What Can Time-Series Regressions Tell Us About Policy Counterfactuals?

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Policy evaluation through time series regressions

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  a) **Identification result**: give conditions s.t. impulse responses to *multiple distinct* policy shocks allow us to construct Lucas critique-robust rule counterfactuals
  b) **Empirical method**: estimate IRFs to multiple policy shocks, then combine them to approximate the desired counterfactual rule
  Application: **Romer-Romer** + **Gertler-Karadi** to predict counterfactual propagation of **inv. shock**.

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- Many linearized models have these features [RBC, NK-DSGE, HANK, ...]
- Sufficient statistic argument: **method applies to a class of models**
  - Do not need to take a stand on deep structural features of the economy

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(ii) **Linearity**

- Essential in practice though not conceptually necessary
- Restrictions on counterfactuals: don't deviate too far from **baseline** dynamics
  E.g.: can study alternative interest rate rules, but not large changes in π^*

Visual illustration

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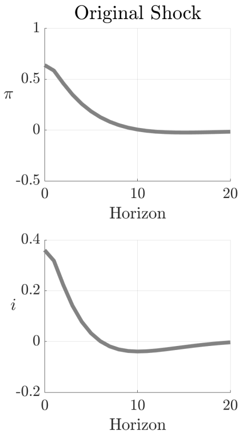
Q: How would this **cost-push shock** have propagated in the absence of a monetary reaction?



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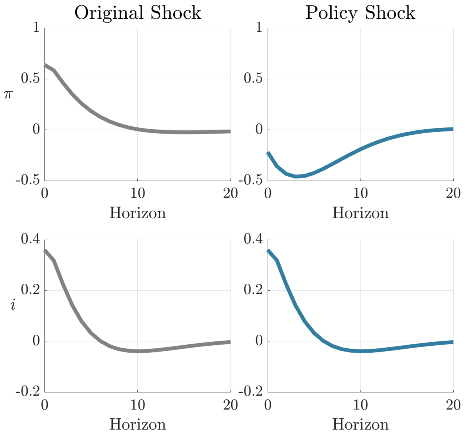

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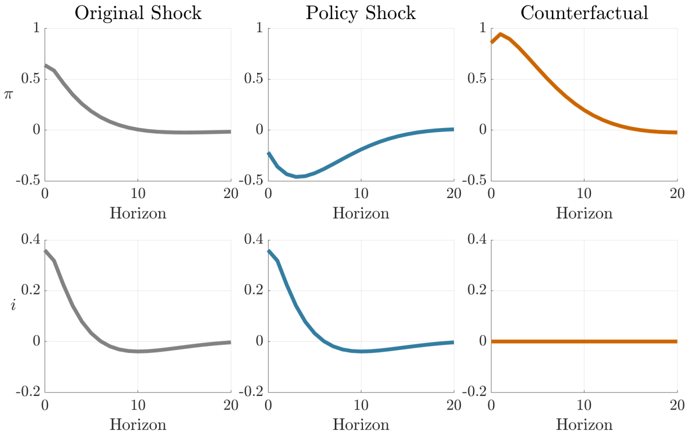

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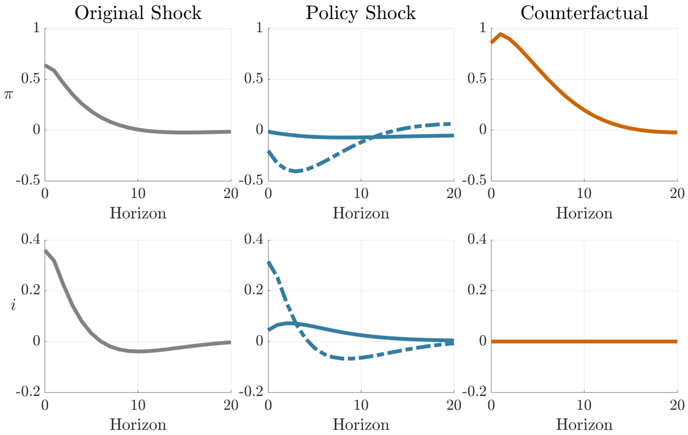

