Special Repo Rates and the Cross-Section of Bond Prices

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[1]The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

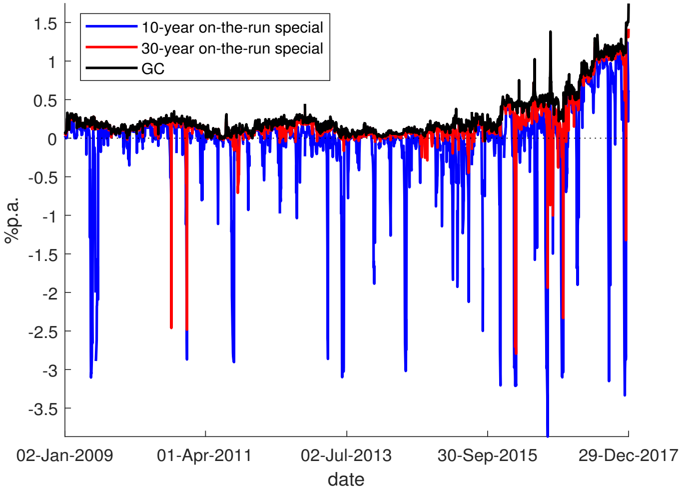
Motivation: Treasury Market Anomalies

- An "anomaly" is when two assets or trades have virtually the same cash-flows but very different prices.
- Example: the "on-the-run" premium
  - On-the-run Treasury bond is the most recently issued bond of that maturity.
  - On-the-run Treasuries trade at higher prices (lower yields) than older Treasuries of equal duration.
  - Most of the term-structure literature throws these bonds out (Gürkaynak Sack & Wright 2007).
- Other examples:
  - TIPS-Treasury bond puzzle
  - TIPS relative illiquidity
  - Negative interest rate swap spread

What We Do

- Estimate a DTSM directly on observed Treasury bond prices
  - i.e., not the estimated zero-coupon yields from a different model (such as Gürkaynak Sack & Wright 2007)
- Two advantages of using observed rather than estimated bond prices:
  - Link cash prices to overnight repo rates in the bilateral "special" repo market.
  - Using the full cross-section allows us to identify latent pricing factors without estimating the time-series parameters of the model.
- Special repo rates capture the value of each bond as collateral, beyond just its coupon and principal payments.
- Most important: collateral value is risky and includes a **risk premium**.

On-the-Run Special Rates



other maturities   first off-the-run

What We Find

- We account for the collateral value of Treasuries within a DTSM, effectively linking the pricing in the cash and repo markets.
- Analyze the extent to which cash price differences in the cross-section are consistent with observed special collateral (SC) rates.
- Derive a time-varying risk premium associated with the SC rate and verify its ability to explain price anomalies across Treasury markets.
- At the 10-year maturity, it explains 74%–90% of the on-the-run premium, about 68% of the TIPS-Treasury Bond puzzle, and about 58% of TIPS relative illiquidity.

Summary Statistics 1/2/2009—12/29/2017

Maturity at Issuance	Avg # Bonds	% On Special (off-the-run)	Avg Spread (off-the-run)	Avg Spread (on-the-run)
All	220	84.7	4.79 (6.16)	19.7 (41.6)
2	11.2	87.3	5.37 (8.8)	20.5 (38.7)
3	21.7	88.9	5.72 (7.6)	21.1 (36)
5	46.9	84.5	3.84 (6.09)	24.5 (41.7)
7	47.8	86.4	4.85 (5.47)	6.18 (11)
10	35.2	83.2	3.44 (3.69)	35.4 (66.5)
30	58.6	82.4	5.86 (6.43)	8.97 (23.2)

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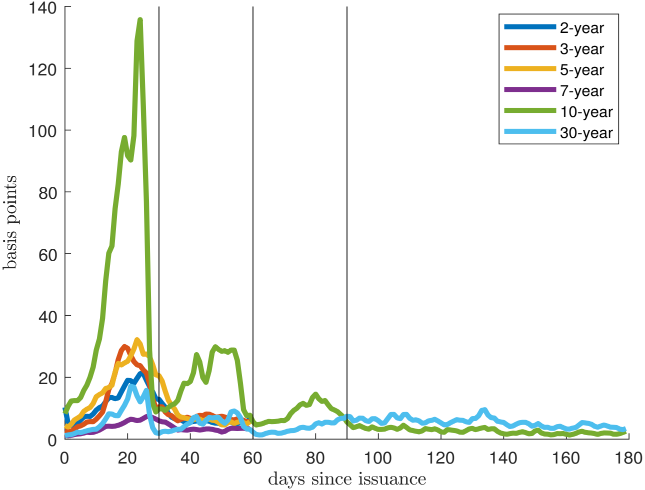
Special Spreads over the Auction Cycle



variation

Special Spreads are a Dividend Proportional to Price (1)

- GC rate is R_t, special rate is $r_t < R_t$.
- At t I borrow P_t against my special collateral at rate r_t, and lend an amount Δ at the GC rate R_t.
- At $t+1$ earnings are $(1+R_t)\,\Delta - (1+r_t)\,P_t$.
- Choose Δ such that no gain or loss at $t+1$: $\Delta = \frac{1+r_t}{1+R_t} P_t < P_t$.

