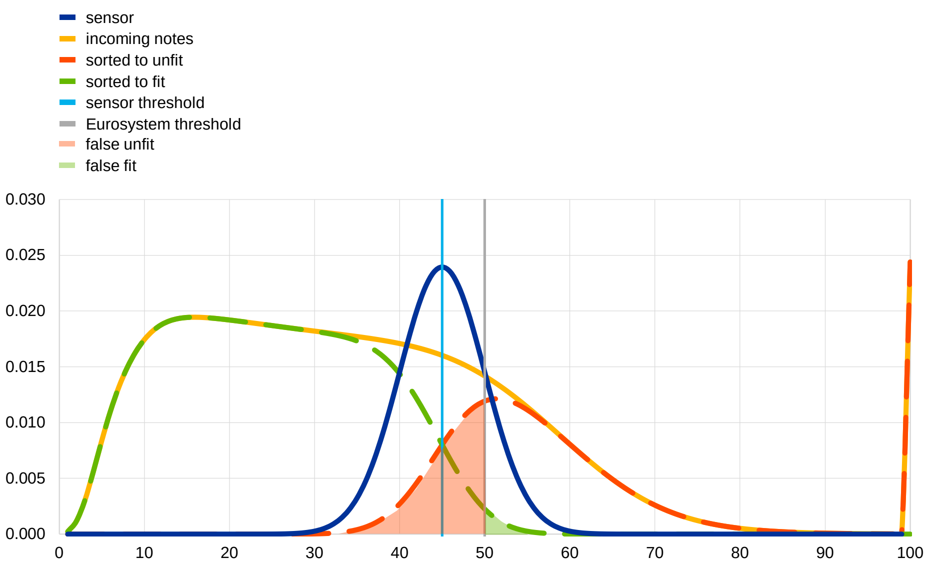
3.2.2 Sorting of banknotes

The sorting step for both NCBs and CHs is simulated by applying a model sensor with inaccuracies following a Gaussian distribution (see Chart 8) onto the fitness profile of incoming banknotes. The inaccuracy of the model sensor is expressed in standard deviations (SD), measured in fitness levels. Chart 10 shows how a fitness sensor, operating at a sensor threshold of 45 fitness levels and having an inaccuracy simulated by a SD of five fitness levels, separates a typical note circulation profile (with 20% unfit banknotes (i.e. fitness level of >50)) into fit and unfit banknotes. In the example presented, a small number of the banknotes sorted to fit are more soiled than the Eurosystem threshold (false-fit, 0.4%[12]) due to sensor inaccuracies and the distance between the sensor and Eurosystem threshold, but a substantial number of banknotes sorted to unfit are less soiled than the Eurosystem threshold (false-unfit, 9%).

Chart 10

Schematic depiction of the sorting step in Model A

(x-axis: note fitness level (1: new, 50: Eurosystem threshold, 100: very soiled); y-axis: frequency)



Source: ECB Banknote Circulation Model.
Note: The spike at the 100 fitness level denotes defective and very unfit notes.

3.2.3 Inflows and outflows of banknotes

Model A allows banknote inflows and outflows to be simulated. Modelled banknote inflows can have any fitness profile; both the note volume and associated fitness profile need to be specified. For banknote outflows only the volume needs to be specified, as the fitness profile of outflows is assumed to be the same as that of the current banknote circulation in the cash cycle. Both inflows and outflows are modelled to be neutral to the banknote circulation volume. For outflows, this is

12 Share of total sorted banknotes, same for false-fit.

achieved by replacing the missing banknotes in each model cycle with additional new banknotes issued by the NCB. For inflows, the NCB new note replacement volume in each cycle is reduced accordingly.

3.2.4 Circulation volume increase

An annual increase or decrease in the number of banknotes in circulation can be specified as a percentage of the active circulation. Based on this figure, the corresponding number of banknotes is added to or removed from the number of new banknotes added in each iteration. Any banknotes added for volume increases are not counted in this paper towards the quoted replacement costs, as they are not – in the actual sense – used to replace unfit banknotes from circulation. In addition, the model does not simulate seasonal changes in the circulation volume (see Chart 1), as they do not have a substantial impact on the note quality in circulation. Furthermore, such fluctuations are in most cases not covered by the issuance of new banknotes but by fit banknotes from the NCBs' stocks[13].

In order to reach steady-state conditions, any increase in note volume circulation (specified as the annual volume increase in %) is equivalently followed in our model by a change in the NCB and CH sorting volumes (otherwise, for example, at an annual note volume increase of 10% with constant NCB sorting volume, the quality in circulation would constantly degrade).

3.2.5 The value of banknotes and the costs of sorting

Total cash cycle costs are modelled as the sum of NCB sorting costs plus note replacement costs. As shown in Figure 3, the banknotes which have to be replaced consist of unfit banknotes destroyed by the NCB, outflows and any increases in circulation.[14] Model inputs for the two cost components are the NCB sorting costs (per 1,000 banknotes sorted)[15] and the issuance costs for new banknotes.[16] A third component of the total cost of cash is the processing costs for commercial cash handlers. This is not modelled, as no consolidated data exist. Considering that the ECB's Recirculation Decision is voluntarily applied by CHs and that there is rapid growth in banknote handling machines, supporting recirculation, and a strong

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13 In reality, banknote growth is not constant over time, and as such no steady state is ever achieved in real life. However, although money growth fluctuates seasonally, banknote quality eventually settles down to a repeating seasonal pattern. The Eurosystem has higher banknote demand around the festive season and also the summer holiday season. This is covered by the NCBs issuing additional banknotes from their stocks. As these stocks are of better quality than the average banknote circulation (because they do not contain unfit banknotes), the result is a slight increase in banknote quality around the period of higher banknote demand.

14 For more in-depth studies, the model also outputs the residual value of false-unfit banknotes using a linear depreciation from fitness level 1 to a definable residual value at the Eurosystem threshold (50). This aspect is not covered in this paper.

15 Including all NCB costs for lodging, unpacking, processing, destruction of unfit, repackaging of fit banknotes, storing and reissuance.

16 Including production, transport, storage and handling costs.

increase in recirculation volumes (see Chart 2 and Chart 3), the authors argue that banknote recirculation reduces the societal costs of cash overall.

3.2.6 Sorting policies

While in this paper we present steady-state results achieved at a fixed NCB sorting threshold, Model A also allows three other NCB sorting policies to be defined. These are:

1. sorting at constant NCB note destruction volumes (shred rate);
2. sorting at a constant share of false-fit reissued;
3. sorting to achieve a defined quality in circulation.

Although the model's steady-state results are identical when the same conditions are specified[17] via either policy, these additional sorting policies have proven useful in studying changes in the cash cycle at constant note quality in circulation (see for example Section 4.1.3.2) or in studying dynamic cash cycle changes requiring NCB sensor adjustments over time.[18] Further background on the different sorting policies can be found in the manual published together with our model.

3.3 Model B: Modelling of national cash cycles using real-life data

While Model A is designed as a generally applicable, theoretical model of banknote circulation, Model B is more focused on accurately simulating country-specific cash cycles. To achieve this, it is designed and calibrated based on collected data on the circulation in specific countries.

In the following subsections, we provide information on the data collection during the external circulation trial (ECT), which was the data source used. We also discuss how the available data were processed, in order to characterise some of the stochastic processes of the cash cycle.

3.3.1 The circulation data used to model the cash cycles

The ECB organised an ECT in three countries, which ran from October 2014 to October 2016. The intent was to study various aspects of the note cycle, including but not limited to the durability of the banknotes, the return frequency and cash usage. The ECT lasted two years and was conducted in Austria, Ireland and the

17 If, for example, a modelled cash cycle at an NCB sensor threshold of 50 has 10% unfit banknotes in circulation, an NCB shred rate of 5% and 5% false-fit banknotes in the banknotes reissued, specifying an NCB shred rate of 5%, will deliver an NCB threshold of 50.

18 Not covered in this paper.

Netherlands. At the start of the trial, between 1.0 and 1.1 million €5 banknotes of the Europa series, with registered serial numbers, were issued in each of the countries within a two-week period. Each time that these banknotes were sorted in these three countries' NCB sorting centres, they were logged in a database for future processing. We refer to the event of the banknote returning to the sorting centre as a "banknote return". Banknotes can have multiple returns, as they are put back into circulation if they are classified as fit by the sorting machine. For each return of a banknote, the following information relevant to this study was logged:

- the time of return, measured in weeks from the beginning of the trial;
- the soil level in native machine units of the sensor used in the country in question;
- a range of defect parameters and a combined binary defect/no-defect judgement.

In the analysis of the data we encountered the following challenges:

1. The soil level of the measured banknotes is given in native machine units, which differ widely depending on the sorting centre and machine they come from (see Section 2.2). To keep within the notation of the 1-100 soil range, any soil data reported in this paper were converted to the normalised scale by processing a standardised soil test deck on each sorting machine and correlating the machine soil levels to the standardised scale.
2. Depending on the workload, the banknotes returning to the sorting centre were often stored for several weeks before being processed. Sometimes this led to uncontrolled backlogs and fluctuations in the reported return volumes. This was taken into account by excluding subsets of the data which were known to be affected by such irregularities.
3. During the processing of the soil measurements of the sorting machines, we observed sensor artefacts due to technical limitations. The first sensor artefact was an observed "note cleaning": there were many cases where the soil level of the returning banknotes decreased, as if the banknotes were becoming cleaner by use in circulation. The cause of this paradox is believed to be the reduction of the gloss on the banknote as it circulates; due to the imaging system of the fitness sensor not using a perfectly diffuse light source, this was incorrectly interpreted as the note becoming less soiled. The second artefact found was a case of non-continuity of machine units for one of the fitness sensors used in the ECT. Observations from the ECT data, together with further analysis of sensor performance, indicate that some unfit banknotes jumped directly to the minimum soil level, without going through the intermediate unfit values.

3.3.2 Model B structure

Model B treats each banknote as an independent agent with a number of attributes: soil value, age (since issuance) and time since last sorting. This allows for more complex relationships to be modelled, and other attributes can be added easily, together with any interactions between them. A population of such banknotes is created at the beginning and acts as a representative sample of the banknotes in circulation. Each banknote in this population evolves independently, following the different stochastic steps of the cash cycle.

The basic steps of Model B are in accordance with the general flowchart in Figure 3. Every banknote starts with a fitness level that is drawn from a normal distribution corresponding to the production variations (new note profile). At every cycle of the model, the banknote has a certain probability of returning to the NCB for sorting, either directly from circulation or through a CH (as unfit or fit surplus). Depending on its soil value, the banknote is sorted to unfit (at the NCB or the CH) by a sensor that follows a Gaussian model as depicted in Chart 8. If a banknote is shredded at the NCB, it is replaced with a new banknote with fitness again randomly drawn from the new note profile; otherwise it is reissued into circulation. In each iteration the banknote becomes more soiled following a probabilistic ageing rate and has a probability of suffering a defect.

The probabilities that govern these basic steps of the cash cycle are country-specific. We characterise these basic steps for a specific country by drawing on data from the ECT. In the following subsections, we discuss how the available data were processed in order to characterise the processes associated with the return of a banknote to the NCB and ageing – both accumulation of soil and defects – addressing at the same time the effects of recirculation. Model B does not account for banknote migration (inflows and outflows) because the ECT did not monitor for this (other than between the three participating countries) and no accurate real-life data on migration are available at present. Model B therefore follows the simplifying assumption that the whole banknote population is actively circulating: no migration, hoarding or change in a country's circulation volume is directly modelled as such, but these will need to be considered when the return probability is determined.

3.3.3 Return probability

Each week, the banknotes have a certain probability of returning to an NCB, where they will be sorted. The return probability is defined as the probability of a banknote in circulation returning to the NCB for sorting. However, not all the population of banknotes in the ECT are actively circulating; a share of the population is hoarded or migrates outside the country. Since these effects are not directly modelled here, in order to find the probability of a banknote in circulation to return to the NCB, it is important to estimate the sub-population of the ECT that is actively circulating, i.e. that will eventually return to the NCB. We denote this as V_∞. The volume V_∞ can be estimated by looking at the cumulative volume of banknotes returning in every country over the duration of the ECT. To simplify the task we have taken into account only the first return of each banknote. We project the volume of returning banknotes to obtain the asymptotic value V_∞, the volume of all issued ECT banknotes that will return at least once to the sorting centre.

The weekly return probability to the NCB, P_{NCB}^{i}, can then be determined in relation to the volume still actively circulating in a given week (for the first return). With V_i we denote the number of banknotes that return on week i. Then each week i the banknotes still actively circulating are

$$V_\infty - \sum_{j=1}^{j=i-1} V_j$$

The probability of a banknote returning on a given week i is then equivalent to the share of banknotes that return on week i, as a percentage of the volume of banknotes remaining in active circulation on week i.

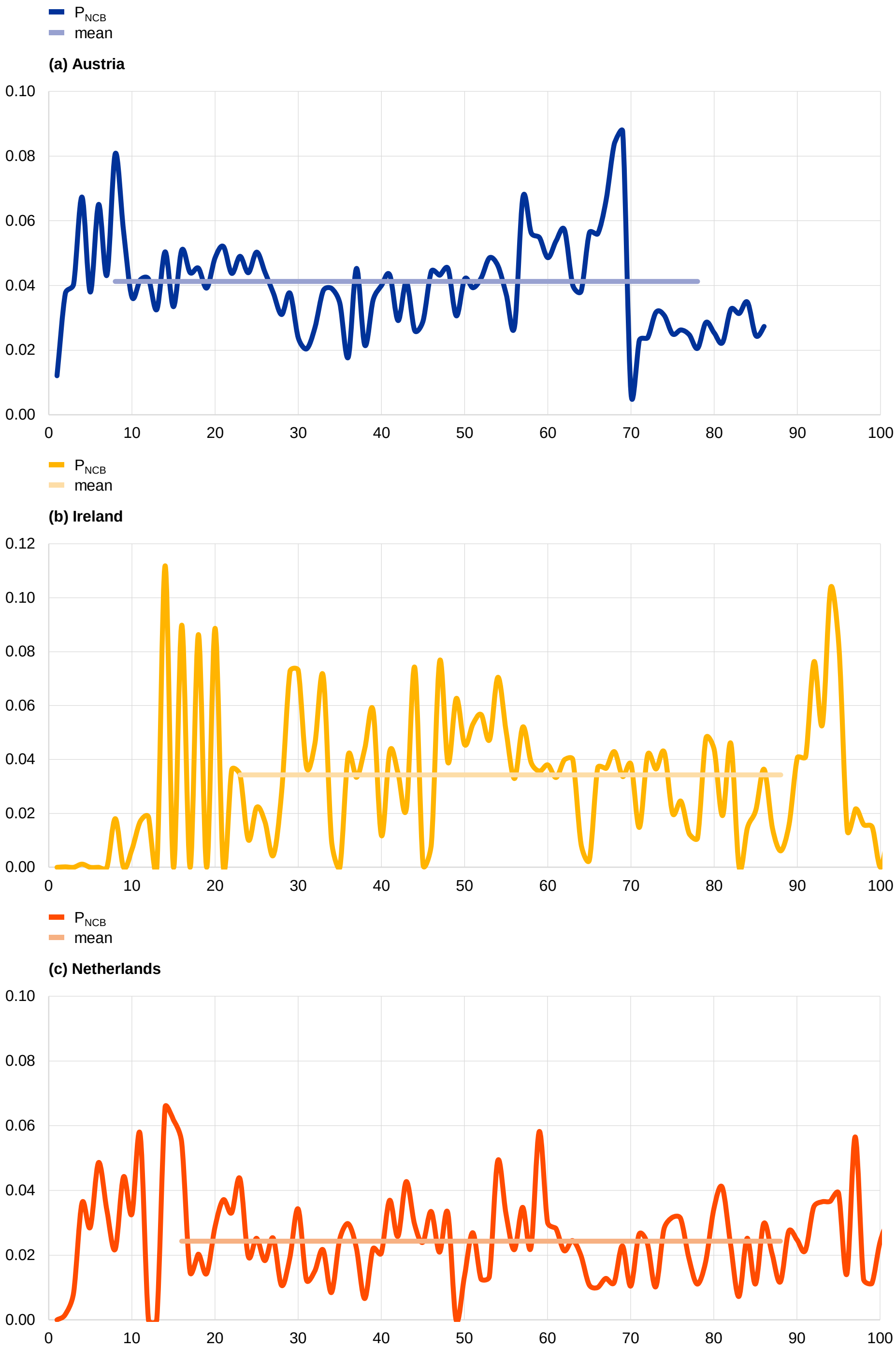
$$P_{NCB}^{i} = \frac{V_i}{V_\infty - \sum_{j=1}^{j=i-1} V_j}$$

The percentage of the actively circulating banknotes returning every week, i.e. the return probability, for the three countries is depicted in Chart 11. No systematic dependence of the probability of return on the time that a banknote has been in circulation can be seen, which is consistent with previously published and internal studies (Den Butter & Coenen, 1982), (Koeze, 1979), (Martin & Meuer, 2001). The fluctuations observed in Ireland in particular during the first few weeks were found to be due in part to backlogs in the sorting centres. In the absence of any visible systematic dependencies, we model the return probability of a banknote at any given week as a constant probability, P_{NCB}. This is the mean return rate observed in the graphs in Chart 11, excluding the unstable returns of the first weeks.

Chart 11

Share of banknotes returning every week, normalised by the volume of banknotes in active circulation, considering only the first return

(x-axis: time in circulation in weeks; y-axis: frequency)



Source: Eurosystem ECT.
Note: The dashed line denotes the mean weekly return rate for each country.