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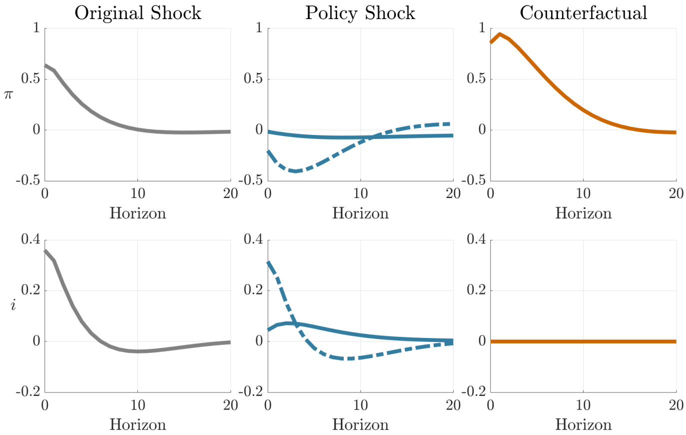

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- Emp. method: enforce **cnfct'l rule** *as well as possible* using linear combo of policy shocks

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$\{\boldsymbol{\pi}(\boldsymbol{\varepsilon}), \boldsymbol{i}(\boldsymbol{\varepsilon})\}$	IRFs of endogenous variables, $\boldsymbol{\pi}$, and policy instruments, $\boldsymbol{i}$, to $\boldsymbol{\varepsilon}$
$\boldsymbol{\nu}$	Vector of policy shocks = deviations in policy at different horizons
$\Theta_{\boldsymbol{\pi},\boldsymbol{\nu}}, \Theta_{\boldsymbol{i},\boldsymbol{\nu}}$	Matrices of IRFs mapping policy shocks to $\boldsymbol{\pi}$ & $\boldsymbol{i}$

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- Counterfactual rule expressed as restrictions on IRFs

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- Object of interest

$\{\tilde{\boldsymbol{\pi}}(\boldsymbol{\varepsilon}), \tilde{\boldsymbol{i}}(\boldsymbol{\varepsilon})\}$	IRFs of $\boldsymbol{\pi}$ and $\boldsymbol{i}$ under counterfactual rule

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For any $\{\mathcal{A}_\pi, \mathcal{A}_i\}$ *that induces a unique eq'm, we can recover the counterfactuals* $\tilde{\boldsymbol{\pi}}(\boldsymbol{\varepsilon})$ *and* $\tilde{\boldsymbol{i}}(\boldsymbol{\varepsilon})$ *as the impulse responses under the baseline rule to* $\{\boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$*, where* $\boldsymbol{\nu}$ *solves*

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- **Contrast with Sims-Zha (1995)**: date $t = 0$ shocks not ex post shocks

Counterfactuals with "a few" shocks

In practice you observe just a few **policy shocks**, giving the columns of the (small) IRF matrices $\{\Theta_{\pi,\nu}, \Theta_{i,\nu}\}$. What can you do with those?

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  - Solve problem:
  $$\min_{\nu} \underbrace{\left|\left|\mathcal{A}_{\pi}\left(\boldsymbol{\pi}(\boldsymbol{\varepsilon}) + \Theta_{\pi,\nu} \times \boldsymbol{\nu}\right) + \mathcal{A}_{i}\left(\boldsymbol{i}(\boldsymbol{\varepsilon}) + \Theta_{i,\nu} \times \boldsymbol{\nu}\right)\right|\right|}_{\text{rule inaccuracy}}$$
  This selects linear combo of shocks to implement rule *as well as possible*
  - Will show through applications: existing evidence is enough to enforce at least some counterfactuals with a high degree of accuracy

Applications

Systematic Monetary Policy Rule Counterfactuals

A review of empirical evidence

What can we get from the **empirical monetary policy shock literature**?