Special Spreads are a Dividend Proportional to Price (2)

- Gain at t of $\left(1 - \frac{1+r_t}{1+R_t}\right) P_t$.
- Define

$$y_t \equiv \log \frac{1 + R_t}{1 + r_t} \geq 0$$

- Then the price of a zero-coupon bond on special with special spread y_t is

$$\begin{aligned} P_t &= \left(1 - e^{-y_t}\right) P_t + E_t^* P_{t+1} \\ &= e^{y_t} E_t^* P_{t+1} \end{aligned}$$

- Model: link dynamics of y_t to other factors that influence Treasury bond prices.

Repo Specials and Price Residuals, GSW Model

Maturity at Issuance	Avg Spread (on-the-run)	Avg Price Res (% of par)
All	19.7 (41.6)	0.185 (0.592)
2	20.5 (38.7)	0.00738 (0.0512)
3	21.1 (36)	0.0516 (0.1)
5	24.5 (41.7)	0.109 (0.126)
7	6.18 (11)	0.0469 (0.173)
10	35.4 (66.5)	0.676 (0.903)
30	8.97 (23.2)	0.217 (0.953)

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Prices and Special Spreads: Reduced-Form Evidence

$$\eta_{i,t} = \alpha_i + \beta_1 y_t^i + \beta_2 \eta_{i,t-1} + \xi_{i,t}$$

y_t^i	0.00280***	0.000340***	0.00231***	0.000382***
	(5.121)	(5.704)	(4.970)	(5.326)
$\eta_{i,t-1}$		0.902***		0.866***
		(105.7)		(69.47)
R^2	0.007	0.822	0.006	0.759
Observations	496,420	495,792	496,420	495,792
CUSIP FE	NO	NO	YES	YES
# of CUSIP	628	628	628	628

t-statistics in parentheses, clustered at CUSIP level

*** p<0.01, ** p<0.05, * p<0.1

Model: Special Spreads

- Non-negative special spread on bond i at time t:

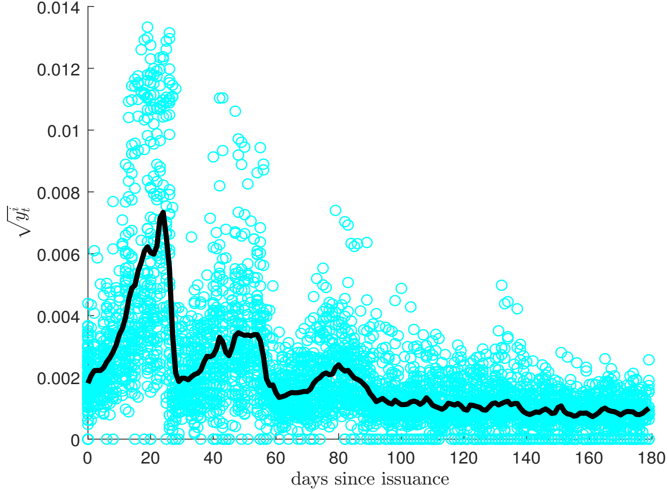
$$y_t^i = \left[\underbrace{y^D_{\tau(t,i)}}_{\text{deterministic component}} + \underbrace{y_t^S + x_t^i}_{\text{stochastic component}} \right]^2$$

where

- $y^D_{\tau(t,i)}$ is the deterministic component of the square root of the special spread that depends on time since issuance τ
- y_t^S is a stochastic aggregate repo factor
- x_t^i is a stochastic bond-level residual that follows

$$x_{t+1}^i = \rho x_t^i + \sigma_x \varepsilon_{t+1}^i$$

Special Spreads over the Auction Cycle (10-Year)



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Observed Repo Factor

- Repo factor: average deviation from auction cycle:

$$y_t^S = \frac{1}{n_t} \sum_{i=1}^{n_t} \left[\sqrt{y_t^i} - y_{\tau(t,i)}^D \right]$$

  where we include on-the-run and first off-the-run bonds of varying maturities.

- Idiosyncratic factor is the residual:

$$x_t^i = \sqrt{y_t^i} - y_t^S - y_{\tau(t,i)}^D$$

- x_t^i and $y_{\tau(t,i)}^D$ appended to the state vector X_t, bond by bond.

Model: Dynamics and Risk Prices

- Aggregate state is a VAR(1): state variables

$$X_{t+1} = \mu + \Phi X_t + \Sigma \varepsilon_{t+1}$$

where $\varepsilon_{t+1} \sim \mathbb{N}(0, I)$.