3.3.3.1 Recirculation and return to the NCB

In the previous subsection we determined that the probability at each cycle that a banknote returns to the NCB, P_{NCB}, is constant. However, when CHs are part of the cash cycle, P_{NCB} cannot be directly applied to a banknote in circulation. As seen in the flowchart in Figure 3, the total volume returning every cycle, V_{NCB}, is the volume returning directly from circulation, $V_{NCB,Sort}$, and the volume that has already been pre-sorted (a mix of the unfit and fit surplus), $V_{CH,ToNCB}$: $V_{NCB} = V_{NCB,Sort} + V_{CH,ToNCB}$. Consequently, based on P_{NCB}, we have to determine the probability that a banknote will be processed by a CH, $P_{CH,Sort}$, and the probability that a banknote will return to the NCB directly from circulation, $P_{NCB,Sort}$. In the absence of per-banknote data directly collected from CHs, we have estimated these two probabilities using the reported total national volumes of banknotes sorted by the NCB and the CHs, as well as the volume of banknotes recirculated by CHs. The relevant model inputs are summarised in Annex B.

3.3.4 Modelling of note ageing

The ageing rate of a banknote, expressed as a weekly soil increment, is not constant, but follows a certain probability function, as discussed previously (see Section 3.2.1). For Model B, we use the data from the circulation trial to determine the underlying statistical function governing this ageing rate. At the beginning of the trial and each time a banknote returned to the sorting centre, its soil value was measured and transformed to standardised fitness levels ([1-100]). This allowed us to calculate the increase in each note's soil value, dS, between two returns. Assuming that the soiling has taken place gradually and uniformly across the weeks that the banknotes have been in circulation, we can estimate the weekly soil increment, dS_{week}, per country.

Caution is needed to ensure that the soil increment, dS_{week}, is real and unaffected by sensor artefacts. Occurrences where the soil measured is believed not to match the true soil level should not be included in the analysis. We therefore excluded from the ageing study all events that were suspected of being affected by the sensor artefacts mentioned in Section 3.3.1.

3.3.4.1 Ageing rate probability

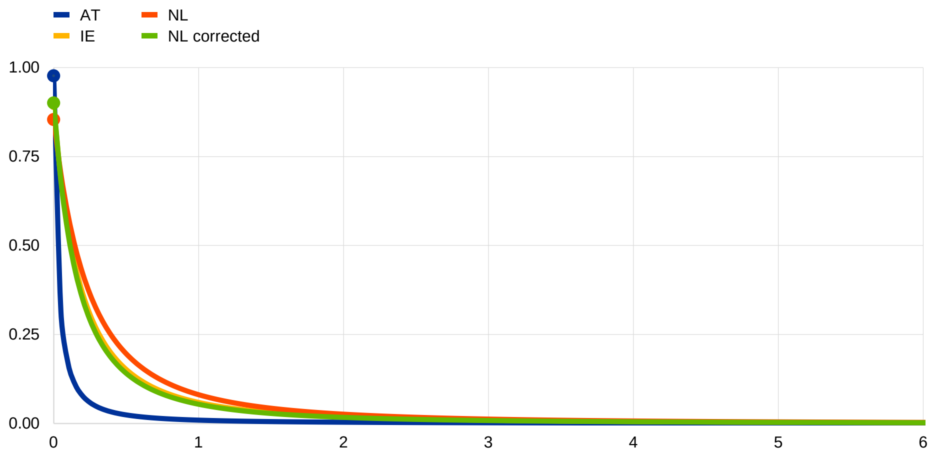
Analysing the pre-processed data, we modelled the frequency of weekly soil increments, dS_{week},. The best fit is found to be an inverse power function of the form $y = a(x + b)^{-c}$, where x is the soil increment per week, or dS_{week}, and y is the probability of this soil increment being observed. An exponential function and a Poisson function were also tested, but neither matched the tail of the data well. The functions that govern the ageing rate, dS_{week}, for the three countries are depicted in visual form in Chart 12 and reported in Annex B. In the figure, a corrected function for the Netherlands is also plotted; this is explained in the next subsection.

In this comparative graph, banknotes in the Netherlands appear to age faster than those in the other two countries, as larger soil increments have a higher likelihood of occurring. In fact the average dS_{week} observed in the ECT data is the highest for the Netherlands, with an ageing rate of approximately 0.5 normalised fitness levels per week. Such fast ageing is unexpected, as it would result in a high number of unfit banknotes in circulation or very high replacement by new banknotes. However, this is not supported by the annual QS in the Netherlands.

Chart 12

The ageing functions derived for the three countries analysed

(x-axis: soil increment per week (dS_{week}); y-axis: probability (dS_{week}))



Source: Eurosystem ECT.

The observation of unexpectedly fast ageing in the Netherlands is believed to be at least partially due to the CHs' involvement. The Netherlands has a large share of CH processing. According to the data reported by CHs and the NCB for 2015, almost the entire volume of €5 banknotes reaching the NCB sorting centre had already been pre-sorted by CHs. This has the effect that the banknotes that reach the NCB sorting centre and are registered in the ECT do not represent the quality in circulation, but are predominantly the CH unfit banknotes. In the ECT, as all the banknotes have been in circulation for the same duration, the banknotes returning to the sorting centre are therefore only those that have aged faster than the average population.

3.3.4.2 Ageing in cash cycles with high CH involvement

The ageing function for the Netherlands could be corrected if we knew, in the ECT, the difference between the fitness of the whole banknote population and the fitness of the share that is sorted at the NCB. Unfortunately, there were no measurements in our trials that would allow this difference to be quantified. Instead, we estimated this difference using the model. We did this by simulating the conditions of the ECT in the

Netherlands[19] and monitoring the fitness of: (i) the whole population; and (ii) the population that is sorted at the NCB (most of which has been pre-processed by the CHs).

The model shows that the fitness of these two samples differs increasingly as time progresses, from three normalised fitness levels (in the first 20 weeks) to nine fitness levels at steady state; the fitness of the banknotes sorted at the NCB is therefore 3-9 fitness levels worse (more soiled) than the population average. We use this information to correct the ageing function for the Netherlands: we subtract the average of six normalised fitness levels[20] from all the ECT soil increments that are registered. Following the same process for calculating the ageing rate probability, as outlined in Section 3.3.4, we recompute the ageing function for the Netherlands and receive the corrected function (see Chart 12 and Annex B).

The correction of the ageing rate, as described here, is only an approximation. Ideally, the banknotes being handled by CHs or a sample from the real circulation would be monitored so that more reliable ageing functions can be calculated.

3.3.5 Modelling of defects

Banknote soiling is assumed to be a continuous process: we can assume that the soil increases gradually between two measurements of the same banknote, enabling us to estimate the average ageing per cycle. However, as discussed earlier, defects are binary events that happen at an unknown point in time between two returns of a banknote. This makes modelling the probability of defects per cycle a challenging task, considering also that different types of defects (e.g. dog-ears and tape) would in reality follow different likelihood distributions. Despite some attempts, we did not manage to extract from the available ECT data the defect probability per cycle, which is used in Model A as an arbitrary input parameter.

We therefore followed a different approach to introduce defects into Model B: when we examine a population of banknotes with a certain age/soil distribution, we can determine the percentage of banknotes that are defective. The largest such population that we have and that reflects the steady-state population is the 2015 QS. Using this data we can determine the dependency between soil and defects. This relationship for Austria can be seen in Chart 13. Each point marks the percentage of banknotes with soil in the specific fitness bin that also have a defect. The observed dependency can in this case be modelled as a Gaussian cumulative distribution function (CDF), with a low error for all studied countries. The three defect functions that correspond to Austria, Ireland and the Netherlands in normalised fitness levels are reported in Annex B.

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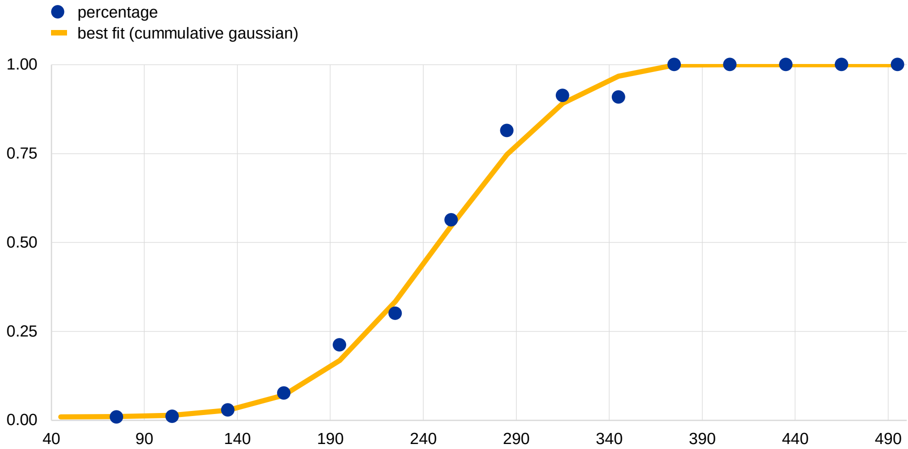
[19] The inputs of the model corresponding to the return probability of banknotes to the NCB and CHs, the ratio of CH processing to NCB processing and the ageing rate have been set to the Netherlands values, as estimated in the previous subsections.

[20] An average of six was chosen, as in the ECT, banknotes returned throughout the whole trial, from Week 2 to the end of the trial.

Chart 13

The probability of defects in a population of banknotes depending on the soil level,[21] modelled as a cumulative Gaussian distribution

(x-axis: sorting machine specific soil units; y-axis: unfit probability due to defects)



Source: 2015 quality survey, data for Austria.

It must be stressed that, in contrast to Model A, the depicted probability of defects of a banknote with a soil value of s here is not the probability that this banknote suffered a defect in the last/current cycle, but the total probability that the note has suffered a defect at some unknown point before the observation time of the QS. Therefore, this profile cannot be applied in every cycle, to every note in circulation. Instead, it is only applied temporarily when a population of banknotes is examined (i.e. processed by the NCB or CHs). At this point the defects are sorted out from the population.

We tried to validate the dependence of defects on soil from the ECT data. Even though the general trend of the Gaussian cumulative distribution function was repeated, the error was slightly higher for high soil levels. Additionally, the mean and standard deviation of the best-fitting Gaussian CDF are different from those obtained from the QS data. These discrepancies might indicate that defects depend on other factors as well, such as age. However, as we do not know the time in circulation of banknotes in the QS, we cannot confirm whether this is the underlying reason behind these differences. Moreover, unlike the QS, the ECT population is not a steady-state population, meaning that a one-to-one comparison is not straightforward. Finally, it must be taken into account that additional analysis of the ECT data identified a misclassification between soil and stain, which also adds to the discrepancies observed.

[21] Fitness here is reported in machine-specific soil units, instead of normalised fitness levels. This is because the unbound range of machine soil values allows for a better visual observation of the trend.

3.4 Concluding remarks on the two modelling approaches

The two models presented have conceptual and practical strengths and weaknesses. Model A incorporates more key elements, such as a changing circulation volume (economic growth, inflows and outflows). Using this model in theoretical cash cycles with well-defined inputs allows for a comprehensive study of the influence of key parameters on the cash cycle. Some aspects of such a study would not be possible with Model B, as it does not cover certain elements of the cash cycle.

If performing the same study on a real cash cycle, the inputs might not be well defined, or even not obtainable at all, such as the active circulation or the banknote lifetime. In this case, the inputs for Model A are calibrated on an expert estimation basis, which carries the danger of inaccurate estimations. In this aspect, the merit of Model B is that it is founded on the collection and analysis of real cash cycle data. It can therefore simulate the cash cycle of a specific country without relying on estimates. Its limitation is that simulating all the relevant aspects of a banknote cycle can be challenging due to the lack of accurate data, such as detailed data on CH sorting. There is currently a clear trend towards new sorting machines collecting and storing detailed multi-dimensional data per banknote, opening up the possibility for full exploitation in the future using Model B’s concept of multiple attributes per note.

4 Results

In this section, we apply our two models to study different aspects of a cash cycle. Using Model A, Section 4.1 presents an analysis of two theoretical cash cycles, both of which resemble typical national cash cycles of different NCBs and/or denominations. Following a scenario analysis conducted on two “base cases”, a more detailed discussion on the key factors is provided. In Section 4.2, Model B is used to simulate specific national cash cycles, and the predicted results are validated against known figures for the same national cash cycles.

4.1 Results for Model A: Application to two theoretical cash cycles

4.1.1 Definition of two cash cycles

To study the behaviour of Model A, we define two example cash cycles, which act as the cash cycles of two theoretical countries. These theoretical cash cycles are identical in all aspects (e.g. theoretical lifetime of banknote, accuracy of sorting sensors used), except for the involvement of the CHs and the NCB. In Cash Cycle 1, the NCB is more actively involved, sorting more banknotes than the CHs, while Cash Cycle 2 represents a country where CHs process a larger share of the banknotes. Both modelled cash cycles resemble real-life scenarios in the sense that they reflect the cash cycles of some larger euro area countries regarding the return frequency of banknotes and the NCB-to-CH sorting ratio for transactional denominations. The input parameters of the two cycles are shown in Table 1. We do not define any inflows or outflows in the base scenario in either cash cycle. The model iteration time per step in all cases was seven days; the Pearson correlation coefficient at which steady state was assumed was $\rho_{Notes_{Start},Notes_{End}} = 0.999999995$.

Table 1

Overview of the two base cash cycles used in Model A

Input Parameter	Cash Cycle 1	Cash Cycle 2
New note issuance costs	€50 per 1,000 notes	
New note fitness level	1	
New note fitness variation (SD)	5 fitness levels	
Notes in active circulation	1 billion	
Annual change in circulation volume	5%	
Note inflows/outflows	None	
Ageing model	Poisson	
Theoretical note lifetime (due to soiling)	24 months	
Defect likelihood per year	10% (increasing linearly with the fitness level)	
CH sorting volume per year	2 billion	5 billion
CH share of fit notes sent to NCB (as surplus)	25%	
CH sorting threshold	70	
NCB sorting threshold	50 (i.e. at Eurosystem threshold)	
CH and NCB sensor inaccuracy (SD)	10 fitness levels	
NCB sorting costs	€10 per 1,000 notes	
NCB sorting volume per year	5 billion	2 billion
Eurosystem fit/unfit threshold	50	

Notes: Rounded Eurosystem averages and estimates were used for all input parameters. The estimates for new banknote issuance costs and NCB sorting costs in this example are the same for both theoretical cash cycles and do not include any economies of scale due to different NCB sorting volumes or different annual banknote replacement volumes.

Using the theoretical model, we receive the results shown in Table 2 and the fitness profiles of banknotes in circulation as shown in Chart 14. The banknote flows as depicted in Figure 3 are provided in Annex A.

Table 2

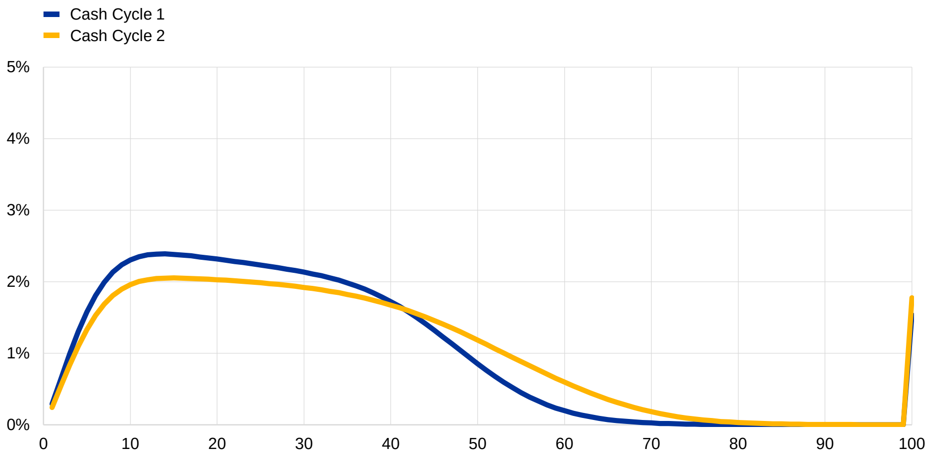
Results of the two base cash cycles

Model results	Cash Cycle 1	Cash Cycle 2
Technical		
% unfit notes in circulation	6.80%	14.70%
of which defects	1.54%	1.78%
NCB destruction (shred) rate	11.50%	23.80%
Annual note replacement volume[22]	574.6m	476.5m
Average note lifetime in circulation	20.9 months	25.2 months
Financial		
Annual note replacement costs	€28.7m	€23.8m
Annual NCB sorting costs	€50m	€20m
Total costs (sum)[23]	**€78.7m**	**€43.8m**

Chart 14

Banknote fitness profiles in circulation for Cash Cycle 1 and Cash Cycle 2

(x-axis: fitness level (1=superfit, 100=defects and superunfit; y-axis: frequency)



Source: Model A.
Note: The peak at fitness level 100 indicates defective notes.

The two cycles studied, despite using banknotes with the same theoretical note life, produce different note lifetimes and quality in circulation. In Cash Cycle 2, fewer banknotes return to the NCB (2 billion against 5 billion in Cash Cycle 1). These banknotes are, however, more soiled; this can be seen from the NCB destruction (shred) rate, which is two times higher in Cash Cycle 2. Even though a higher percentage of NCB-sorted banknotes are destroyed in Cash Cycle 2, the absolute volume of shredded banknotes is still lower, resulting in lower annual replacement with new banknotes of 98.1 million banknotes. As less banknotes are replaced in

22 The note replacement volume (and costs) quoted here does not include banknotes needed to increase the circulation volume (as they are not replacing unfit banknotes). The additional costs due to new banknotes needed to increase circulation volume is, in this case, €2.5 million in the first year for both cash cycles ([1 billion banknotes in circulation] increased by [5%], at [€50 per 1,000 new banknotes]).