- Key point: policy is **multi-dimensional** and so are IVs for policy shocks
  - For our main applications we will use two canonical monetary shock series:
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- Aside: similar range of shock series in the **fiscal policy literature**
  Ramey, Ramey-Zubairy, Mertens-Ravn

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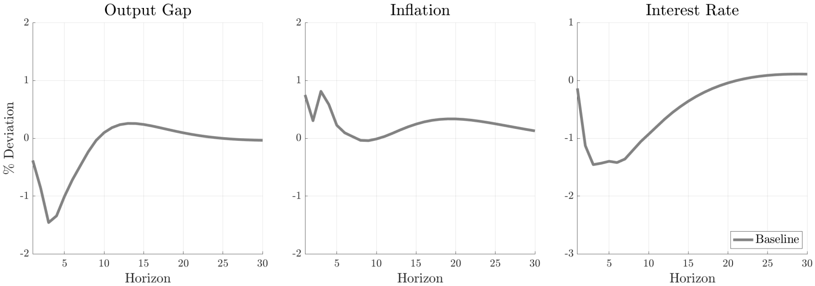

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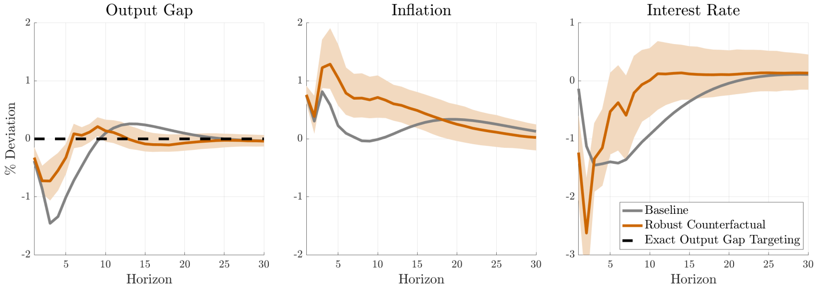

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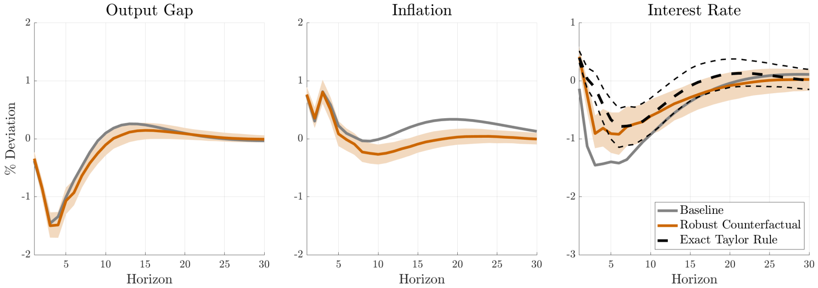

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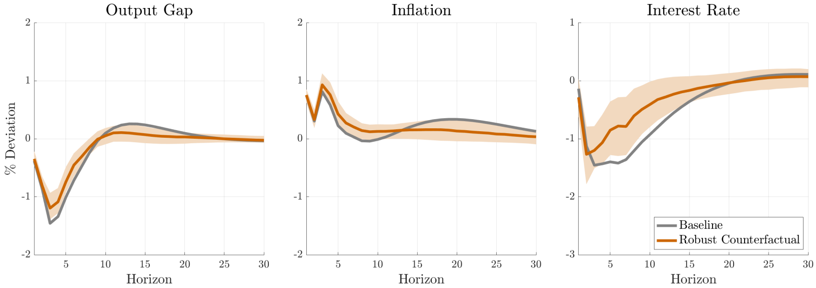
- **Results**: **optimal dual mandate policy** ▸ Other Applications & Robustness



Conclusions

Takeaways

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- **Why we think this matters:**
  1. **Method:** construct counterfactuals for systematic policy changes with weaker structural assumptions without violating Lucas critique
  2. Theory ahead of measurement? Causal effects of more **policy paths** would be valuable
     Future emp. work: should focus more on the instrument paths that correspond to a given shock.