- Short rate of interest is affine in the state:

$$\log(1 + R_t) = \delta_0 + \delta_1' X_t$$

- Stochastic discount factor exposed to ε_{t+1} through $\lambda_t \equiv \lambda + \Lambda X_t$:

$$\log \frac{M_{t+1}}{M_t} = -\delta_0 - \delta_1' X_t - \frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1}$$

- Special spread is quadratic in the state:

$$y_t^i = X_t' \Gamma_i X_t$$

Bond Prices

- Result: a zero-coupon bond with n days to maturity has log price given by

$$\begin{aligned}\log P_t^{(n)} &= X_t' \Gamma X_t + \log E_t \left\{ \frac{M_{t+1}}{M_t} P_{t+1}^{(n-1)} \right\} \\ &= A_n + B_n' X_t + X_t' C_n X_t\end{aligned}$$

- With loadings given by $A_0 = 0$, $B_0 = \vec{0}_{k \times 1}$, $C_0 = \vec{0}_{k \times k}$, and loadings given by loadings

Measurement

- All bonds in the sample pay coupons, making them portfolios of zero-coupon bonds:

$$P_t^i = \sum_j c_j \exp\left\{A_{m_j} + B'_{m_j} X_t + X'_t C_{m_j} X_t\right\}$$
$$\equiv P^Z\left\{\vec{c}\,(i)\,, \vec{m}\,(i)\,, X_t\right\}$$

- Stacking all bonds for a given t gives the measurement equation

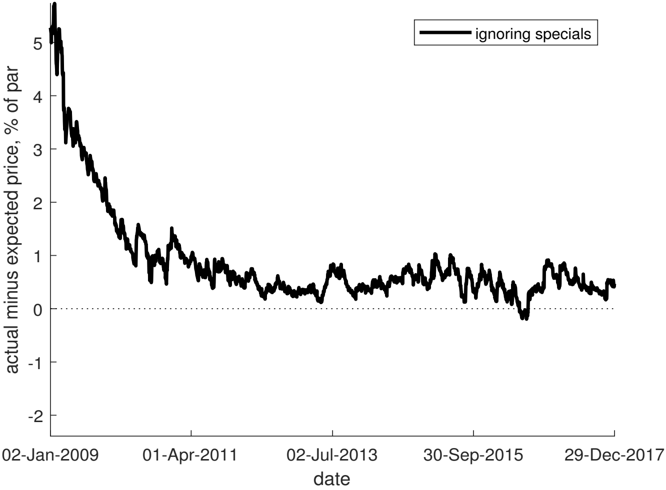
$$\vec{P}_t = \begin{bmatrix} P^Z\left\{\vec{c}\,(1)\,, \vec{m}\,(1)\,, X_t\right\} \\ P^Z\left\{\vec{c}\,(2)\,, \vec{m}\,(2)\,, X_t\right\} \\ \dots \\ P^Z\left\{\vec{c}\,(n_t)\,, \vec{m}\,(n_t)\,, X_t\right\} \end{bmatrix} + \vec{\eta}_t$$

- n_t is typically greater than 100, so we can estimate the X_t for each t using NLLS without using a nonlinear Kalman filter.

Estimation Steps

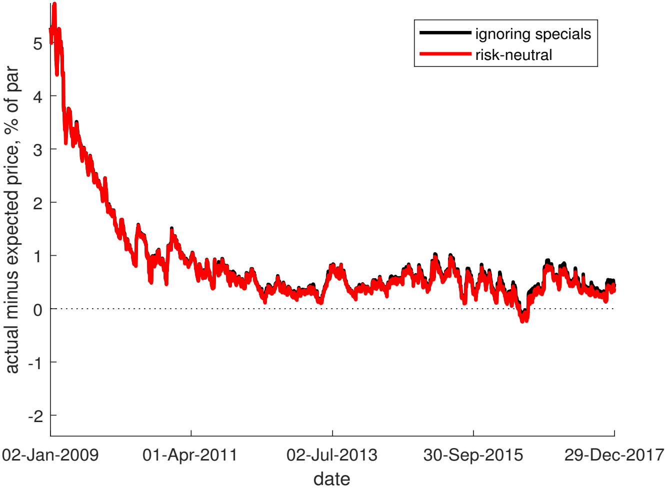
- Split sample into 10-year on-the-run and first off-the-run ("special") bonds, and all others ("normal" bonds).
- Estimate a 3-factor model on "normal" bonds ignoring special spreads.
  - 10 parameters
  - $\rightarrow X_t$ at each date t.
- 4-factor models
  1. Price on-the-runs and first off-the-run, ignoring their special spreads.
  2. Price their special spreads risk-neutral (no risk premia earned on repo factor, repo factor doesn't affect other risk premia)
  3. Allow a constant repo risk premium (one additional parameter)
  4. Allow a time-varying risk premium (four additional parameters)

Price Residuals, Special Bonds (ignoring repos)