23 Excluding CH processing costs.

Cash Cycle2, banknotes stay in circulation for about 4.5 months longer, resulting in a substantially lower quality in circulation (14.7% unfit compared with 6.8% for Cash Cycle 1). This is expected, as the sorting is performed primarily by CHs at a “dirtier” threshold (70) than that of the NCB (50). Cost-wise, Cash Cycle 1 has almost twice the annual costs to the central bank of Cash Cycle 2. This is in small part due to the increased replacement costs, and primarily to the substantially higher NCB sorting volume causing additional annual costs of €30 million. The costs of Cash Cycle 2 – in the absence of reliable data – do not include the increased sorting costs for CHs compared to Cash Cycle 1. The two cash cycles show clearly that the return frequency of banknotes to CHs and NCBs is one of the key drivers for quality and costs. We will discuss the impact of the return frequency in more detail in Section 4.1.5.

4.1.2 A sensitivity analysis of the model based on the two cycles

All of the input parameters affect the final quality in circulation and total costs, but to what extent? This section seeks to determine the sensitivity of the model results to variations in the input parameters and to identify the parameters that are key drivers of note quality and cash cycle costs. By applying the input parameter variations to the two theoretical cash cycles in the same way, it immediately becomes visible how the NCB and CH sorting ratio affects the model’s sensitivity to changes in the other parameters. For this analysis, the input parameters for the two cash cycles as defined above were modified within ranges considered to be either within the inaccuracy of that parameter or within the expected range in which they can be adjusted by an NCB. Parameters that are either fixed (e.g. the NCB or CH sorting volume) or that have no impact on quality (e.g. banknote replacement or sorting costs where the impact is linear to the processing volume and replacement volume) were not included in this analysis. An overview of the changed parameters and the resulting cash cycle changes in terms of quality and total costs are shown in Table 3.

Table 3

Sensitivity analysis on key model parameters for the two cash cycles (changes given in p.p. for unfit banknotes in circulation and in EUR millions for total costs)[24]

			Results			
			Cash Cycle 1		Cash Cycle 2	
			Unfit notes in circulation	Total costs	Unfit notes in circulation	Total costs
Base case results			**6.8%**	**78.73**	**14.7%**	**43.82**
Scenario input	**Base cash cycle value**	**Scenario value**				
Note production parameter						
New note fitness variation (SD in fitness levels)	5	1	-0.5	-1.96	-0.6	-1.04
		15	+1.2	+5.60	+3.5	+4.43
Note circulation parameters						
Notes in active circulation	1 billion	0.9 billion	-0.9	-2.33	-1.3	-1.86
		1.1 billion	+0.9	+2.27	+1.2	+1.82
Annual change in circulation volume*	+5%	0%	+0.3	+1.02	+0.8	+1.01
		+15%	-0.5	-1.90	-0.9	-1.37
Note inflow (5% unfit)	0	0.25 billion	+1.5	-7.31	+3.4	-8.10
Note inflow (20% unfit)	0	0.25 billion	+2.3	-5.87	+4.7	-6.67
Note outflow	0	0.25 billion	-1.4	+7.44	-3.2	+8.17
Note lifespan parameters						
Theoretical note life (due to soiling) in months	24	18	+2.8	+6.66	+4.6	+5.96
		30	-1.7	-4.57	-2.5	-3.27
Defect likelihood per year	10%	5%	-0.5	-1.09	+0.2	-0.84
		15%	+0.3	+0.86	+0.1	+1.14
CH parameters						
CH share of fit notes sent to NCB (as surplus)	25%	15%	-0.1	+0.02	-0.3	+0.08
		35%	+0.1	-0.02	+0.4	-0.09
CH sorting threshold	70	60	-0.3	+0.12	-2.3	+0.71
		80	+0.1	-0.04	+1.7	-0.24
CH sensor inaccuracy (SD in fitness levels)	10	5	+0.1	-0.03	+0.8	-0.21
		15	-0.1	+0.05	-0.7	+0.27
NCB parameters						
NCB sorting threshold	50	45	-2.6	+2.88	-2.6	+2.54
		55	+3.5	-3.38	+4.0	-1.35
NCB sensor inaccuracy (SD in fitness levels)	10	5	+1.0	-2.87	+1.0	-0.41
		15	-1.0	+3.60	-0.1	+1.86
Model parameters						
Note ageing model	Poisson	Exponential	-0.0	-0.09	+0.2	+0.21
		Gamma (α=1)	-0.0	-0.14	+0.3	+0.28
		Weibull (k=1.5)	-0.0	+0.07	+0.0	+0.02
		LogNormal (σ=0.8)	-0.0	-0.28	+0.3	+0.32
Defect profile	Linear increase with fitness level	Constant	+0.3	+0.74	+0.6	+0.71

[24] For the scenario dealing with the annual change in the circulation volume, see also footnote 21. Additional costs for new banknotes due to a circulation increase for the two scenarios would be zero (for the no growth scenario) and €7.5 million (for the +15% scenario).

The model identified the NCB sorting threshold and theoretical banknote lifetime as strongest drivers to control quality (and costs) of notes in circulation – for both cycles. These parameters are studied in more detail in Sections 4.1.3.1 and 4.1.3.4 respectively.

Increasing the accuracy of NCB fitness sensors (i.e. reducing their inaccuracy expressed as SD of fitness levels) at first glance has a negative impact on note quality. This counter-intuitive behaviour is discussed in more detail in Section 4.1.3.2. Section 4.1.3.3 analyses in more detail the impact of changes to the CH sorting threshold, which is especially relevant for Cash Cycle 2. The inaccuracy of the CH fitness sensors and the share of CH fit banknotes sent back to the NCB have only a small influence in our simulations on note quality in circulation, and even more so on cash cycle costs.

Inflows and outflows are one of the strongest external factors for a national cash cycle. Note inflows (even those of good quality) have a negative impact on the national note quality, as they restrict an NCB's scope to issue new banknotes. The opposite is the case for outflows, which, in our model, are compensated for by additional new banknotes issued to keep the number of banknotes in circulation constant. We present a more detailed analysis in Section 4.1.3.5.

The new note fitness variations, which in the model are expressed as SD of fitness levels, also significantly affect quality and costs. For Cash Cycle 1, changing the variation of new banknotes (in SD) between one fitness level (highly uniform production) and 15 fitness levels (substantial production variations) can result in either savings of €1.96 million or additional costs of €5.60 million per year compared with the base case. The overall range covers about 10% of the total cash cycle costs. In addition, the simulated increase in new note production variations has a negative impact on quality in circulation, adding an additional 1.2% of unfit banknotes to circulation. The same trend is visible for Cash Cycle 2, albeit with a slightly different magnitude for quality and costs.

The number of banknotes in active circulation – which is difficult to determine – has a significant impact on the model results. A larger active note circulation volume results in banknotes being returned less frequently to the NCB or CHs; this in turn leads to more unfit banknotes in circulation. Subsequently, the NCB note destruction volume increases, resulting in additional replacement costs. The behaviour is similar for both cycles. A good knowledge of the active circulation volume is therefore required to accurately model any specific national cash cycle.

The annual increase in the note circulation volume has the expected slight "cleaning" effect, as more new banknotes need to be injected into the cash cycle. The effect on note quality is stronger in Cash Cycle 2 due to its lower base quality, with the financial impact being comparable in both cases. The reason why the model results denote a cost saving where there is a higher annual growth rate is that the costs for the additional new banknotes injected into the cash cycle due to growth are not included in the replacement costs (see also Section 3.2.4 and footnote 21).

The impact of the ageing model is small. Changes in the note quality and total costs are smaller than 0.3 p.p. and €0.3 million p.a. respectively in the worst case. The impact of the modelling of the defect likelihood in relation to note fitness (increasing or constant) is also small and within the general inaccuracy of the model.

In general we can see that the relationships between the model inputs and results, while a basic trend can be estimated, differ in their magnitude for the specific cash cycle, are of a non-linear nature and are difficult to predict with any reasonable accuracy using simplified approaches.

4.1.3 Detailed analysis of key cash cycle parameters

The sensitivity analysis above changed individual model parameters, but left the NCB sorting threshold at the Eurosystem unchanged (50). In reality, however, an NCB has the possibility of modifying the sorting threshold on its machines to adjust the quality in circulation. For policy decisions it is therefore necessary to know how any cash cycle changes affect quality and costs and whether, if the quality is maintained by moving the NCB sorting threshold, there are still cost savings. This is studied in detail in this section.

4.1.3.1 NCB sorting threshold

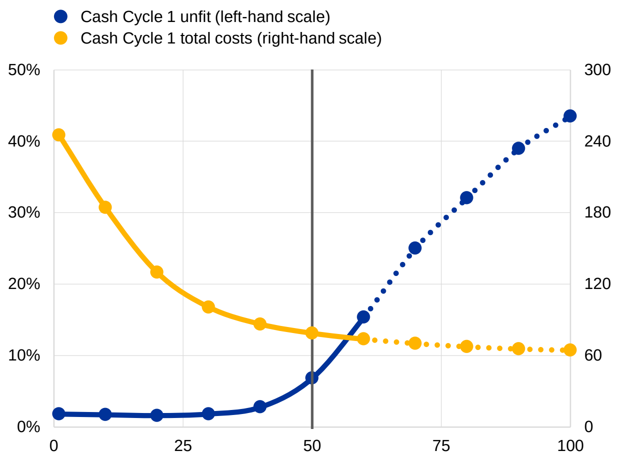
An adjustment of the sorting threshold by an NCB has an impact on both the quality in circulation and replacement costs. In real life, NCBs select a sorting threshold that meets the requirements of the cash cycle in their countries, as cash cycles differ due to geographical, cultural and societal differences. Chart 15 shows the effect of such an adjustment to our two base cash cycles. The dotted part of the curves indicates NCB sensor thresholds where the banknotes reissued by the NCB would contain too many unfit banknotes (false-fit > 8%) and would no longer conform to the minimum note quality allowed within the Eurosystem.[25]

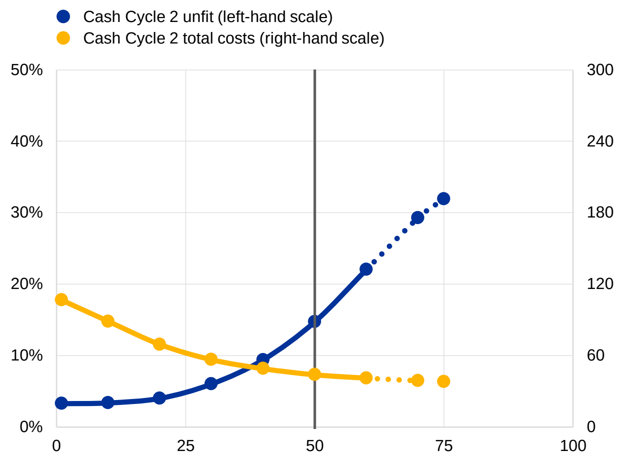
[25] Furthermore, for Cash Cycle 2 no steady-state condition could be derived at NCB sensor thresholds greater than 75, as the resulting quality in circulation would be too low and the number of banknotes returned by CHs as unfit would be higher than the total NCB sorting capacity of 2 billion banknotes p.a.

Chart 15

Quality in circulation and total costs as a function of the NCB sorting threshold for Cash Cycle 1 (left) and Cash Cycle 2 (right)

(x-axis: NCB sorting threshold; left y-axis: percentage of unfit notes in circulation; right y-axis: total cash cycle costs in EUR millions)





Source: Model A.
Notes: The dotted parts of the lines correspond to NCB sensor thresholds where the notes reissued by the NCB would contain too many unfit notes (false fit > 8%). The vertical lines at 50 refer to the base cases as defined in Section 4.1.1. Circles denote calculated scenarios.

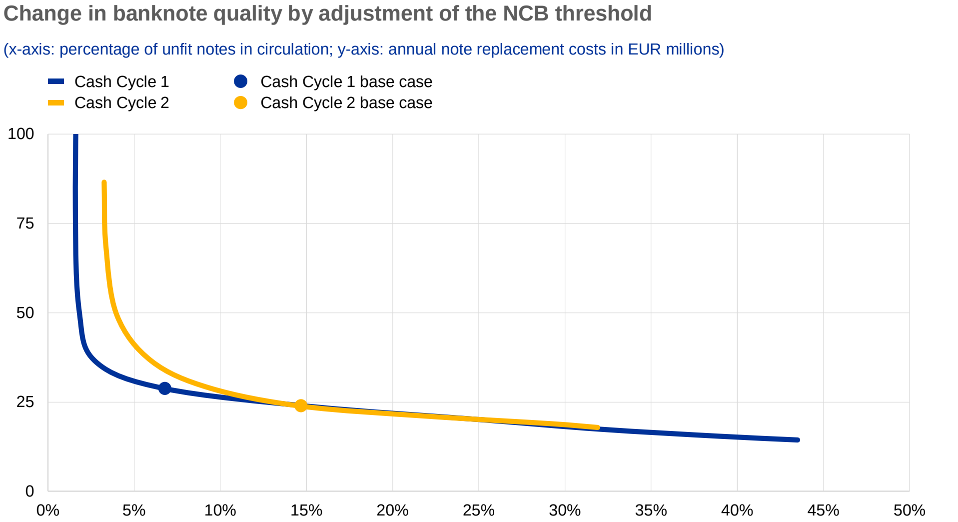
As expected, for more severe (lower) sorting thresholds, the total cash cycle costs increase (due to the higher number of banknotes removed from circulation) and note quality improves in both cases. However, the magnitude is very different in the two cases. While in Cash Cycle 1, the quality can be improved with more severe sorting to about 2% unfit in circulation, even with complete destruction of all banknotes received and replaced with new banknotes[26], the best quality that can be reached in Cash Cycle 2 is about 4% unfit in circulation. The total costs for Cash Cycle 1 remain higher than for Cash Cycle 2 in all cases due to the constant difference of €30 million p.a. for the additional NCB processing in Cash Cycle 1 (5 billion banknotes compared with 2 billion for Cash Cycle 2).

The question that arises now is: what is the benefit of a higher NCB sorting volume? This becomes evident when comparing just the replacement costs for the two cash cycles (see Chart 16 and also Section 4.1.5).

[26] By sorting at an NCB fitness threshold of 1.

Chart 16

Annual banknote replacement costs as a function of banknote quality for the two investigated cash cycles



Source: Model A.
Note: The circles denote the results of the base cash cycles as defined in Table 1.

As long as there are more than about 10% unfit banknotes in circulation, the replacement costs are identical in both cycles. However, with a higher return frequency of banknotes to the NCB (as in Cash Cycle 1), the quality of banknotes in circulation can be raised to about 5% with only a linear increase in replacement costs. Where NCB processing volumes are lower (Cash Cycle 2), the point where any further improvement in note quality comes at exponentially higher replacement costs is already at about 10% unfit in circulation. In other words, the return frequency of banknotes to the NCB puts an effective ceiling on the quality achievable for a specific cash cycle. The chart also shows that the two base cash cycles as defined in Table 1 are within this linear part of the replacement cost curve.

Understanding the relationship between note return frequency to an NCB and the achievable note quality in circulation is especially relevant for the Eurosystem. Here, the replacement costs for banknotes are shared by an allocation of the total annual banknote production volume according to each NCB's share of the ECB's capital, using a key that is linked to the respective country's share of the total population and gross domestic product of the EU (European Central Bank, 2017a). The note processing costs, conversely, are covered by each NCB. In short, therefore, greater involvement in note processing allows an NCB to maintain higher quality in circulation without using an excessive share of the Eurosystem banknote production volume.

4.1.3.2 The benefit of more accurate fitness sensors

Several authors have studied banknote processing and proposed improvements to the underlying fitness algorithms (Geusebroek, Markus & Balke, 2011), (Kwon, Pham, Park, Jeong & Yoon, 2016), (Buitelaar, 2008). Moreover, substantial work has been carried out within the Eurosystem to better assess the different fitness sensors installed. Analysing the data of Eurosystem internal studies we found that on average a current fitness sensor has an inaccuracy (SD) of about ten fitness levels, with better models having only about three fitness levels inaccuracy and worse sensors having an inaccuracy as low as 20 fitness levels.[27] The question for the NCB is whether the savings achieved by a better sensor that destroys fewer banknotes incorrectly would outweigh the investment required for the sensor hardware.

In the sensitivity analysis conducted above, it was found that having a sensor with a lower error rate will result in lower replacement costs, but also in lower quality in circulation. But why does a better sensor cause lower quality in circulation? The probability of a sensor making an error is symmetrical in both the fit and unfit sides around the sensor threshold (see Chart 10): the modelled sensor therefore has the same probability of classifying a fit note as unfit (false-unfit) as of classifying an unfit note as fit (false-fit). However, the majority of banknotes sorted by an NCB are fit in both base cash cycles. This means that – in absolute terms – the error of the sensor is dominated by false-unfit banknotes. A positive side-effect of this sensor error is that the circulation is also “cleaned” of fit banknotes that have relatively high soil values, near the Eurosystem threshold. This results in good overall quality of banknotes in circulation (of course at the expense of destroying and replacing “barely fit” banknotes). A “sharper” sensor will make fewer such a false-unfit errors, reducing the aforementioned side-effect and resulting in lower overall quality of fit banknotes reissued.