Appendix

Model examples

1. **HANK model**
   - Generalized IS curve

$$\boldsymbol{c} = \mathcal{C}(\boldsymbol{y}, \boldsymbol{\pi}, \boldsymbol{i}, \boldsymbol{\varepsilon}^d) = \mathcal{C}_y \boldsymbol{y} + \mathcal{C}_\pi \boldsymbol{\pi} + \mathcal{C}_i \boldsymbol{i} + \boldsymbol{\varepsilon}^d$$

2. **Behavioral models**
   - Various behavioral frictions correspond to simple adjustments of the matrices in $\mathcal{H}$
   - Example: sticky information in consumption decisions

$$\tilde{\mathcal{C}}_p(t,s) = \sum_{q=1}^{\min(t,s)} [\mathcal{E}(q,s) - \mathcal{E}(q-1,s)] \mathcal{C}_p(t-q+1, s-q+1)$$

where

$$\mathcal{E} = \begin{pmatrix} 1 & 1-\theta & 1-\theta & \dots \\ 1 & 1 & 1-\theta^2 & \dots \\ 1 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

back

Identification result: proof

- We claim that we can find the right counterfactual as the solution of

$$\begin{pmatrix} I & 0 & -\Theta_{x,\nu} \\ 0 & I & -\Theta_{z,\nu} \\ \tilde{\mathcal{A}}_x & \tilde{\mathcal{A}}_z & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{z} \\ \boldsymbol{\nu} \end{pmatrix} = \begin{pmatrix} \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) \\ \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) \\ 0 \end{pmatrix}. \tag{1}$$

- The equilibrium system under the new policy rule can be written as

$$\begin{pmatrix} \mathcal{H}_w & \mathcal{H}_x & \mathcal{H}_z \\ 0 & \tilde{\mathcal{A}}_x & \tilde{\mathcal{A}}_z \end{pmatrix} \begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -\mathcal{H}_\varepsilon \\ 0 \end{pmatrix} \boldsymbol{\varepsilon} \tag{2}$$

  This system by assumption has a unique solution $\{\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$.

- Since (**??**) also holds under the initial policy rule, and since the last line of (1) imposes the new policy rule, any $(\boldsymbol{x}, \boldsymbol{z})$ that are part of a solution of (1) are also part of a solution of (2). Thus (1) has at most one solution.

back

Identification result: proof

- Remains to show that (1) has a solution. Consider the candidate

$$\{\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{\nu} = (\tilde{\mathcal{A}}_x - \mathcal{A}_x)\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) + (\tilde{\mathcal{A}}_z - \mathcal{A}_z)\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$$

- Since $\{\boldsymbol{w}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$ solve (2), they also solve

$$\begin{pmatrix} \mathcal{H}_w & \mathcal{H}_x & \mathcal{H}_z \\ \mathbf{0} & \mathcal{A}_x & \mathcal{A}_z \end{pmatrix} \begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = - \begin{pmatrix} \mathcal{H}_\varepsilon \boldsymbol{\varepsilon} \\ (\tilde{\mathcal{A}}_x - \mathcal{A}_x)\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) + (\tilde{\mathcal{A}}_z - \mathcal{A}_z)\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) \end{pmatrix} \quad (3)$$

- (3) has a unique solution, so $\{\boldsymbol{w}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$ is that solution
- Finally, by definition of $\Theta_{\mathcal{A}}$, this tuple also solves (1) □

back

Identification results: second-moment properties

- Under invertibility, our informational requirements also suffice to recover counterfactual second-moment properties:

Proposition

Suppose that the VMA process for observables $\{x_t, z_t\}$ *under the baseline rule is invertible with respect to the shocks* $\{\varepsilon_t, \nu_t\}$*.*

Then, for any $\{\tilde{\boldsymbol{A}}_{\boldsymbol{x}}, \tilde{\boldsymbol{A}}_{\boldsymbol{z}}\}$ *that induces a unique eq'm, we can recover the second-moment properties of* $\{x_t, z_t\}$ *under the counterfactual rule.*

- **Proof sketch**
  - Basic idea: apply result for shock path $\boldsymbol{\varepsilon}$ to the Wold errors
  - Can show: under invertibility this gives the same result as directly mapping the true structural shocks to their counterfactual propagation

back

Identification results: optimal policy rules

- The same logic identifies **optimal policy rules** given a (quadratic) loss function:

  To be clear: still need theory to learn about mapping from aggregates to welfare (= loss function).