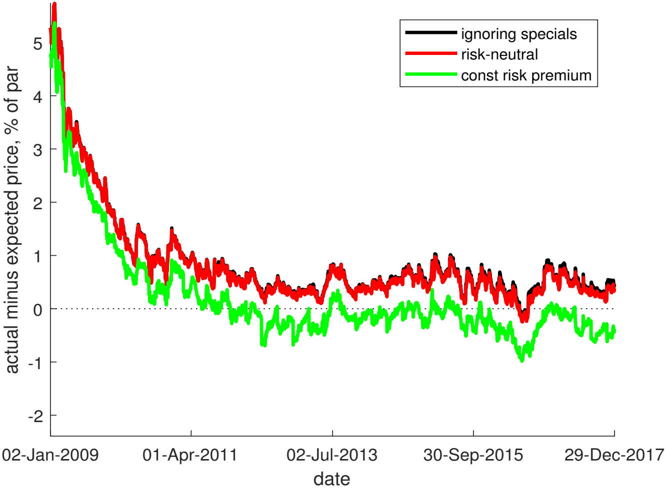
sum of squared residuals: 0.379

Price Residuals, Special Bonds (risk-neutral)



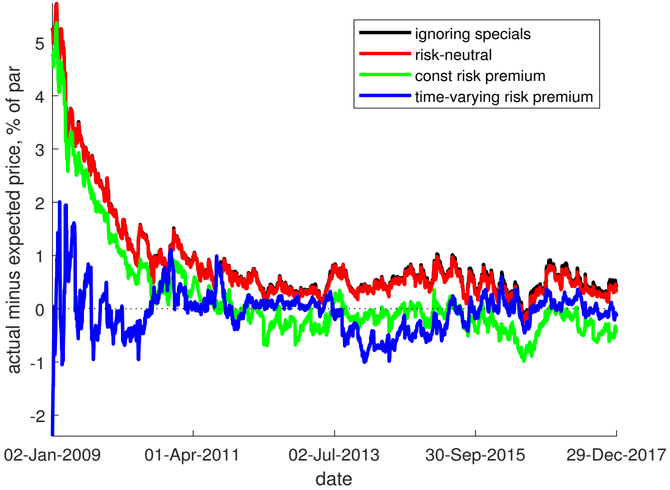
sum of squared residuals: 0.37. $R^2 = 0.024$.

Price Residuals, Special Bonds (constant risk premium)



sum of squared residuals: 0.238. $R^2 = 0.374$.

Price Residuals, Special Bonds (time-varying)



sum of squared residuals: 0.042. $R^2 = 0.889$.

Repo Risk Premium

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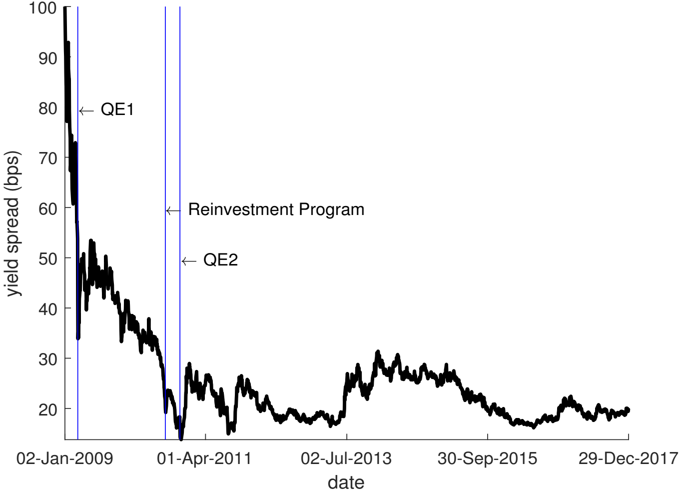

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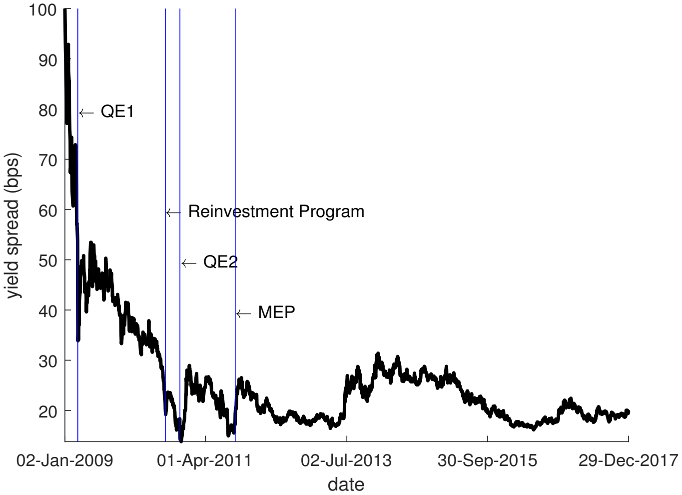

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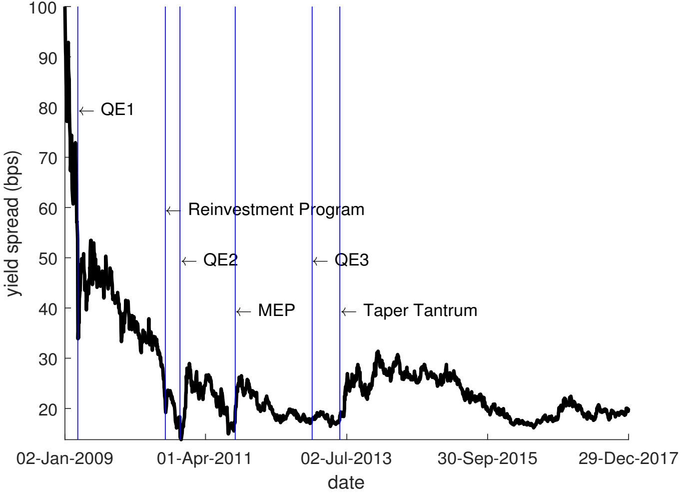