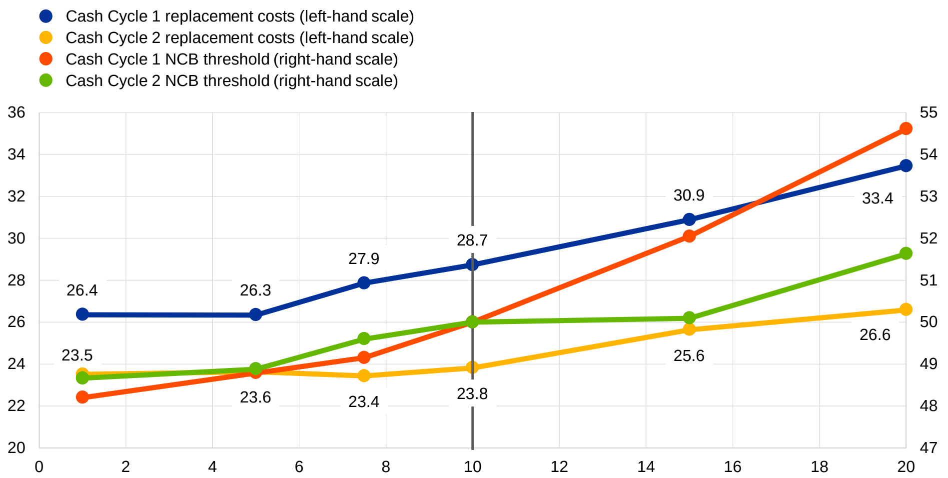
This lowering of the quality in circulation has to be counteracted by applying a more severe sorting threshold when using an improved sensor. The question to answer then becomes: what would the change in the note replacement costs be if sensors with different accuracies were to be operated (by adjusting the NCB threshold) so that they all deliver constant quality in circulation? This can be answered by running Model A and specifying the expected note quality in circulation while varying the sensor accuracy. Chart 17 shows that for Cash Cycle 1, which has stronger NCB involvement and better note quality in circulation, sensor performance plays a substantial role in annual replacement costs at stable quality.

[27] When modelling real-life cash cycles, the sensor error put into the model not only needs to include the error of a new sensor, but also needs to take into account the influence of dust, maintenance intervals, etc. on the performance of an NCB’s sensor population.

Chart 17

Annual replacement costs and NCB sensor threshold for sensors with varying accuracy at constant banknote quality in circulation (6.8% for Cash Cycle 1, 14.7% for Cash Cycle 2, see Table 2)

(x-axis: sensor SD (in fitness levels); left y-axis: annual replacement costs (EUR millions); right y-axis: sensor threshold)



Source: Model A.
Notes: The vertical line refers to the base cycles as defined in Table 1, with an NCB sensor SD of ten fitness levels. Circles denote calculated scenarios.

If a central bank were to use sensors with errors as assumed in the base case (ten fitness levels), upgrading to better sensors with errors of, for example, five fitness levels would reduce annual replacement costs by €2.4 million (from €28.7 million to €26.3 million), accompanied by a slight adjustment of the NCB sorting threshold from 50 to 49 units. For Cash Cycle 2, however, an improvement in the central bank sensor performance does not lead to any substantial savings on the replacement cost[28]. This difference versus Cash Cycle 1 is due first to the lower return frequency to the central bank and the resulting lower processing volume at NCBs, and second to the fact that banknotes sorted by the NCB contain already more unfit banknotes. In this specific modelling case, investing in better sensors appears therefore to be most beneficial for those NCBs that are actively involved in the cash cycle, whereas for other central banks the cost-benefit needs to be specifically evaluated.

4.1.3.3 Changing the fitness thresholds for cash handlers

The main reason why recirculation has been adopted so quickly within the Eurosystem is that it provides an economic benefit to CHs compared with returning banknotes to the NCB. Recirculation allows the majority of banknotes to be directly reissued to customers, with only unfit banknotes and any surplus needing to be returned, resulting in a substantially lower number of secure transports and lower associated costs. This in turn makes the investment in banknote handling machines

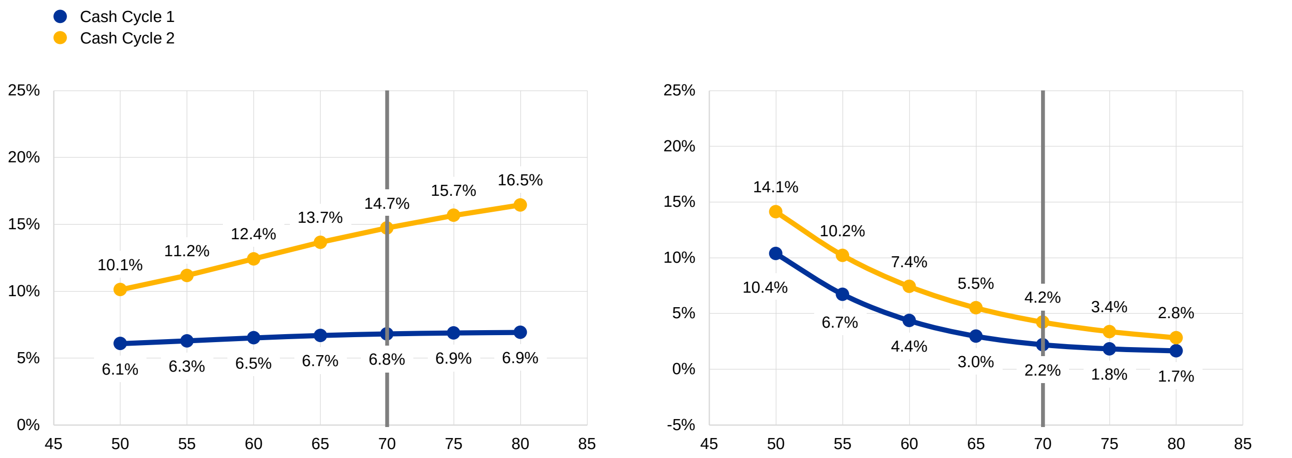
28 The reason why in some cases annual replacement costs increase slightly despite better sensors is due to model inaccuracies introduced by determining a steady state that is slightly off from the actual result.

that are compliant with the Recirculation Decision an economically advantageous choice. The current implementation of the Recirculation Decision allows CHs to establish a viable business model based on increased recirculation of fit banknotes. Now, of course, the quality in circulation can be improved by setting tighter standards for CHs and applying a more severe fitness threshold, yet this has also other effects, as we will show. Chart 18 shows the impact on cash handlers of applying more severe fitness criteria.

Chart 18

Impact of a change in the recirculation fitness threshold on banknote quality (left) and the share of banknotes sorted to unfit by CHs (right)

(x-axis: fitness level; left y-axis: unfit notes in circulation; right y-axis: CH unfit rate)



Source: Model A.
Notes: The vertical lines refer to the base cycles as defined in Table 1, with a CH sensor threshold of 70 fitness levels. Circles denote calculated scenarios.

As expected, the number of unfit banknotes in circulation drops in Cash Cycle 2 (which has a substantial recirculation share) from 14.7% unfit (under the base case with a CH fitness threshold of 70) to 12.4% at a CH sorting threshold of 60, whereas the impact of the same change in Cash Cycle 1 is negligible. In both cases, the improvement in quality comes at the expense of higher replacement of unfit banknotes by new banknotes at the NCB. The second chart shows that in both cash cycles the percentage of CH unfit banknotes that are sorted out and must be returned to the central bank in turn rises sharply. We can see that, cost-wise, any more severe standards for note recirculation would have a negative impact on CHs' current business models, as they would create more unfit banknotes and subsequently require additional transports to the NCB. In addition, there is a ceiling to the severity that can be requested from CHs as all (or at least the vast majority) of banknotes issued by the NCBs need to be fit by their standards.[29] Applying tighter recirculation thresholds for all or larger cash handlers must be decided on a national level. This step is most effective in cases of a lower quality in circulation and a large share of recirculation. The drawback of higher unfit rates at the cash handler needs to be weighed against the total benefits gained from such a national policy change.

29 This is not the case for some select large-volume CHs in some Eurosystem cash cycles, where the NCB has removed itself significantly from the cash cycle and has – in close cooperation with the CHs – set criteria that are as strict as NCB standards.

Box 1

Dual processing of banknotes by NCBs and CHs

Note in particular in Chart 18 that even if CHs operate at the NCB sorting threshold (at a fitness level of 50), a 4 percentage point difference in the quality of the banknotes in circulation remains (6.1% for Cash Cycle 1, against 10.1% for Cash Cycle 2) despite the total (NCB + CH) sorting volume being identical (7 billion banknotes p.a.). The reason is that (see also Figure 3) the increased volume of CH unfit banknotes at lower CH thresholds reduces the NCB's capacity (which is constant in our model) to sort banknotes from circulation. In other words, there is less benefit for note quality in circulation if an NCB processes a note already found unfit by a CH a second time than if it would instead sort a note coming directly from circulation. For the same reason, the return of additional CH fit banknotes to the NCB as surplus has a relatively small but negative impact as again this fraction of the banknotes is processed twice in each iteration. While such "dual processing" of banknotes is in most cases unavoidable due to the NCB's task to authenticate banknotes and destroy only genuinely unfit banknotes, the effect should be kept in mind when assessing the benefits of CH recirculation.

4.1.3.4 Increasing the banknote lifetime

As already mentioned, Model A has as an input the theoretical note life. An increase in this parameter either corresponds to an increase in the soil resistance of a note or can be attributed to the public treating banknotes more carefully.[30] The actual average lifespan of a note until destruction is then dependent on the return frequency to the NCB and the NCB's sorting threshold.

Chart 19 shows the – unsurprisingly – very positive effect of an increased theoretical note life on total cash cycle costs, due to reduced note replacement needs, as well as a significant increase in the note quality in circulation. In the event of an increase in the theoretical note lifetime from 24 to 36 months, the note replacement costs drop from €28.7m to €20.9m for Cash Cycle 1 and from €23.8m to €17.8m for Cash Cycle 2, together with improvements in quality of 2.8 and 4.9 percentage points respectively.[31]

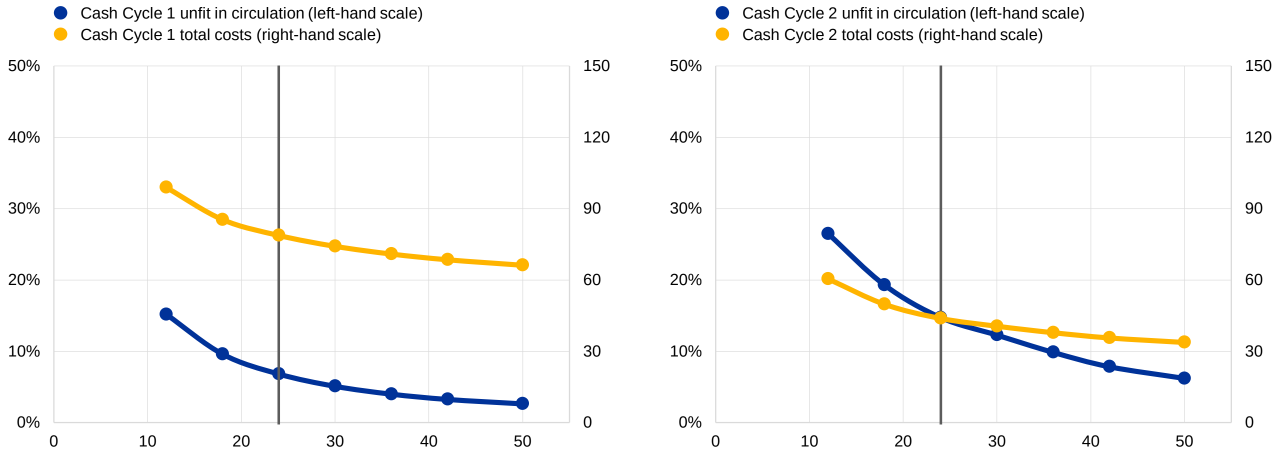
[30] As an example, an increase in this parameter could also be achieved by an advertising campaign to treat banknotes with greater care.

[31] The improvement in quality is greater in Cash Cycle 2 due to the lower note quality of the base case.

Chart 19

Impact of an increase in banknote lifetime on quality and total annual cash cycle costs for the two cycles studied

(x-axis: theoretical note life in months; left y-axis: percentage of unfit notes in circulation; right y-axis: total annual cash cycle costs (NCB processing and replacement costs) in EUR millions)



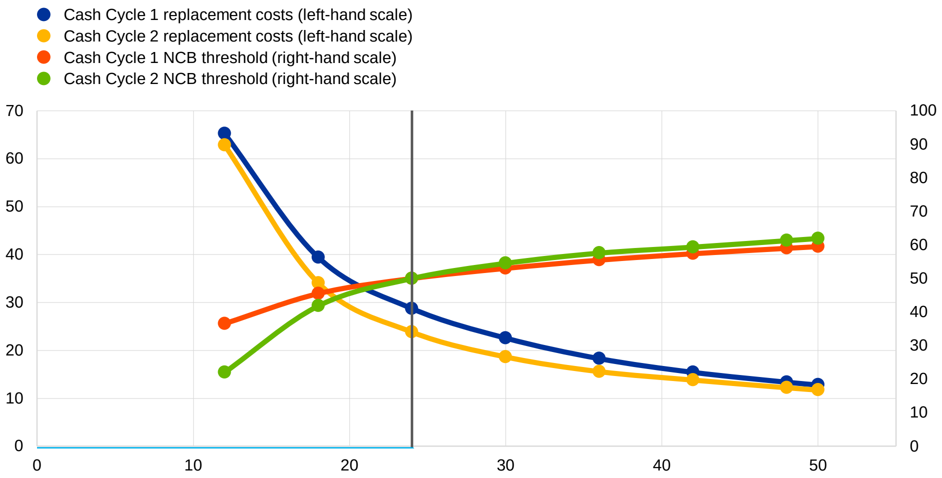
Source: Model A.
Notes: The vertical lines refer to the base cycles as defined in Table 1, with a theoretical note life of 24 months. Circles denote calculated scenarios.

The total cost savings from an extended note lifetime could be even greater if the NCB were to decide to keep the quality in circulation stable despite issuing banknotes with a longer lifetime. An NCB can achieve this by adjusting its sorting threshold to settings that are more lenient than the Eurosystem threshold. This change in sorting policy is of course only possible within the limit agreed by the Eurosystem of 8% false-fit banknotes reissued by the NCB. Chart 20 shows that by implementing such a policy, the same increase in note life (from 24 to 36 months) reduces replacement costs for Cash Cycle 1 further, down to €18.3m (-€2.5m from the case at constant NCB threshold) and to €15.6m (-€2.2m) for Cash Cycle 2. To reap these benefits, the NCB would need to adjust its sorting threshold from 50 to about 55 fitness levels. If the note life could be further increased to, for example, 48 months, additional savings would be achievable at constant quality if the NCB sorting threshold were increased to about 60. However, due to the lower involvement of the central bank and the lower quality in circulation (14.7% unfit at the base scenario), it would not be possible to take advantage of these savings for Cash Cycle 2, as the banknotes reissued by the NCB would then contain too many unfit banknotes. In practical terms, this means for Cash Cycle 2 that in the event of a substantial increase in the banknote lifetime, the note quality would improve from its (rather low) level.

Chart 20

Impact of a change in theoretical banknote lifetime on annual banknote replacement costs for the two cycles studied at constant quality in circulation (6.8% for Cash Cycle 1 and 14.7% for Cash Cycle 2), indicating also the required adjustments to the NCB sorting threshold

(x-axis: theoretical note life in months; left y-axis: NCB sensor threshold; right y-axis: replacement costs in EUR millions)



Source: Model A.
Notes: The chart shows the impact of an increase in the theoretical note life on replacement costs at constant quality in circulation. For Cash Cycle 2, unfeasible scenarios (more than 8% unfit in notes reissued by the NCB) are indicated by dotted curves. The vertical line refers to the base cycles as defined in Table 1, with a theoretical note life of 24 months. Circles denote calculated scenarios.

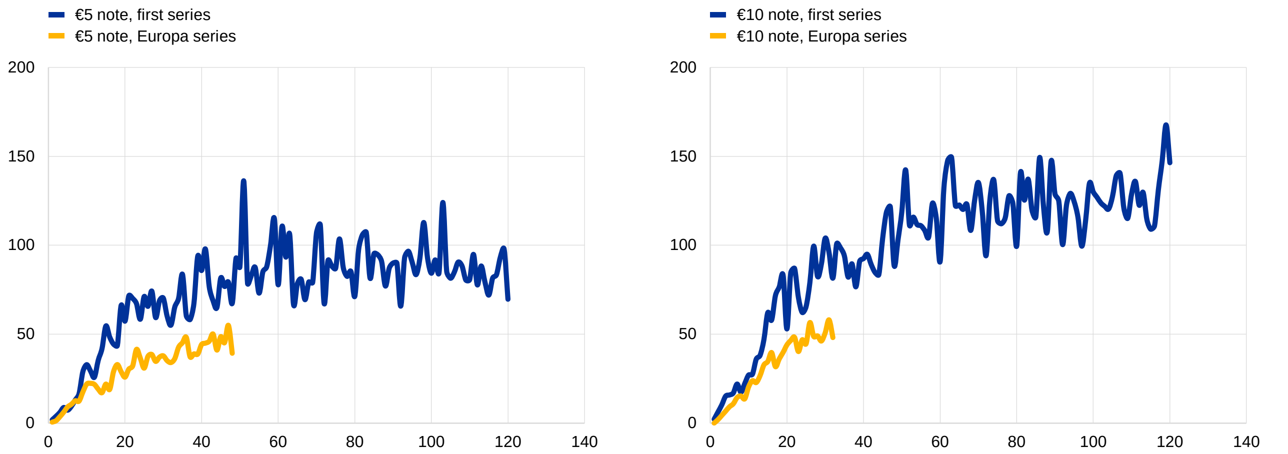
Note that the above example is somewhat unrealistic, as any increase in note production costs to achieve this longer note life (for example by applying a protective coating or moving to more durable substrates) is not considered. Clearly, when a central bank is considering switching to a new substrate, the different note production costs as well as costs related to the adaptation of NCB and CH equipment need to be considered. There is a rule of thumb that the use of banknotes with a longer theoretical lifetime is economically advantageous as long as the relative increase in note life is higher than the increase in the total note production costs; our modelling results show this to be only a rough approximation. The actual savings are in fact strongly dependent on the cash cycle and the NCB's related adjustment (or not) of its sorting policy.