Proposition

Consider a policymaker with loss function $\mathcal{L} = \sum_{i=1}^{n_x} \lambda_i \boldsymbol{x}_i' W \boldsymbol{x}_i$. The optimal policy rule $\boldsymbol{\mathcal{A}}_{\boldsymbol{x}}^*$ for such a policymaker is given as

$$\sum_{i=1}^{n_x} \underbrace{\boldsymbol{\Theta}_{\boldsymbol{x_i},\boldsymbol{\nu}}' \lambda_i W}_{\boldsymbol{\mathcal{A}}_{\boldsymbol{x_i}}^*} \boldsymbol{x}_i = 0 \tag{4}$$

Given this optimal rule, we can use (3) to recover counterfactuals for the shock path $\boldsymbol{\varepsilon}$, denoted $\boldsymbol{x}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$ and $\boldsymbol{z}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$.

Simple intuition: shocks are enough to figure out what paths of x we can implement via z.

▸ back

Identification results: optimal policy rules

- Solution to true optimal policy problem is characterized by FOCs

$$\mathcal{H}_w'(I \otimes W)\boldsymbol{\varphi} = \mathbf{0} \tag{5}$$

$$(\Lambda \otimes W)\boldsymbol{x} + \mathcal{H}_x'(I \otimes W)\boldsymbol{\varphi} = \mathbf{0} \tag{6}$$

$$\mathcal{H}_z' W\boldsymbol{\varphi} = \mathbf{0} \tag{7}$$

Denote the (unique) solution by $\{\boldsymbol{x}^*(\boldsymbol{\varepsilon}), \boldsymbol{z}^*(\boldsymbol{\varepsilon}), \boldsymbol{\varphi}^*(\boldsymbol{\varepsilon})\}$.

- The problem of choosing the best "errors" $\boldsymbol{\nu}$ gives

$$\mathcal{H}_w'(I \otimes W)\boldsymbol{\varphi} = \mathbf{0} \tag{8}$$

$$(\Lambda \otimes W)\boldsymbol{x} + \mathcal{H}_x'(I \otimes W)\boldsymbol{\varphi} + \mathcal{A}_x' W\boldsymbol{\varphi}_z = \mathbf{0} \tag{9}$$

$$\mathcal{H}_z'(I \otimes W)\boldsymbol{\varphi} + \mathcal{A}_z' W\boldsymbol{\varphi}_z = \mathbf{0} \tag{10}$$

$$W\boldsymbol{\varphi}_z = \mathbf{0} \tag{11}$$

Thus $\boldsymbol{\varphi}_z = \mathbf{0}$, so the equivalence follows.

back

Identification results: optimal policy rules

- The constraint set of the ν-problem can be written as

$$\begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = \Theta_{\mathcal{A}} \times \begin{pmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\nu} \end{pmatrix} \tag{12}$$

- The FOCs then become

$$\sum_{i=1}^{n_x} \lambda_i \Theta'_{x_i,\nu} W \boldsymbol{x}_i = 0 \tag{13}$$

back

Global identification argument

- **Setting**
  - Consider a T-period economy with stochastic event ω_t each period, with histories denoted by $\omega^t \equiv \{\omega_0, \omega_1, \cdots, \omega_t\}$. Let boldface denote vectors over dates and states.
  - Write the equilibrium system as

$$\mathcal{H}(\boldsymbol{x}, \boldsymbol{z}) = \boldsymbol{0} \quad (14)$$

$$\mathcal{A}(\boldsymbol{x}, \boldsymbol{z}) + \boldsymbol{\nu} = \boldsymbol{0} \quad (15)$$

  with solution $\boldsymbol{x} = x(\boldsymbol{\nu})$, $\boldsymbol{z} = z(\boldsymbol{\nu})$