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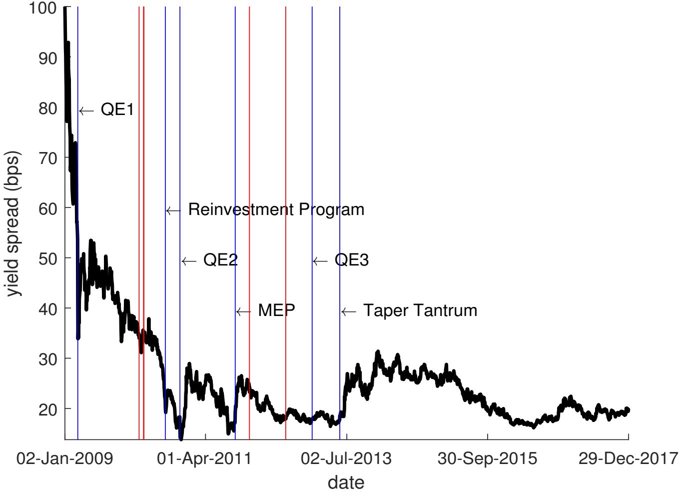

Other Anomalies

- SC risk premium correlates with other Treasury anomalies.
- Often, anomalies are defined using the 10-year on-the-run note.
- Some work “controls” for repo specials but implicitly assumes risk-neutral pricing.

Anomaly	10-Year Only		10-Year (pooled)	
	Corr	R^2	Corr	R^2
GSW 10-Year On-the-Run	0.91	0.83	0.9	0.81
Off Note-Bond Spread	0.79	0.62	0.73	0.53
TIPS-Treasury Puzzle	0.82	0.68	0.78	0.61
TIPS Liq Premium	0.76	0.58	0.78	0.62
Nominal Fitting Errors	0.6	0.36	0.52	0.27

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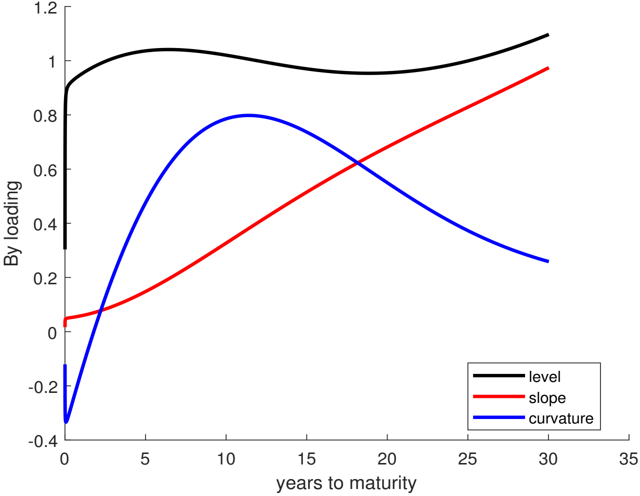
Anomaly	10-Year Only		10-Year (split)	
	Corr	R^2	Corr	R^2
GSW 10-Year On-the-Run	0.91	0.83	0.75	0.56
Off Note-Bond Spread	0.79	0.62	0.68	0.46
TIPS-Treasury Puzzle	0.82	0.68	0.71	0.5
TIPS Liq Premium	0.76	0.58	0.63	0.4
Nominal Fitting Errors	0.6	0.36	0.6	0.36

Conclusion and Further Work

- Cash prices for highly-valued 10-year Treasury securities are consistent with their SC repo rates only after incorporating a time-varying risk premium on the repo factor.
- This repo risk premium can explain a significant portion of price anomalies across Treasury cash, repo, and derivatives markets.
- This suggests a common underlying economic mechanism for these anomalies, linked to the collateral value of high-quality securities.
- Future work:
  - Explore special spreads on seasoned bonds and delivery fails.
  - Jointly price Treasury futures along with spot prices and repo rates.
  - Link this factor to other price anomalies.