These model results are very much in line with the Eurosystem experience with the Europa series €5 (introduced in May 2013) and €10 banknotes (introduced in September 2014), which have been protected against soiling by an additional varnish layer. Varnishing has resulted in a substantial decrease in the note replacement volume to about 50% of the previous figure, resulting for the €5 note in annual savings of about 500 million new banknotes at stable quality in circulation, as established by the QS. A total of 1.10 billion first series €5 banknotes were destroyed in the 2012 annual destruction, whereas from May 2016 to April 2017 only 0.57 billion Europa series €5 banknotes had to be replaced, at a similar note volume in circulation. Chart 21 shows this reduction, comparing the monthly note destruction volumes in the months after first issuance. A similar reduction in the replacement rate is currently emerging for the €10 note.

Chart 21

Monthly destruction of first series €5 and Europa series €5 banknotes (left) and €10 banknotes (right) in the months after first issuance (January 2002 and May 2013 respectively for €5; January 2002 and September 2014 for €10)

(x-axis: months after first issuance of the series; y-axis: monthly note destruction in millions)



Source: Currency Information System 2.
Note: Data for Europa series up to (including) April 2017

For the Eurosystem, the coating on the Europa series has been beneficial: the savings have substantially outweighed the additional production costs.

4.1.3.5 The impact of net inflows and outflows

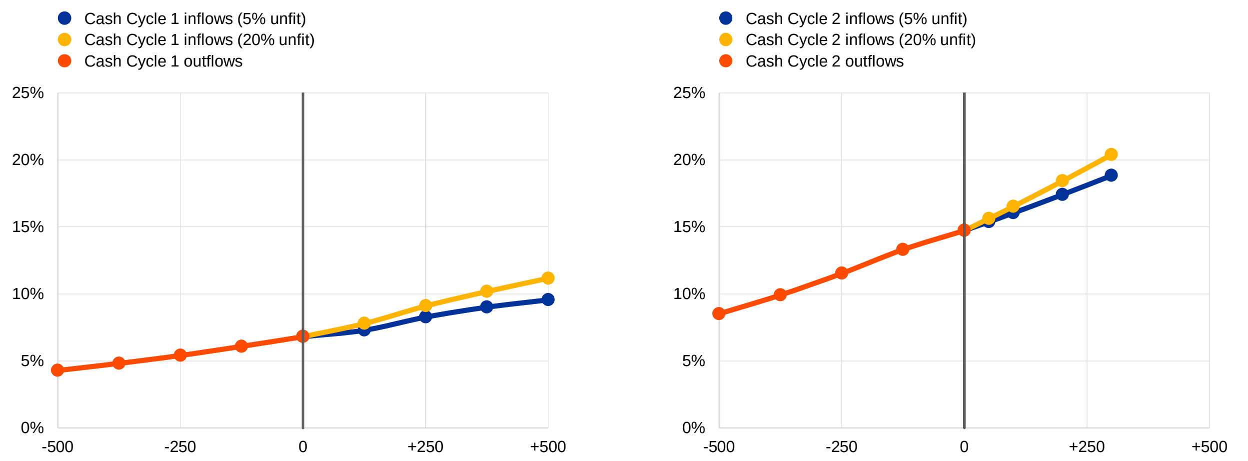
Inflows and outflows of banknotes into and out of a cash cycle play a substantial role for the national cash cycles of the Eurosystem. While we treat them as neutral with regard to the active circulation volume (see Section 3.2.3) they do affect the note quality of the specific cash cycle; be they inflows from other countries or outflows leaving the euro area. As the note quality of the inflowing banknotes can also vary, we studied two different scenarios with 5% and 20% unfit rates in the inflowing note population. For outflows, the model assumes that the note quality is identical to that of the banknotes in circulation. Chart 22 below shows the impact on note quality of inflows and outflows of up to 50% of the number of banknotes in circulation (1 billion) for the two cash cycles per year[32], whereas Chart 23 shows the corresponding change in note replacement costs. Note that – contrary to the cases presented in this section – in reality any cash cycle has a mix of inflows and outflows. While this can simulated in Model A, acquiring this data is often difficult or even impossible.

[32] Calculations based on estimated national circulation and a comparison of the national net issuance levels with total banknote circulation growth have shown that such inflows and outflows do occur in some national cash cycles.

Chart 22

Impact of inflows (+) and outflows (-) on banknote quality for Cash Cycles 1 and 2

(x-axis: number of banknotes in millions; y-axis: unfit in circulation)

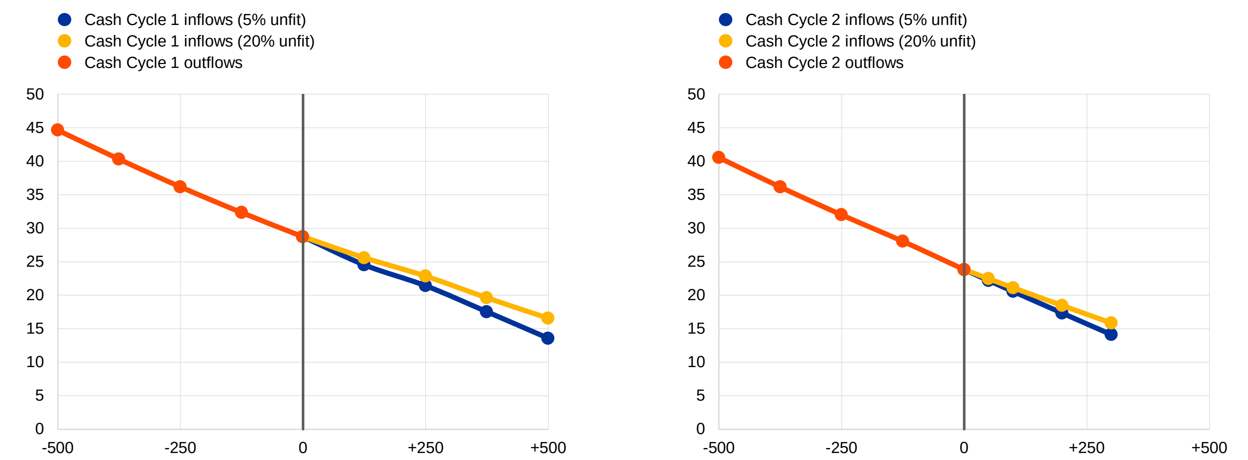


Source: Model A.
Note: For Cash Cycle 2 only inflows up to 300 million notes per year could be simulated. Higher inflows resulted in the number of new notes added by the NCB in each cycle to become negative. Circles denote calculated scenarios.

Chart 23

Impact of inflows (+) and outflows (-) on annual note replacement costs for Cash Cycles 1 and 2

(x-axis: number of banknotes in millions; y-axis: EUR millions)



Source: Model A.
Note: For Cash Cycle 2 only inflows up to 300 million notes per year could be simulated. Higher inflows resulted in the number of new notes added by the NCB in each cycle to become negative. Circles denote calculated scenarios.

The lower the quality of inflowing banknotes, the more it will – unsurprisingly – degrade the quality in circulation. The difference is, however, only about 2 p.p. for the largest modelled inflow scenario and therefore much less than the difference in the quality of the two inflow populations. It should be noted that any inflow has a negative effect on circulation quality, as the inflow reduces commensurately the number of new banknotes that can be issued by the NCB. This is also why inflows of more than 0.3 billion banknotes could not be modelled in Cash Cycle 2, as the number of new banknotes added in each iteration then becomes negative. In reality this means that at this point[33] fit banknotes pile up at the NCB, which then need to be shipped to a net issuing NCB (see also Section 2.1)[34]. While inflows restrict the issuance of new banknotes and make it more difficult for an NCB to adjust its circulation quality, they do reduce national replacement costs substantially (see Chart 23). Although the effective replacement cost reduction looks dramatic in the figure, it must be borne in mind that the savings for this particular national cash cycle give rise to additional costs on a similar scale for the “outflowing” cash cycle providing the banknotes. The size of the impact is very similar for both cash cycles.

In this respect, the model results confirm the assumption that outflows have a positive effect on a national circulation, as these outflows can be replaced with new banknotes or good-quality fit banknotes from stock, but that they cause a corresponding increase in banknote replacement costs.

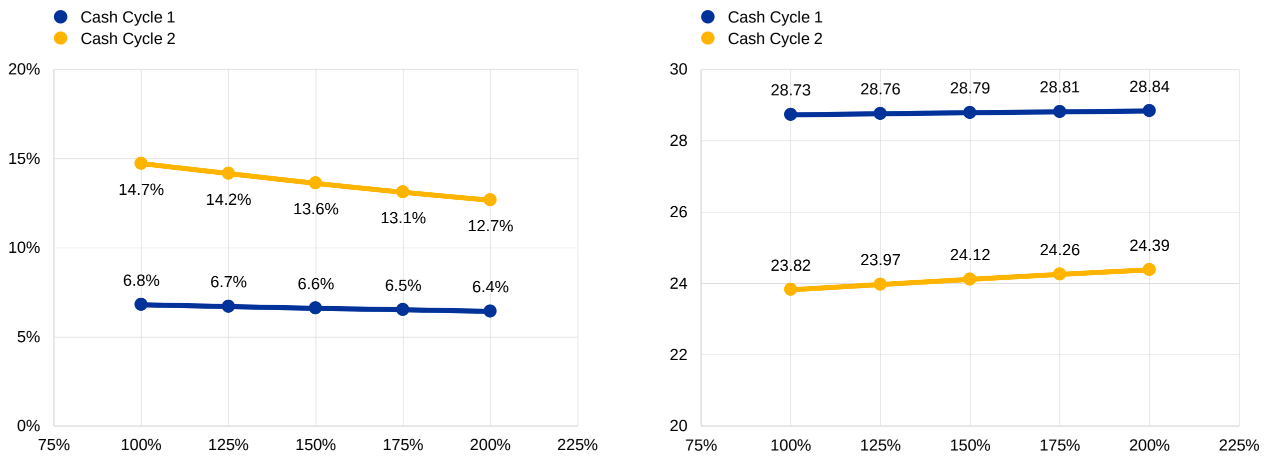
4.1.3.6 Future cash cycle trends – the impact of increased recirculation volumes

Considering the rapid pace of recirculation within the Eurosystem, this trend is expected to continue. As can be seen from Chart 24, increased recirculation, even at the current sorting thresholds, which are lower than those of a central bank, leads to an increase in note quality.

Chart 24

Impact of increased CH recirculation on note quality in circulation (left) and replacement costs (right)

(x-axis: CH sorting volume in relation to NCB sorting volume; left y-axis: unfit in circulation; right y-axis: EUR millions)



Source: Model A.
Notes: For Cash Cycle 2 scenarios with CH sorting volumes of more than 150% of the base scenario, the share of fit banknotes sent back to the NCB as surplus had to be reduced (from 25% to 17% at the 200% scenario) as the CH banknotes provided to the NCB would otherwise exceed the NCB sorting capacity. Circles denote calculated scenarios.

33 For Cash Cycle 1, this point is at inflows of about 0.6 billion banknotes p.a.

34 For modelling real-life cash cycles this should be reflected by lowering the ‘new notes fitness level’ of net-issuing countries, as they would then also issue not only new notes, but also excess fit notes from countries having a substantial inflow of notes.

This positive effect is more pronounced in Cash Cycle 2, which has a larger recirculation share. The increase in quality comes with a slight increase in replacement costs due to the increased volume of unfit banknotes returned by CHs to the NCBs as a consequence of their more extensive sorting. Overall, the expected increase in recirculation will have a mildly positive effect on circulation quality, accompanied by a minor increase in replacement costs for NCBs.

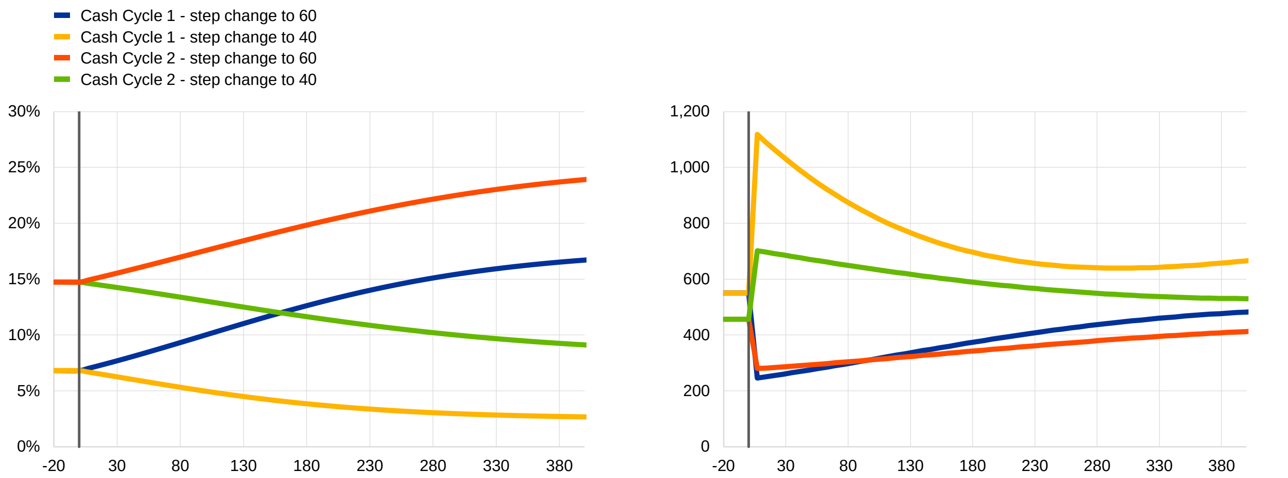
4.1.4 Dynamic modelling of step changes to a cash cycle

So far, all the cases presented have shown the steady-state condition at given cash cycle parameters. However, understanding the time behaviour of step changes applied to a cash cycle is valuable for central banks in deciding when to introduce changes to their sorting policies. Both our models also allow step changes to be simulated to see how quickly a cash cycle reacts to changes in its input parameters. While the models support step changes to all parameters, the most direct (and interesting) is to change the NCB sorting threshold for the two base cases' steady-state conditions. One possible application is for an NCB to determine how to modify its sorting policy in order to adjust the circulation quality based on the most recent QS. Another application, as was done for the introduction of the Europa series, can be to ensure optimal stock management and consumption of first series banknotes before introducing a new series. Most Eurosystem countries lowered their sorting standards in the months before the introduction of the Europa series denominations in order to save banknotes, knowing that the effect on circulation quality would be delayed and would not be visible to the public. Chart 25 shows the effect on note quality and note replacement costs in the weeks after a step change to the NCB sorting threshold for the two base cases to either more severe (threshold of 40) or more lenient (threshold of 60) standards.

Chart 25

Impact on quality in circulation (left) and weekly replacement costs (right) of more and less severe NCB sorting on Cash Cycles 1 and 2

(x-axis: days after step change; left y-axis: unfit notes in circulation description, right y-axis: k EUR)



Source: Model A.
Note: The vertical gray line denotes the application of the step change.

When more lenient sorting standards are applied, the quality in circulation decreases for both cash cycles only very gradually. Even after six months the change in the unfit in circulation does not exceed 5 p.p. (or 10 p.p. in 12 months) in either case. Meanwhile, such a step change in the NCB's sorting policy leads to an immediate decrease in the NCB unfit rate, reducing the weekly replacement costs from €550k to €250k for Cash Cycle 1 and from €460k to €280k for Cash Cycle 2.[35]

4.1.5 Frequency of note return to CHs and NCBs – a detailed analysis

The base results of the two theoretical cash cycles have already shown that changes in the frequency of note return to the NCB (f_{NCB}) and CHs (f_{CH}) are one of the key factors for a cash cycle. The comparison showed that, at similar total sorting volumes, markedly better quality in circulation is achieved with a higher NCB sorting share (Cash Cycle 1), due to the NCB sorting threshold being stricter than the CH thresholds. This comes at the expense of a higher note replacement volume (see Table 2). This section provides a more detailed view on the return frequency to the NCBs and CHs. For the CHs, return frequencies from 1 to 5[36] were modelled. The minimum NCB return frequency modelled was 1.5, as lower return frequencies would not allow the NCB to process all banknotes returning from CHs in cases with high CH sorting volumes. All figures used to create the resulting contour plots are provided in Annex A. The top plot in Chart 26 below shows the note quality in circulation as a function of the NCB and CH return frequency when both NCB and CH operate at the nominal sorting thresholds of 50 and 70 respectively[37]; as already seen from the results of the two base cases, a higher return frequency to the NCB results in better note quality in circulation. The impact of CH sorting has only a secondary effect and diminishes with higher NCB sorting (at f_{NCB}= 1.5 a change of f_{CH} from 1 to 5 results in a reduction in unfit banknotes from 22.6% to 17.6% [-5 Percentage points]. At f_{NCB}= 5, the same f_{CH} change reduces unfit banknotes only from 1.73% to 1.15% [-0.58 percentage points]).