- **Identification result**: counterfactuals for alternative rule $\tilde{\mathcal{A}}(\boldsymbol{x}, \boldsymbol{z}) = \boldsymbol{0}$
  - Construct counterfactual as $x(\tilde{\boldsymbol{\nu}}) = \tilde{\boldsymbol{x}}$, $z(\tilde{\boldsymbol{\nu}}) = \tilde{\boldsymbol{z}}$ where $\tilde{\boldsymbol{\nu}}$ solves

$$\tilde{\mathcal{A}}(x(\tilde{\boldsymbol{\nu}}), z(\tilde{\boldsymbol{\nu}})) = \boldsymbol{0} \quad (16)$$

  - Can show: solution $\tilde{\boldsymbol{\nu}}$ to this system exists and indeed generates $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{z}})$

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Rule dependence of expansion point

- Key assumption for us: private-sector block $\{\mathcal{H}_x, \mathcal{H}_z, \mathcal{H}_\varepsilon\}$ does not depend on policy rule $\{\mathcal{A}_x, \mathcal{A}_z\}$
- Lucas island model can illustrate two ways in which this can fail:
  1. Change in long-run avg. inflation ($=$ inflation target)
  2. Rule coefficients affect solution to filtering problem

back

Lucas island model

- The policy rule is

$$x_t = \delta + \phi_y y_t + x_{t-1} + \boldsymbol{\varepsilon}^m$$

  We want to predict the effects of changes in:

  1. δ: mean nominal demand growth = avg. inflation
  2. ϕ_y: policy feedback coefficients

  Is evidence on the propagation of $\boldsymbol{\varepsilon}^m$ enough?

- The price level and nominal/real output are related via

$$y_t = x_t - p_t$$

- The Lucas supply curve is

$$y_t = \theta(p_t - \bar{p}_t)$$

  where $\bar{p}_t = \mathbb{E}_{t-1}(p_t)$, $\theta = \frac{\tau^2}{\tau^2+\sigma_p^2}$, τ is exogenous and σ_p is the volatility of p_t

back

Lucas island model

- The model is closed with an equation for $\bar{p}_t$. Guess that

$$p_t = \alpha_0 + \alpha_1 x_t + \alpha_2 x_{t-1}$$

We can confirm this guess with $\alpha_0 = \frac{\theta\delta}{1+\theta}$, $\alpha_1 = \frac{1}{1+\theta}$, $\theta_2 = \frac{\theta}{1+\theta}$.

- Plugging this in we get the last equation as

$$\bar{p}_t = \delta + x_{t-1}$$

- Finally, solving for the price variance, we get

$$\sigma_p^2 = \left(\frac{1}{1+\theta}\right)^2 \text{Var}(\phi_y y_t + \boldsymbol{\varepsilon}^m)$$

and

$$y_t = \frac{1}{1 - \frac{\theta}{1+\theta}\phi_y} \frac{\theta}{1+\theta} \boldsymbol{\varepsilon}^m$$

back

Lucas island model

- The Lucas island model has revealed two ways in which the coefficients of the non-policy block can depend on the policy rule:
  1. The average growth rate of nominal demand shows up directly in the equation for future prices (δ). This has changed the steady state.
  2. The policy rule coefficient ϕ_y affects the volatility of prices, thus the solution to the household filtering problem, and so the coefficient θ
- Thus in both cases our key separation assumption is violated

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Alternative empirical strategies

1. **Refinement** of Sims-Zha (1996): enforce rule with strictly smaller ex-post surprises
   - Diagnostic: Sims-Zha counterfactual is less credible if it relies on large *ex-post* surprises (dated $t > 0$)
   - With multiple shocks: can minimize the norm of date-$t > 0$ shocks subject to the counterfactual rule holding perfectly
2. **Hybrid**: trade off rule inaccuracy and ex-post surprises
   - Let $\{\Omega_{x,\mathcal{A}}^{(h)}, \Omega_{z,\mathcal{A}}^{(h)}\}$ denote impulse responses to policy shocks that materialize at horizon h
   - Then solve simple ridge regression problem:

$$\min_{\{\boldsymbol{s}^h\}_{h=0}^{H}} ||\tilde{\mathcal{A}}_x(\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \sum_{h=0}^{H} \Omega_{x,\mathcal{A}}^{(h)} \times \boldsymbol{s}^h) + \tilde{\mathcal{A}}_z(\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \sum_{h=0}^{H} \Omega_{z,\mathcal{A}}^{(h)} \times \boldsymbol{s}^h)|| + \psi \sum_{h=1}^{H} ||\boldsymbol{s}^h|| \tag{17}$$