Estimated Yield Loadings $-\frac{1}{n}B_n$



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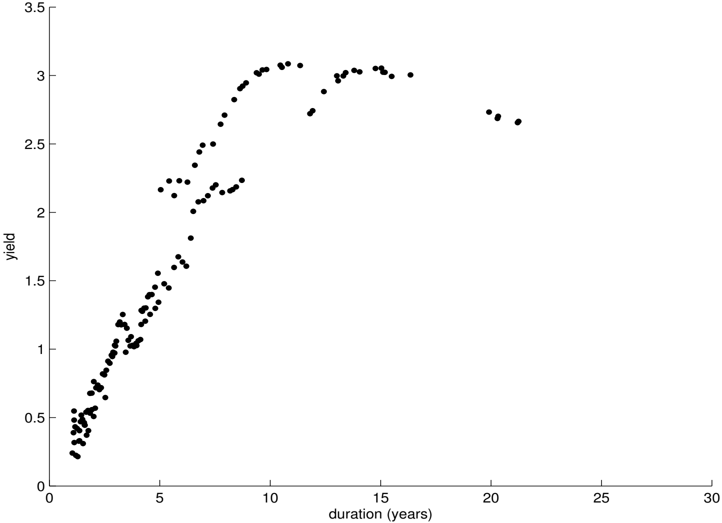
Yield Curve in Late December 2008



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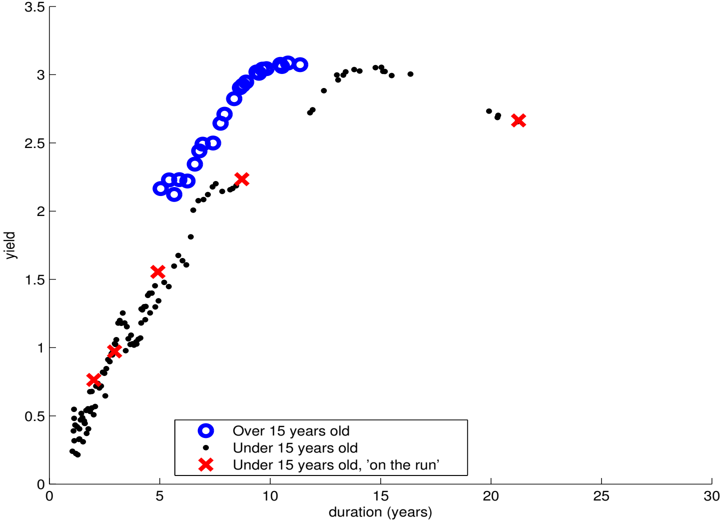

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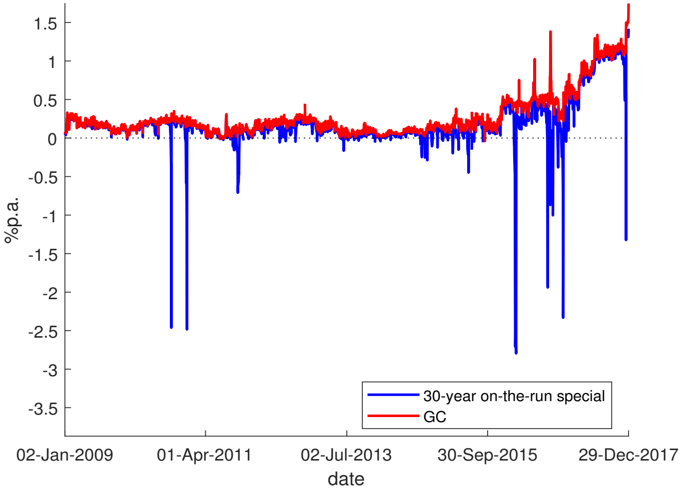
30-Year On-the-Run Special Rate



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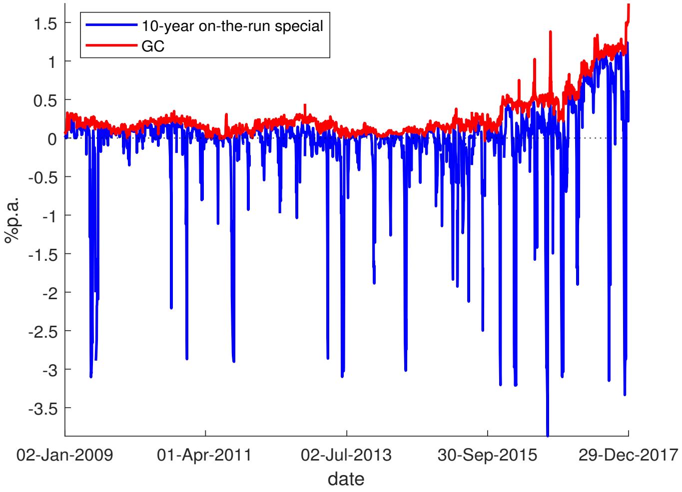
10-Year On-the-Run Special Rate



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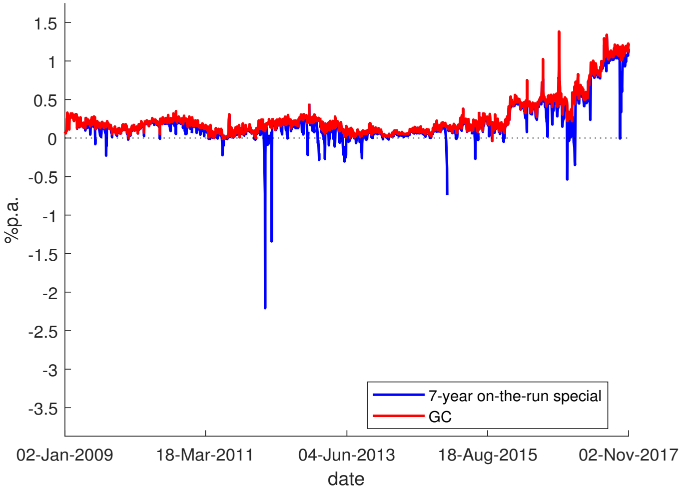
7-Year On-the-Run Special Rate