35 The lower savings for Cash Cycle 2 are due to the lower return frequency of banknotes to the NCB.

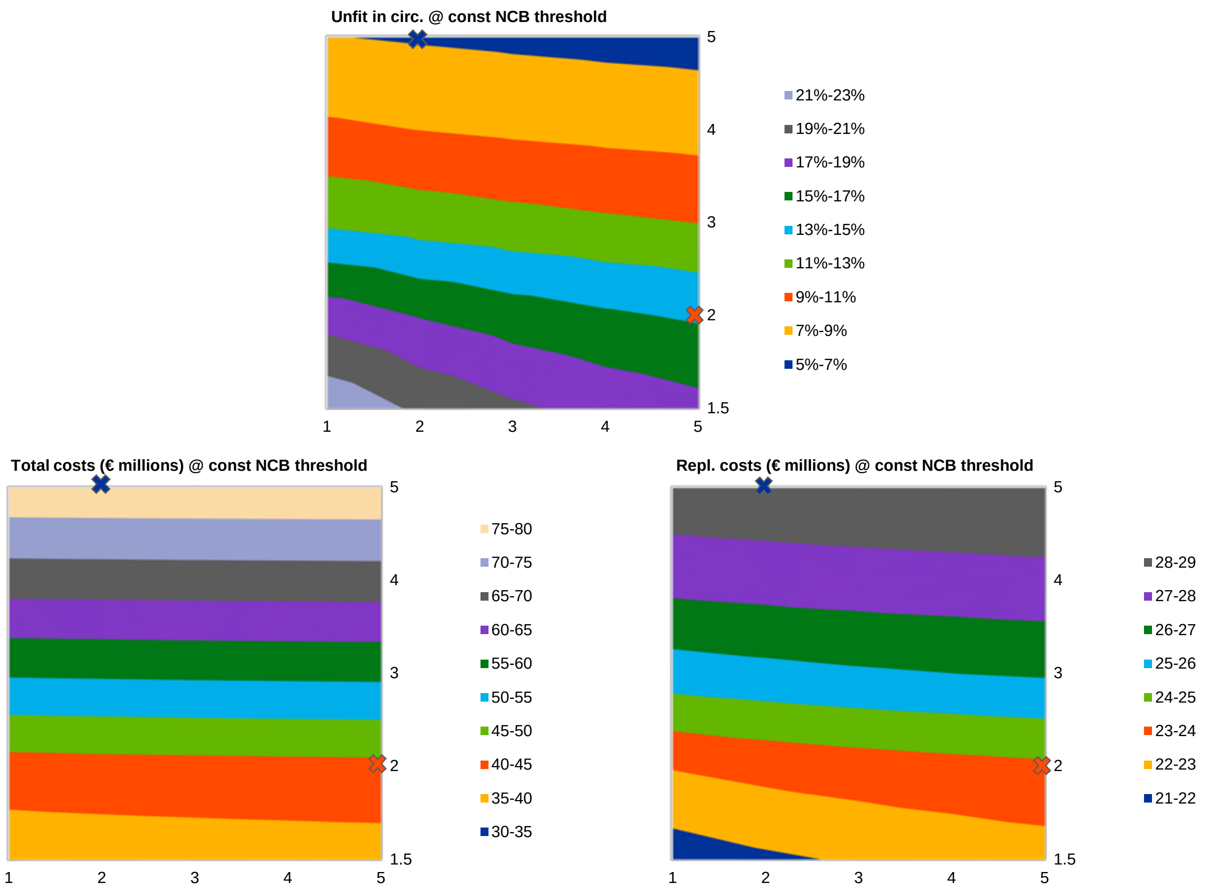
36 As the base cash cycles have an active circulation volume of 1 billion banknotes, the return frequency is equal to the NCB or CH sorting volume in billions.

37 As the modelled circulation volume is 1 billion banknotes, the given NCB and CH sorting volume in billions is identical to the annual return frequency.

Chart 26

Impact of banknote return frequency to NCB and CHs on unfit in circulation (top), annual replacement costs (bottom left) and total annual costs (bottom right)

(x-axis: billion banknotes per year; y-axis: billion banknotes per year; z-axis top (colour): unfit notes in circulation; z-axis bottom left and right (colour): EUR millions)



Source: Model A.
Note: The return frequencies for the two base cash cycles are indicated by blue (Cash Cycle 1) and red (Cash Cycle 2) crosses.

In close combination with the increase in note quality the replacement costs rise and even more so the total costs due to the higher NCB sorting volume. Again, similar to Section 4.1.3.1 (NCB sorting threshold) the question on the benefit of higher return frequency to the NCB arises, which we will study below by looking at two scenarios where we vary $f_{NCB}$and f_{CH} at a stable, defined quality in circulation. For the sake of the exercise we define 10% and 5% unfit in circulation as the target quality for the two scenarios.

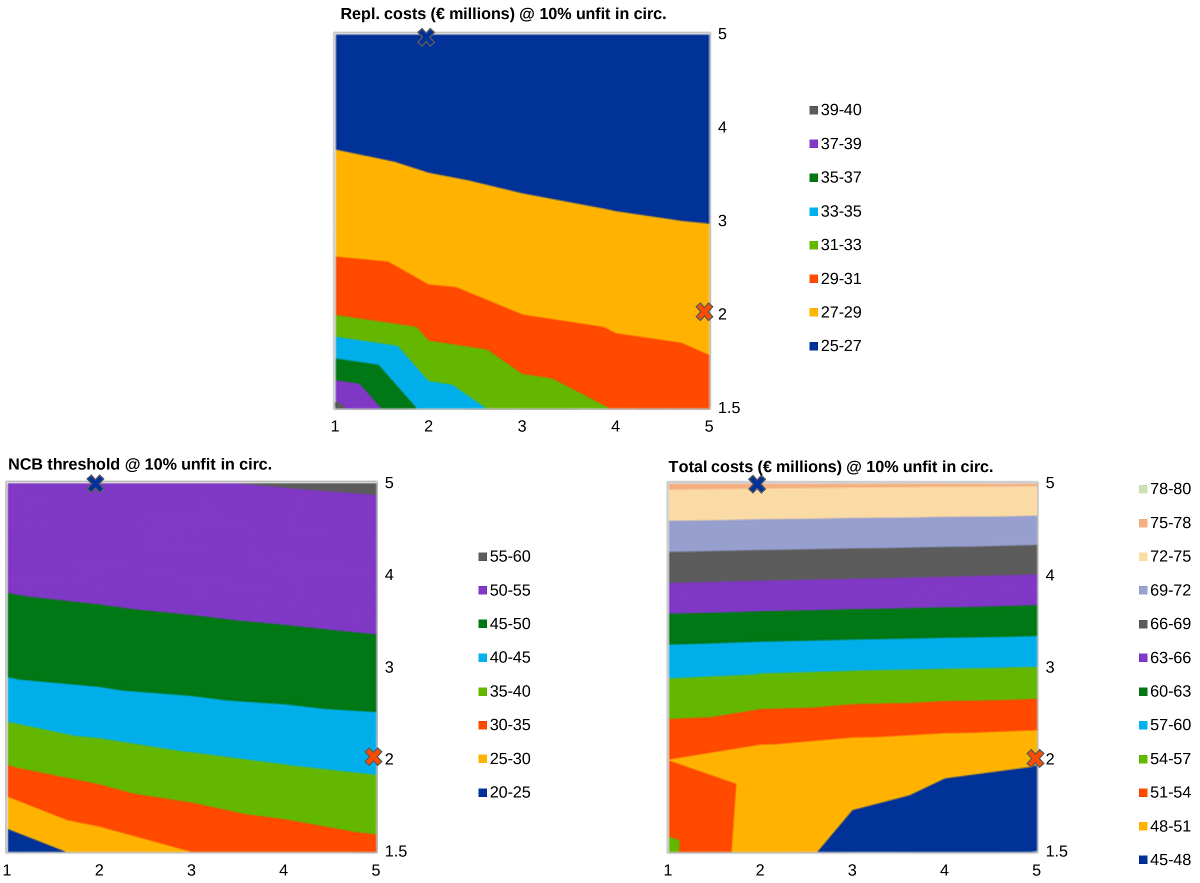
Impact of f_{NCB} and f_{CH} at 10% unfit banknotes in circulation

Chart 27 below shows the note replacement costs for this scenario, together with the required adjustment to the NCB sorting threshold. The total annual costs are also provided. It is apparent that a higher f_{NCB} keeps note replacement costs low. For instance, for f_{NCB} > ~3.5 annual replacement costs are below €27 million p.a. They start to rise gradually with lower values for f_{NCB}, but remain within acceptable limits if f_{CH} is large enough. Only in cases of low NCB and CH involvement does the NCB need to sort very severely (with a sorting threshold at a fitness level of 20-25) destroying a large number of fit banknotes to keep the quality at 10% unfit in circulation. Under such circumstances, note destruction becomes so high that, based on total costs, the cash cycle becomes inefficient[38] and either f_{NCB} or f_{CH} should be increased. For this specific model scenario, the most effective cash cycle would be at f_{NCB} ~1.5–2 and f_{CH} ~3-5.

Chart 27

Impact of banknote return frequency to NCBs and CHs at a target quality of 10% unfit banknotes in circulation on replacement costs (top), required NCB sorting threshold (bottom left) and total costs (bottom right)

(x-axis: billion banknotes per year; y-axis: billion banknotes per year; z-axis top (colour): EUR millions; z-axis bottom left (colour): fitness levels; z-axis bottom right (colour): EUR millions)



Source: Model A.
Note: The return frequencies for the two base cash cycles are indicated by blue (Cash Cycle 1) and red (Cash Cycle 2) crosses.

Impact of f_{NCB} and f_{CH} at 5% unfit banknotes in circulation

This scenario depicts a cash cycle where an NCB strives to maintain substantially better quality in circulation than in the above case. Chart 28 below shows the note replacement costs for this scenario, together with the required adjustment to the NCB sorting threshold. The total annual costs are also provided. White areas in the

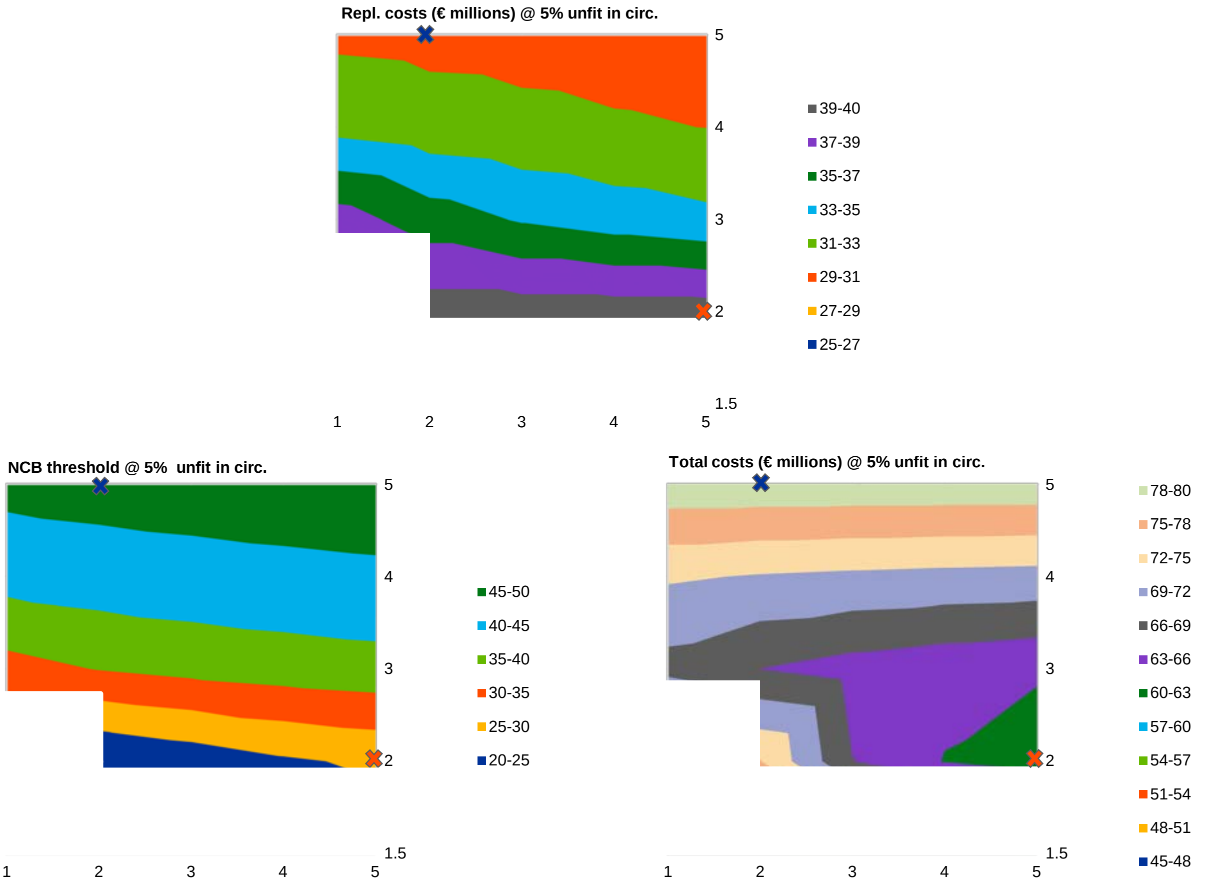
38 See Chart 27, total costs, bottom left part, where total costs start to increase again.

charts indicate conditions where, even at total destruction of all banknotes returned to the NCB, the target quality level of 5% unfit could not be achieved. Naturally, the replacement costs for maintaining higher note quality are increased compared with the 10% unfit scenario. They also rise sharply if f_{NCB} < ~3-3.5. Once again, the bottom left part of the total cost chart shows, now in even more pronounced fashion, that a higher level of NCB involvement is required if such a low number of unfit banknotes in circulation is to be maintained. CHs can only make up for this in part while they continue to apply lower sorting standards than the NCB. For this specific model scenario, the most effective cash cycle would be at f_{NCB} ~2–2.5, requiring a minimum f_{CH} ~3-5. In the event of lower CH involvement, f_{NCB} would need to be increased to 3 to achieve minimum total costs of note processing and replacement. A further increase in f_{NCB} to about 4 would yield a substantial reduction in note replacement costs, albeit accompanied by an increase in total costs due to the increased sorting effort.

Chart 28

Impact of banknote return frequency to NCB and CHs at a target quality of 5% unfit banknotes in circulation on replacement costs (top), required NCB sorting threshold (bottom left) and total costs (bottom right)

(x-axis: billion banknotes per year; y-axis: billion banknotes per year; z-axis top (colour): EUR millions; z-axis bottom left (colour): fitness levels; z-axis bottom right (colour): EUR millions)



Source: Model A.
Notes: The return frequencies for the two base cash cycles are indicated by blue (Cash Cycle 1) and red (Cash Cycle 2) crosses. White areas indicate conditions that do not allow 5% unfit banknotes in circulation to be achieved.

These more specific simulations on return frequencies confirm that good note quality in circulation is primarily driven by the frequency of note return to the NCB and – to a lesser extent – to the banknotes returning to CHs. The higher the targeted note quality, the more NCB involvement is required to ensure that replacement costs remain acceptable, and indeed that the desired quality is achievable. While no specification on minimum or maximum note circulation quality is in place within the Eurosystem, the opinion poll results (see Section 2.4) could serve as an indication of an economical quality level that still ensures public confidence in euro banknotes as well as good machine processability by both CHs and the NCB. Also, the results presented above cannot be taken as a general rule for any national cash cycle, as these are also affected by the other parameters described above – most significantly the note lifetime (which varies from country to country) and inflows and outflows.

4.2 Results for Model B: Simulating three real cash cycles

In this section, Model B, which is calibrated using real data from the ECT, is applied to real country cash cycles to simulate the current circulation. The parameters of the model are specific to the simulated country and are obtained by means of the analysis discussed in Section 3.3. The values for these parameters and other model inputs are summarised in Annex B.

One input that is estimated at this point is the fitness of the new banknotes that are added during the model run. New banknotes are introduced in each iteration to replace the NCB unfit banknotes. These banknotes begin their life in circulation with a soil value that is determined by production variations and is modelled using a normal distribution, with the mean and standard deviation defined in the [1…100] soil range. However, the mean and SD of this distribution are not the same for all the banknotes as, depending on the supply chain, different variants are produced in the Eurosystem, with smaller or larger deviations. At this point we do not know the variant or mix of variants that circulates in every country. Therefore, after some initial tests with different new banknote profiles, we have used a new note profile with a mean of five fitness levels and a standard deviation of six normalised fitness levels, $N(5,6)$, which is well within the estimated production variations in the Eurosystem. The populations with other tested profiles are presented in the following subsections.

We simulate the cash cycles of the three ECT countries – Austria, Ireland and the Netherlands – and validate our results against actual figures from circulation, using validation sources outside the ECT. In the results presented in the following subsections, we refer to validation data originating from the 2015 QS and the processing data reported by CHs and NCBs. Where deemed necessary, the fitness profile of the final population is also compared against the QS population profile.