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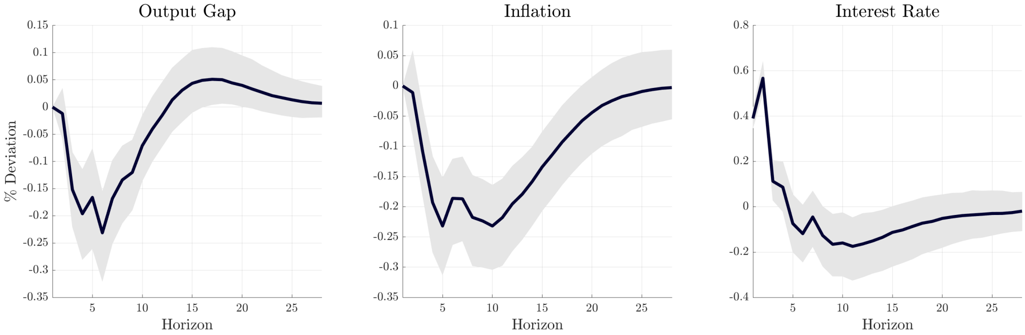
Policy shock causal effects



Romer-Romer

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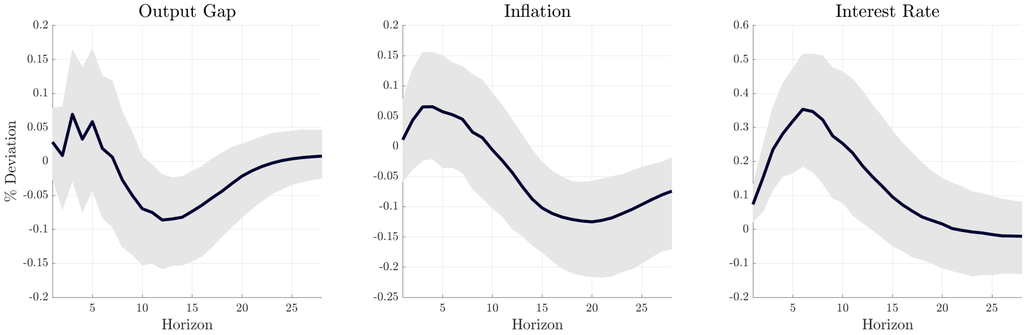
Policy shock causal effects



Gertler-Karadi

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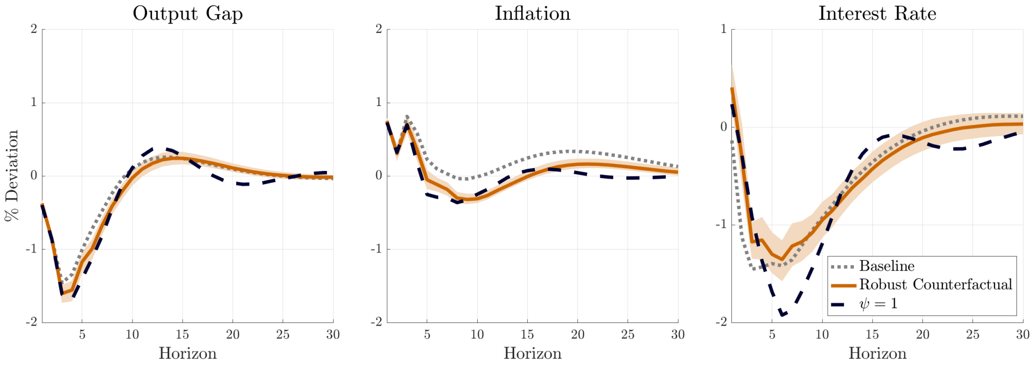
Further applications