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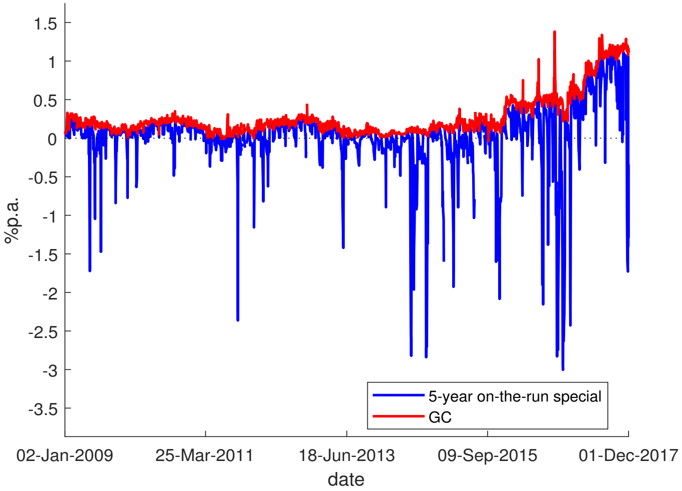
5-Year On-the-Run Special Rate



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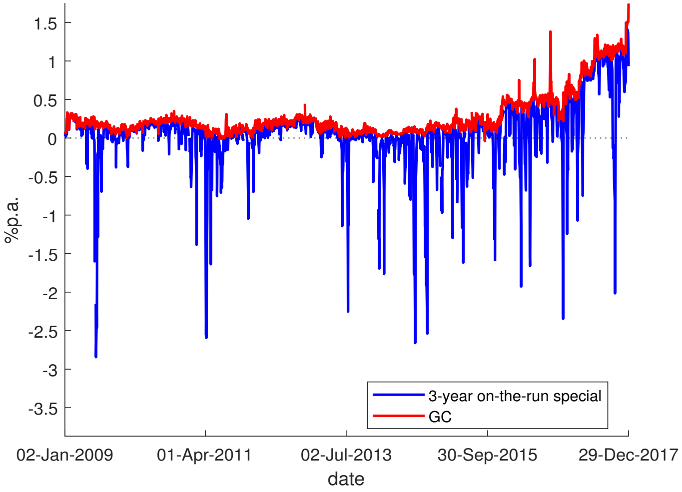
3-Year On-the-Run Special Rate



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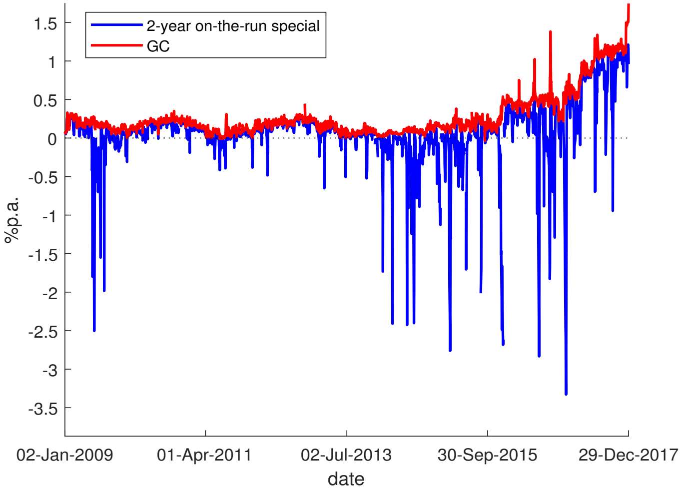
2-Year On-the-Run Special Rate



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Model (Bond Price Loadings)

$$\log P_t^{(n)} = A_n + B_n' X_t + X_t' C_n X_t$$

$$\begin{aligned}
C_n &= \Gamma + \Phi^{*\prime} C_{n-1} D_{n-1} \Phi^* \\
B_n' &= -\delta_1' + \left(2\mu^{*\prime} C_{n-1} + B_{n-1}'\right) D_{n-1} \Phi^* \\
A_n &= -\delta_0 + A_{n-1} + \frac{1}{2} B_{n-1}' \Sigma G_{n-1} \Sigma' B_{n-1} \\
&\qquad + \frac{1}{2} \log |G_{n-1}| + \left(\mu^{*\prime} C_{n-1} + B_{n-1}'\right) D_{n-1} \mu^*
\end{aligned}$$

where

$$\begin{aligned}
G_{n-1} &= \left[I - 2\Sigma' C_{n-1} \Sigma\right]^{-1} \\
D_{n-1} &= \Sigma G_{n-1} \Sigma^{-1} \\
\mu^* &\equiv \mu - \Sigma\lambda \\
\Phi^* &\equiv \Phi - \Sigma\Lambda.
\end{aligned}$$

back

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$$+ \frac{1}{2} \log |G_{n-1}| + \left(\mu^{*\prime} C_{n-1} + B_{n-1}'\right) D_{n-1} \mu^*$$