4.2.1 Results for Austria

Table 4 shows the results of the model for Austria. The results of the model are reasonable overall by comparison with the expected figures. The CH unfit rate closely matches reality, given the uncertainty in the CH sorting threshold. The NCB unfit rate is only slightly lower than expected. Overall, the quality in circulation (total unfit) is close to that reported in the QS, with approximately 3 p.p. deviation, which is within the estimated accuracy of the QS sample[39].

Table 4

Model results for Austria

Output	Result of model	Actual figure	Validation source
NCB unfit rate	12.71%	16.04%	reported by NCB
Unfit in circulation (total)	8.80%	5.09%	QS 2015
Share of notes with soil only	2.09%	1.47%	QS2015
Share of notes with defects only	4.26%	1.03%	QS 2015
Share of notes with soil and defects	2.45%	2.59%	QS 2015
CH unfit rate	8.70%	6%	reported by CHs

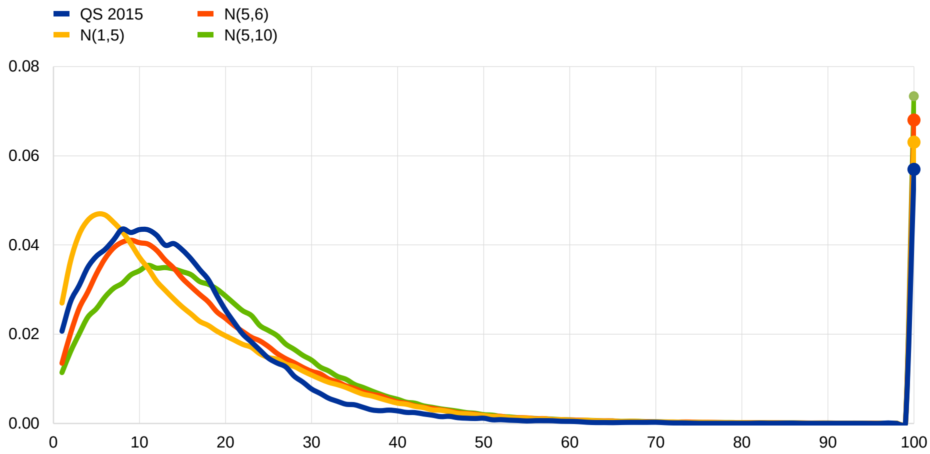
There are various possible reasons for the discrepancies observed. The population of the QS consists of banknotes that are issued and circulate in the country, but also contains banknotes that have migrated into the country. The latter have different fitness profiles, depending on which country they come from. No such inflow is taken into account in Model B. A breakdown of the unfit banknotes in circulation into banknotes that have only high soil values, only defects or both reveals that the result that matches least closely is the percentage of banknotes that only have defects. This is an indication that the modelling of defects can be improved, as discussed in Section 3.3.5. Another possible explanation of part of the deviation is the fitness distribution of new banknotes assumed in our model, which, as discussed previously, can vary significantly. In every country there are multiple production batches from different printing works and paper mills circulating. Chart 29 shows a comparison of the steady-state fitness profiles of the population that result when three different new banknote fitness distributions are used. Note that the spike in the fitness value at fitness level 100, as for Model A, represents the percentage of defects. All the new note fitness distributions used reflect plausible production variations. We plot the 2015 QS population profile alongside these for comparison purposes. It is clear from this chart that changing the new note profile has a significant impact on the results of the model, especially on the final population fitness profile.

[39] The sample consists of 10,000 banknotes per denomination and country which, when applying conventional statistics would result in a very low error. However, the results are also affected by the different soil assessments of the two sorting machines (see Section 2.4) and national differences in how NCBs collect the samples. From a historical analysis of the national QS results, changes of about 2-3 p.p. are attributable to these effects.

Chart 29

Model B final population profiles for Austria, for varying new note fitness profiles, along with QS2015 population profile

(x-axis: normalised fitness levels [1-100]; y-axis: frequency)



Sources: Model B, QS 2015
Notes: All the profiles tested follow the normal distribution N(μ,σ) with varying parameters, μ and σ.

Finally, it should be noted that the QS population profile has been converted from machine units to normalised soil. This process produces a number of banknotes with machine units soil values that correspond to out-of-range values in the normalised fitness range of [1…100]. These soil values are disregarded in the visualisation here. However, this methodology for converting machine soil units can also contribute to discrepancies in the fitness profile shapes we observe, especially in the lower soil values.

4.2.2 Results for Ireland

The second simulated country is Ireland. As Table 5 shows, the quality in circulation is slightly underestimated but well matched. However, there is a major discrepancy on the NCB unfit rate. The inconsistency between the reported quality in circulation and the reported shred rate was also observed when seeking to simulate Ireland’s cash cycle using Model A.

Table 5

Model results for Ireland[40]

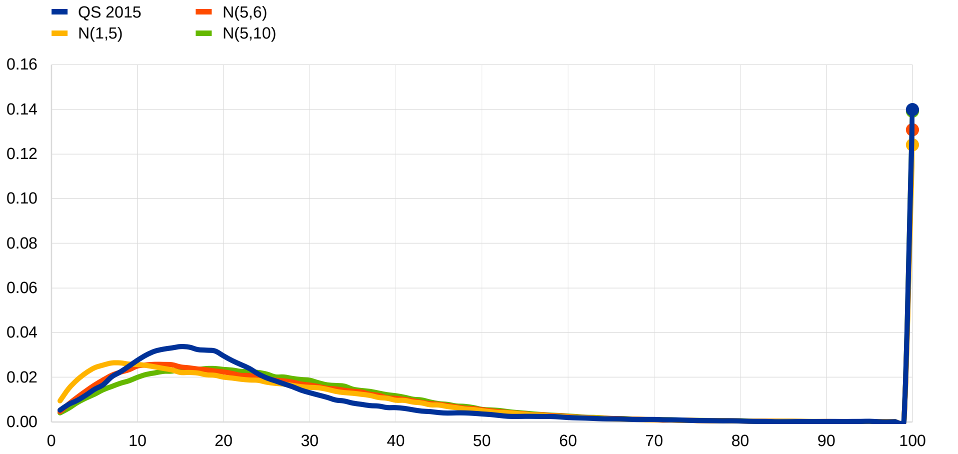
Output	Result of model	Actual figure	Validation source
NCB unfit rate	23.06%	52.12%	reported by NCB
Unfit in circulation (total)	19.20%	16.94%	QS 2015
Share of notes with soil only	6.16%	6.29%	QS 2015
Share of notes with defects only	7.33%	3.85%	QS 2015
Share of notes with soil & defects	5.71%	6.80%	QS 2015

Looking again at the different distributions for the new note profile (see Chart 30), we observe that there is a visible effect in the final population profile, without any of the tested profiles achieving a very close match to the QS. The possible reasons for the discrepancies that were discussed in the case of Austria apply here too.

Chart 30

Model B final population profiles for Ireland, for varying new note fitness profiles, along with QS 2015 population profile

(x-axis: normalised fitness levels [1-100]; y-axis: frequency)



Sources: Model B, QS 2015.
Note: All the profiles tested follow the normal distribution N(μ,σ) with varying parameters, μ and σ.

4.2.3 Results for the Netherlands

The third country that was modelled using the data from the ECT is the Netherlands. The results are presented in Table 6. In this case the unfit in circulation are again slightly overestimated, but are not very far from the QS data. Apart from the discrepancies (and the possible reasons for them), which are in line with the two previous cases, some additional inconsistencies are observed.

[40] Ireland does not recirculate the €5 note, therefore no data for the CH unfit rate is available.

Table 6

Model results for the Netherlands

Output	Result of model	Actual figure	Validation source
NCB unfit rate	36.56%	21.61%	reported by NCB
Unfit in circulation (total)	14.45%	8.87%	QS 2015
Share of notes with soil only	2.53%	2.46%	QS 2015
Share of notes with defects only	8.65%	3.82%	QS 2015
Share of notes with soil & defects	3.27%	2.59%	QS 2015
CH unfit rate	29.25%	45.50%	reported by CHs

The main inconsistency in the case of the Netherlands is the unfit rates of both the NCB and the CHs. For the CHs, a fitness threshold of 35 is applied, based on the information provided by De Nederlandsche Bank (DNB) that its major cash handler, by agreement with DNB, applies this strictness. Even with this low threshold, however, the unfit rate predicted by the model is only close to 30%, and not to the 45.5% rate reported by DNB. For the model to predict such a high recirculation unfit rate, either the quality of banknotes in circulation would have to be lower or the recirculation threshold would have to be even more severe. The former is not considered feasible, as it is not supported by the QS data and the share of unfit banknotes in circulation is already overestimated by Model B. The latter is theoretically possible, and it showcases the need for some degree of CH data monitoring in order to simulate cash cycles with high CH involvement more accurately.

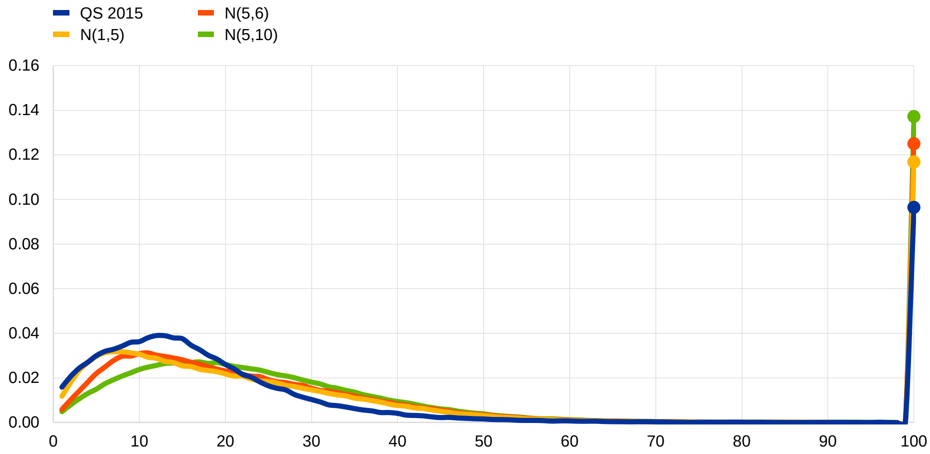
Apart from the recirculation unfit rate, the destruction rate also deviates from the expected value of 21.6%. This is a result of the model predicting worse quality in circulation than is seen in the QS. This is an indication that the ageing calculation in the case of the Netherlands is not very accurate, in spite of our efforts to correct effects linked to the soil evaluation of the sensor used in the ECT and the substantial recirculation share.

As in the previous cases, Chart 31 shows the final population profile for different new note profiles, showcasing the impact of this parameter.

Chart 31

Model B final population profiles for Ireland, for varying new note fitness profiles, along with QS 2015 population profile

(x-axis: normalised fitness levels [1-100]; y-axis: frequency)



Sources: Model B, QS 2015.
Notes: All the profiles tested follow the normal distribution N(μ,σ) with varying parameters, μ and σ.

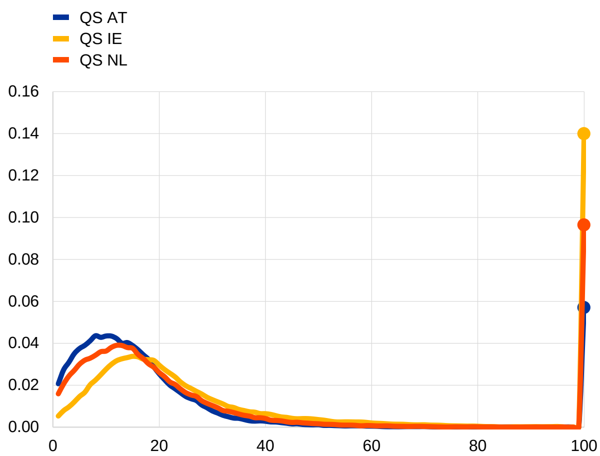
4.2.4 Comparison of the model fitness profiles with real-life note profiles

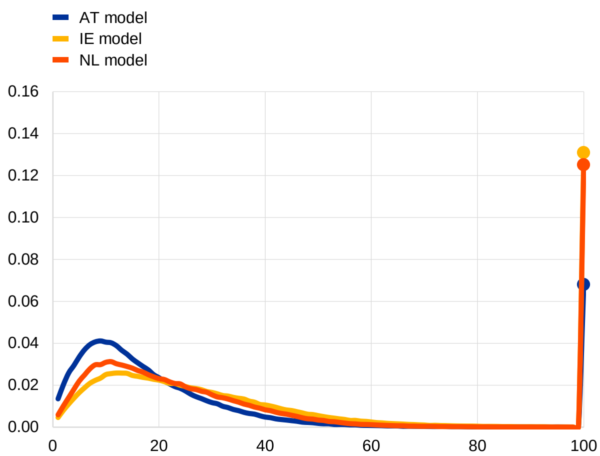
Even though the population profile of every country is not matched perfectly, it is worth examining the extent to which the relationships between the population profiles of the different countries are replicated. Chart 32 illustrates this comparison. It can be seen that the relationship between the fitness profiles of the three countries can be replicated by the model, in both the fit and the unfit ranges. There is a slight mismatch in the relevant positions of the peaks of the population fitness profiles, but overall the actual quality profile of banknotes in circulation is consistent with the QS results.

Chart 32

Comparison between steady-state population profiles from the QS (left) and Model B (right)

(x-axis: fitness level; y-axis: frequency)





Source: 2015 QS, Model B.
Note: In the model, the new note profile of $N(5,6)$ was used for the reported results in this figure.

5 Summary and conclusions

5.1 The main parameters governing a cash cycle

In the simulations of two theoretical cash cycles which differed only in the share of recirculation by cash handlers (40% of the NCB sorting volume in Cash Cycle 1 and 250% in Cash Cycle 2), we identified the following three primary parameters governing note quality and overall cash cycle costs that can be influenced by a central bank in one way or another:

1. **Return frequency of banknotes to the NCB**

   Increasing the frequency at which banknotes are returned to the central bank significantly increases note quality in circulation but also increases the overall cash cycle costs due to additional costs for NCB note processing and transport. CH sorting can only partly substitute note processing by NCBs because CH sorting standards are lower than NCB ones. As changes to this parameter require substantial (policy) changes, an evaluation is needed for each national cash cycle to establish whether changes should be introduced to either reduce the central bank sorting volume (and return frequency) in cash cycles with high note quality or increase the central bank involvement in cash cycles with lower note quality.

2. **Central bank sorting threshold**

   More severe sorting at the central bank increases note quality, up to a ceiling defined by the return frequency. This improved note quality comes at the expense of additional costs for replacement banknotes. We have shown that up to a critical quality level the costs for replacement banknotes increase similarly for different cash cycles. Yet, the return frequency of banknotes to the NCB governs the maximum note quality that can be achieved under economically viable conditions (see Chart 16 and Section 4.1.5). At a critical point, any further increase in the NCB sorting threshold (and unfit rate) will exponentially increase the volume of replacement banknotes without having a substantial further impact on note quality.

   Our modelling approach shows that striking a balance between the ideal NCB return frequency and severity of NCB sorting is specific to each cash cycle.

3. **Banknote durability**

   Increases in a banknote's resistance to soil (i.e. an increase in the average time a banknote can be used in active circulation before being more soiled than the Eurosystem threshold) and defects (e.g. banknotes that are more difficult to tear or crumple) – at a constant NCB sorting threshold – improve the note quality in circulation and reduce the annual replacement costs. Further savings can be achieved if the central bank decides to apply a more lenient sorting policy, forgoing the increase in quality in favour of additional savings in the replacement note volume. The increase in the banknote lifetime typically comes with additional production costs due to the use of more costly substrates or an additional coating step, but could also be achieved in part by educational measures (which of course also have a cost) to the public on how (not) to treat banknotes.

We confirmed that increases in the note lifetime for the Europa series €5 banknotes outweighed the additional costs for the coating and for the Eurosystem overall, providing a substantial saving of more than 500 million banknotes per year.

5.2 Further parameters affecting a cash cycle

Production variations in new banknotes were identified as a secondary parameter determining note quality in circulation. As with the introduction of the Europa series, the Eurosystem has already undertaken further efforts to reduce variations; an assessment is needed as to whether further improvements will result in additional savings.

The impact of the accuracy of the fitness sensors used by the NCBs is dependent on the cash cycle. Our simulations showed that, depending on the current accuracy of the fitness sensor, an increase in its accuracy (or replacement with a more accurate model) yields savings predominantly in cash cycles where the NCB is closely involved in sorting fit banknotes from circulation, where sensor error results in the substantial destruction of fit banknotes. In cash cycles where the central bank predominantly sorts unfit banknotes returned from CHs, we found that sensor improvements have a lesser effect.

In cash cycles with a larger volume of sorting by CHs, applying stricter standards to CH sorting will increase note quality at the expense of substantially more unfit banknotes, which need to be returned to an NCB for destruction. This can have a negative impact on the CHs' business model and increase costs for additional note transports. For each cash cycle, study is needed to establish whether this is a feasible parameter to adjust, either for all CHs or for a certain proportion that process larger volumes.

Furthermore, we found that the cash cycle of a specific country is heavily affected by the active circulation volume, inflows and outflows of banknotes from its area of responsibility due to migration and tourism, and any changes to the actual circulation in a country. While these parameters are purely demand-driven and cannot be controlled by an NCB, they need to be accurately entered into the model in order to allow a reasonable estimation of a cash cycle and any changes applied to it. The determination of these parameters has, however, proven difficult and requires further study.