Nominal GDP targeting

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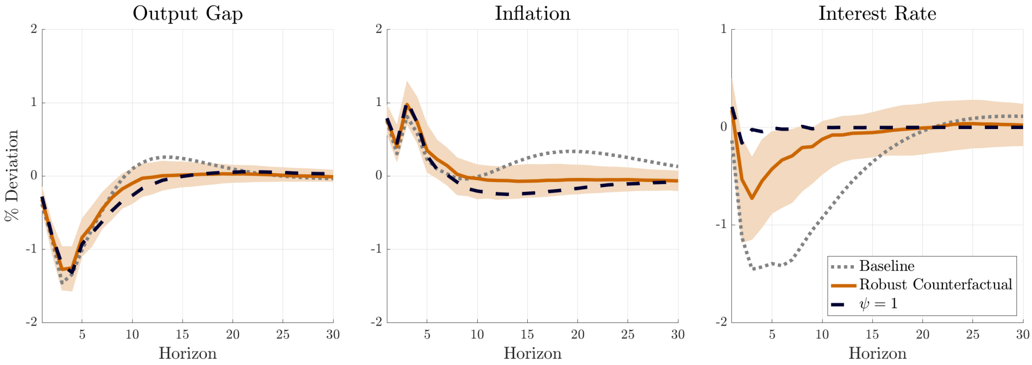
Further applications



Nominal rate peg

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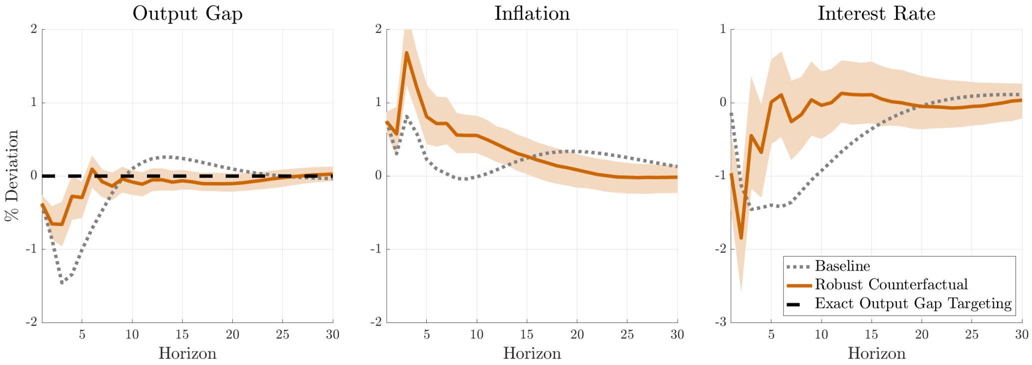
Robustness



Output gap targeting, alternative MP shocks

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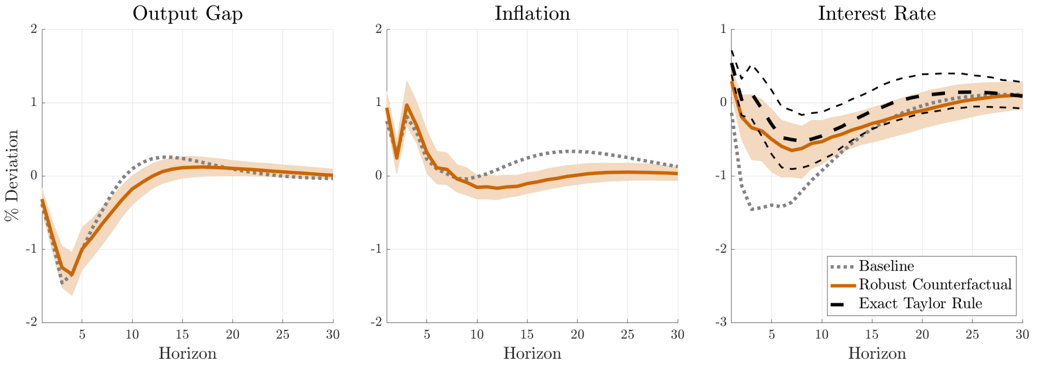
Robustness



Taylor rule, alternative MP shocks

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