By correlating the note quality as determined annually by the Eurosystem with results from country-specific opinion polls (see Chart 6), we found that the public perception of euro banknotes correlates well with the number of banknotes that are unfit by Eurosystem standards. As a first estimate, an unfit level of 10% in circulation seems to be well accepted by the public. While this appears true on an aggregate level, the available data do not allow such an "unfit threshold level" to be clearly established for each country. Once defined on a national level, staying below but close to this quality level would be a target for a cost-effective cash cycle.

Other model parameters like the ageing model and the defect probability at a specific fitness level were found to be less influential for the studied theoretical cash cycles.

In a dynamic modelling example, we showed that for typical Eurosystem cash cycles, the change in circulation note quality has a very long time constant. This allows NCBs to reduce their sorting standards before introducing a new series, in order to save banknotes with only a small drop in note quality which will most likely not have any substantial impact on public perception.

5.3 Application to actual cash cycles

Using the data from an external circulation trial conducted from 2014 to 2016 in the Netherlands, Austria and Ireland, we determined for each cash cycle the key inputs of return frequency and ageing rate based on real-life data. As the defect probability could not be modelled reliably by the collected data due to technical issues in the data collection, we instead used data from each country's QS. We based all other model parameters on gross figures collected by the central banks and previous work carried out in earlier Eurosystem R&D activities. For the production variations of new banknotes, we used estimates originating from earlier Eurosystem studies.

Uncontrolled backlogs at the sorting centre and inconsistent soil values from the different sorting machines in each of the three countries introduced substantial problems in the data analysis and resulted in the exclusion of banknote return events suspected of suffering from these phenomena.

The effects of CH sorting, which results in additional unfit banknotes processed by the NCB and hence makes the NCB fitness profile different from that in circulation, needed to be accounted for. This proved especially difficult for the Netherlands, which has the highest recirculation share of the three ECT countries.

The model results have been validated against known figures, on a number of aspects, covering the quality in circulation and unfit rates in NCB and CH sorting. The overall results are promising, as in most of the cases the quality in circulation could be predicted with reasonable accuracy taking into account the uncertainty of the real-life figures from the QS. Additionally, even though there is not a perfect match with the population profile, the relationships between the three countries' profiles are consistent with those observed in the respective QSs.
However, there are some discrepancies between the model results and the real-life figures that cannot be neglected:

1. **The overestimation of unfit banknotes in circulation.** This is a consistent observation for all three countries. It may be an indication that the ageing function found to best fit the data is not close enough to reality. However, the reason could also be that the sorting machines used for assessing the quality of banknotes in the QS are not the same as those used for determining note fitness in this ECT. As a result, the fitness values used for modelling (in the standard 1-100 scale) could be incorrect either due to a lack of correlation between the QS sorting machines and the ECT sorting machines, but also due to any (unknown and therefore non-compensated) non-linearity between the fitness value delivered by the ECT fitness sensors and the standard 1-100 scale.
2. **The mismatch of the central bank unfit rate in Ireland.** Both the theoretical model and the real-life model find that the target shred rate and quality in circulation cannot be achieved using the best estimates/models for the circulation in Ireland. Reasons for this deviation are not definitively known at present but are assumed to be due to the sum of errors in the model estimates.
3. **Discrepancies in the Netherlands due to the strong recirculation share and also the non-linearity of the fitness sensors used at DNB, account for the bulk of this country's deviations.** We are also aware from an analysis of NCB net issuance data and studies of note migration based on serial number reading that banknote migration plays a substantial role in the Netherlands in particular (ter Huurne, Post, Duijndam, Overakker, Vis & Broeder, 2010). This may account for substantial differences between the note quality found in the QS and the data derived from our model using banknotes only issued locally in the scope of the ECT.

Taking into account that the data collection in the circulation trials was not specifically intended or designed for the purposes of modelling the banknotes in circulation, the results of the model are quite promising. The results are close to the key figures, with some exceptions that have been discussed. It is important to note that the calibration of the model comes directly from the modelling of the circulation and default commonly accepted parameters (e.g. NCB sorting threshold of 50), with minimum intervention or "expert guessing".

5.4 Future work

While we think that the results of our study can already give useful indications for policy decisions and improving the efficiency of cash cycles, we are of the opinion that the following additional work should be undertaken in the future:

1. **Better determine the key parameters of actual circulation and note inflows and outflows:** Further improvements would only be possible with consistent collection of individualised banknote data (e.g. serial number reading) of all banknotes at NCB level, which is not currently the case. However, the current generation of sorting machines which are or will be installed across the

Eurosystem have this capability. It is expected that with larger-scale implementation of serial number reading more accurate data will become available on circulation flows that can be plugged into our model without affecting the anonymity of banknotes.

2. **Improve fitness algorithms delivering accurate fit/unfit classifications and linear fitness values:** In order to produce machine-independent and commonly understood and applicable models, we need to translate machine fitness units into normalised fitness levels. This is difficult with some current fitness sensors, as they rely on non-linear, e.g. fuzzy logic and as such are not designed to deliver a fitness value of a note but in some cases only a binary fit/unfit judgement related to a dedicated training set. The Eurosystem has already conducted substantial developments on machine-independent absolute soil indicators based on colour (RGB) measurements of images, which correlate well with human perception. While the primary aim of any future fitness sensor should be to deliver the most accurate fit/unfit classification, for modelling activities it is essential to also have access to a linear fitness scale that correlates closely to a banknote's soil value during its lifetime.

3. **Include cash handlers in future circulation trials.** As recirculation by CHs is playing a larger role and NCBs are therefore becoming more distant from the actual circulation, in future circulation trials samples of banknotes should also be taken directly at CHs to reduce error and potentially improve the correlation between our models and real-life data.

4. **Carry out further studies on the correlation between quality in circulation and public perception.** The data collected by the opinion polls and the quality survey show a good overall correlation. However, further studies should be undertaken to better determine the limits of the quality in circulation that are acceptable to the public and promote the main task of the Eurosystem of ensuring public confidence in euro banknotes.

References

Balke, P. (2011), From fit to unfit: How banknotes become soiled, Watermark Conference, Rostov on Don.

Bartzsch, N., Rösl, G., and Seitz, F. (2011a), Foreign demand for euro banknotes issued in Germany: estimation using direct approaches, Deutsche Bundesbank Discussion Paper, Series 1, No. 20.

Bartzsch, N., Rösl, G., and Seitz, F. (2011b), Foreign demand for euro banknotes issued in Germany: estimation using indirect approaches, Deutsche Bundesbank Discussion Paper, Series 1, No. 21.

Bouhdaoui, Y., Bounie, D., and Van Hove, L. (2013), When do plastic bills lower the bill for the central bank? A model and estimates for the U.S., Journal of Policy Modeling, Vol. 35, pp. 45 60.

Buitelaar, T. (2008), The Colour of Soil, DNB Cash Seminar 28 29 February, Amsterdam: DNB Cash Seminar.

Cabrero, A., Camba-Mendez, G., Hirsch, A., and Nieto, F. (2002), Modelling the Daily Banknotes in Circulation in the Context of the Liquidity Management of the European Central Bank, ECB Working Paper Series, No 142.

Central Bank of Russia (n.d.), Unfit criteria for banknotes, Retrieved 17 November 2017.

Den Butter, F., and Coenen, R. (1982), The Process of Soiling and the Life of Bank Notes in the Netherlands, Journal of the Royal Statistical Society, Series C (Applied Statistics), Vol. 31, No 3, pp. 226 237.

European Central Bank (2010), Decision of the European Central Bank of 16 September 2010 on the authenticity and fitness checking and recirculation of euro banknotes (ECB/2010/14).

European Central Bank (2017a), Banknotes and coins production.

European Central Bank (2017b), Estimation of euro currency in circulation outside the euro area.

European Central Bank (2017c), Successfully tested types of banknote handling machine.

Federal Reserve Bank Services – Currency Technology Office (2008), Fitness guidelines for federal reserve notes (FRNs).

Geusebroek, J.-M., Markus, P., and Balke, P. (2011), Learning banknote fitness for sorting, Pattern Analysis and Intelligent Robotics (ICPAIR), 2011 International Conference on IEEE, pp. 41 46.

Gillieson, A.H. (1977), Research into the extension of the life of banknotes: Results of 1973, 1975 and 1976 field trials, Bank of Canada Technical Report, No 10.

Janicki, H.P., Main, N.F., Waddle, A. L., and Wolman, A.L. (2007), Currency Quality and Changes in the Behaviour of Depository Institutions. Economic Quarterly, Vol. 93, Fall, pp. 361 391.

Koeze, P. (1979), An accurate statistical estimation of the life-length of f 100 banknotes: A circulation trial with two qualities of currency paper, International Statistical Review/Revue Internationale de Statistique, pp. 283 297.

Kropnick, E. (2012), Measurement of banknote soiling by sorting machines: an empirical study, Billetaria – International Review on Cash Management, Year VI, Issue 11, April, pp. 24 26.

Kwon, S.Y., Pham, T.D., Park, K.R., Jeong, D.S., and Yoon, S. (2016), Recognition of banknote fitness based on a fuzzy system using visible light reflection and near-infrared light transmission images, Sensors, Vol. 16, No 6, p. 863.

Kyrychok, T., Shevchuk, A., Nesterenko, V., and Kyrychok, P. (2014), Banknote Paper Deteriation Factors: Circulation Simulator Methods, bioresources.com, Vol. 1, No 9, pp. 710 724.

Lacker, J.M., and Wolman, A.L. (1997), A simple model of circulation quality, Federal Reserve Bank of Richmond, Mimeo, November.

Martin, J., and Meuer, T. (2001), The Eurosystem's efforts in the search for a longer lasting banknote, Billetaria – International Review on Cash Management , Year V, Issue 9, April, pp. 22 24.

Menzies, G. (2004), Money to burn, or melt? A cost-benefit analysis of Australian Polymer banknotes, North American Journal of Economics and Finance Vol 15, pp. 355 368.

Online survey on the quality of euro banknotes (n.d.).

ter Huurne, D., Post, R., Duijndam, K., Overakker, L., Vis, P., and Broeder, S. (2010), Predictions for Circulation, Migration and Hoarding Amounts of Banknotes in the Netherlands, Amsterdam: VU University.

van der Horst, F., Meeter, M., Theeuwes, J., and van der Woude, M. (2011), What is a fit banknote? The Dutch public responds, DNB Occasional Studies, Vol. 9, No 4.

Van Hove, L. (2015), Modelling banknote printing costs: of cohorts, generations and note-years, Economic Modelling, Vol 46, pp. 238 249.

Annex A
Supplementary results for Model A

In Table A.1 we detail all the steady-state note flows of the two simulated base cash cycles, as analysed in Section 4.1.1. The underlying results that correspond to Chart 26 – Chart 28 are also presented.

Table A.1
Steady-state banknote flows of the two base cash cycles

Model note flows per week at steady state	Cash Cycle 1 (million notes)	Cash Cycle 2 (million notes)
Vinflow	0	0
Voutflow	0	0
VNCB,Sort	85.7	11.35
VCH,Sort	38.4	95.9
VCH,Fit	37.5	91.8
VCH,Unfit	0.8	4
VCH,FitToNCB	9.4	23
VCH,FitToCirc	28.1	68.8
VNCB,Fit	84.9	29.2
VNCB,Unfit	11	9.1
VCircInc	0.9	0.9
VNew	12	10.1
VUnprocessed	876	892.8

Detailed results of the variation of NCB and CH return frequency (results of the base cash cycles coloured blue (Cash Cycle 1) and red (Cash Cycle 2)).

Quality (in % unfit in circulation) @ constant NCB threshold of 50

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	22.6%	20.7%	19.3%	18.4%	17.6%
	2	18.1%	16.9%	16.0%	15.3%	14.7%
	3	12.7%	12.1%	11.7%	11.3%	11.0%
	4	9.3%	9.0%	8.7%	8.4%	8.2%
	5	7.0%	6.8%	6.6%	6.4%	6.3%

Total costs (in € millions) @ constant NCB threshold of 50

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	36.47	36.84	37.11	37.34	37.54
	2	43.06	43.33	43.53	43.69	43.82
	3	55.53	55.71	55.87	56.00	56.11
	4	67.35	67.46	67.56	67.64	67.71
	5	78.66	78.73	78.79	78.84	78.88

Replacement costs (in € millions) @ constant NCB threshold of 50

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	21.47	21.84	22.11	22.34	22.54
	2	23.06	23.33	23.53	23.69	23.82
	3	25.53	25.71	25.87	26.00	26.11
	4	27.35	27.46	27.56	27.64	27.71
	5	28.66	28.73	28.79	28.84	28.88

NCB threshold (fitness level) @ constant quality of 10%

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	21.40	27.00	30.00	32.00	33.52
	2	35.72	37.90	39.34	40.42	41.28
	3	46.10	46.92	47.55	48.07	48.52
	4	50.95	51.45	51.88	52.27	52.67
	5	54.30	54.63	54.92	55.16	55.37

Total costs (in € millions) @ constant quality of 10%

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	54.61	49.36	47.16	45.92	45.09
	2	50.98	49.72	49.01	48.53	48.18
	3	57.79	57.49	57.27	57.10	56.96
	4	66.76	66.54	66.36	66.16	65.91
	5	75.66	75.55	75.46	75.39	75.34

Replacement costs (in € millions) @ constant quality of 10%

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	39.61	34.36	32.16	30.92	30.09
	2	30.98	29.72	29.01	28.53	28.18
	3	27.79	27.49	27.27	27.10	26.96
	4	26.76	26.54	26.36	26.16	25.91
	5	25.66	25.55	25.46	25.39	25.34

NCB threshold (fitness level) @ constant quality of 5%

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	1.00	1.00	1.00	1.00	1.00
	2	1.00	16.50	21.95	24.30	25.82
	3	33.25	35.20	36.50	37.45	38.20
	4	41.92	42.77	43.35	43.84	44.22
	5	46.33	46.75	47.08	47.36	47.63

Total costs (in € millions) @ constant quality of 5%

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	81.57	81.66	81.72	81.76	81.79
	2	106.35	75.16	66.06	62.86	61.03
	3	67.94	65.99	64.82	64.04	63.46
	4	72.38	71.80	71.45	71.17	70.98
	5	80.63	80.47	80.40	80.33	80.25

Replacement costs (in € millions) @ constant quality of 5%

		CH sorting volume (billions)				
		1	2	3	4	5
NCB sorting volume (billions)	1.5	66.57	66.66	66.72	66.76	66.79
	2	86.35	55.16	46.06	42.86	41.03
	3	37.94	35.99	34.82	34.04	33.46
	4	32.38	31.80	31.45	31.17	30.98
	5	30.63	30.47	30.40	30.33	30.25

Annex B
Summary of inputs for Model B

In this Annex we present the best-fit functions for the return frequency, the ageing rate and the defect probability, as they were extracted from modelling the ECT data (Table B.1). A summary of all the inputs used in Model B to simulate the three real cash cycles is also presented (Table B.2).

Table B.1
Best-fitting functions for the return probability, ageing and defects

Return probability to the sorting centre	
AT	$P_{NCB} = 0.04$
IE	$P_{NCB} = 0.025$
NL	$P_{NCB} = 0.035$
Ageing rate function (in normalised fitness levels)	
AT	$P(dS_{week}) = 0.036 * (2.4 * dS_{week} + 0.1)^{-1.4}$
IE	$P(dS_{week}) = 0.74 * (2.4 * dS_{week} + 0.9)^{-2.09}$
NL	$P(dS_{week}) = 0.003 * (0.15 * dS_{week} + 0.08)^{-2.3}$
NL corrected	$P(dS_{week}) = 0.0019 * (0.15 * dS_{week} + 0.05)^{-2.06}$
Defects function (in normalised fitness levels)	
AT	$P(Defect\|Soil = x) = cdf(65,24.4)$
IE	$P(Defect\|Soil = x) = cdf(67,26.6)$
NL	$P(Defect\|Soil = x) = cdf(60.4,26.6)$

Table B.2

Model inputs for Model B used to simulate the cash cycles of the three countries

Input	AT	NL	IE	source
Ageing model	inverse	inverse	inverse	ECT modelling
	(0.0355,0.1,1.445)	(0.0019,0.05,2.06)	(0.74,0.9,2.09)	
Return probability	constant	constant	constant	ECT modelling
	0.04	0.025	0.035	
Defect model	normal CDF	normal CDF	normal CDF	QS 2015
	(65,24.4)	(60.4,26.6)	(67,26.6)	
New note profile	normal	normal	normal	Estimated small deviation
	(5,6)	(5,6)	(5,6)	
Third party processing ratio[41]	0.28	2.23	-[42]	CIS2 recirculation
				H1 2015
Recirculation ratio[43]	0.29	0.52	-	CIS2 recirculation
				H1 2015
NCB sensor	(50,15)	(50,15)	(50,15)	estimated
Recirculation sensor	(60,15)	(35,15)	-	estimated

41 $\boldsymbol{R}_{processed,3rd\ party} = \frac{V_{CH,Sort}}{V_{NCB}}$.

42 Ireland does not recirculate €5 banknotes.

43 $\boldsymbol{R}_{recirculation,3rd\ party} = \frac{V_{CH,FitToCirc}}{V_{CH,Sort}}$.

Abbreviations

CDF	Cumulative density function
CH	Cash Handler
CIT	Cash In Transit
DNB	De Nederlandsche Bank
ECB	European Central Bank
ECT	External Circulation Trial
MFI	Monetary Financial Institution
NCB	National Central Bank
QS	Quality Survey
SD	Standard Deviation

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