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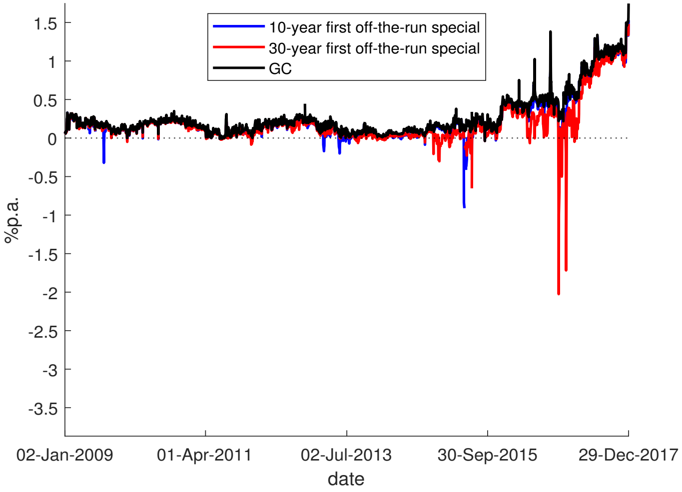
First Off-the-Run Special Rates



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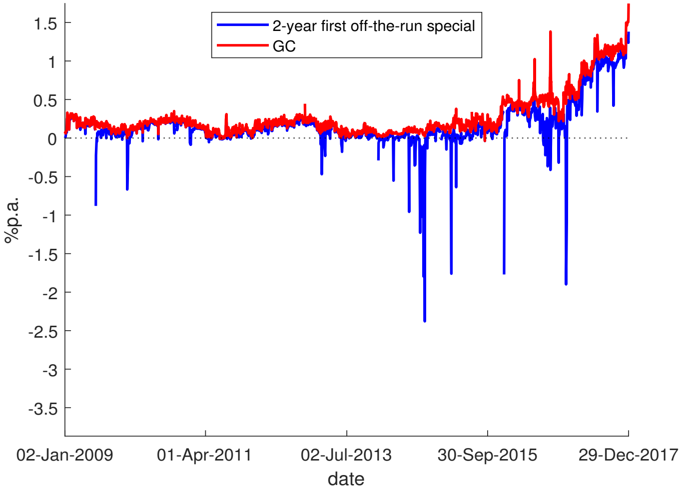
2-Year First Off-the-Run Special Rates



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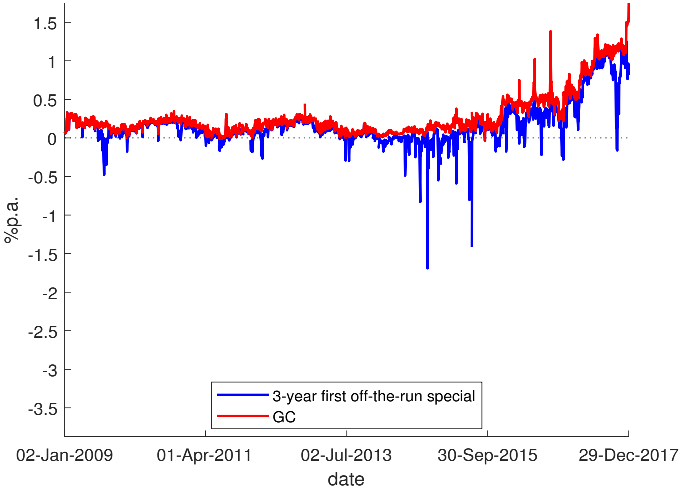
3-Year First Off-the-Run Special Rates



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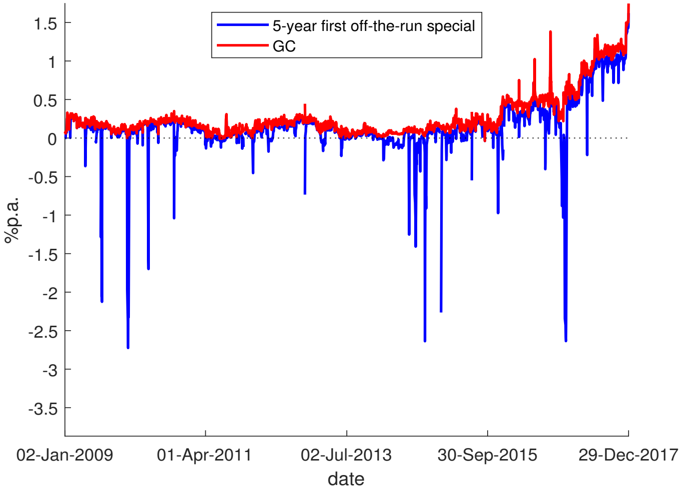
5-Year First Off-the-Run Special Rates



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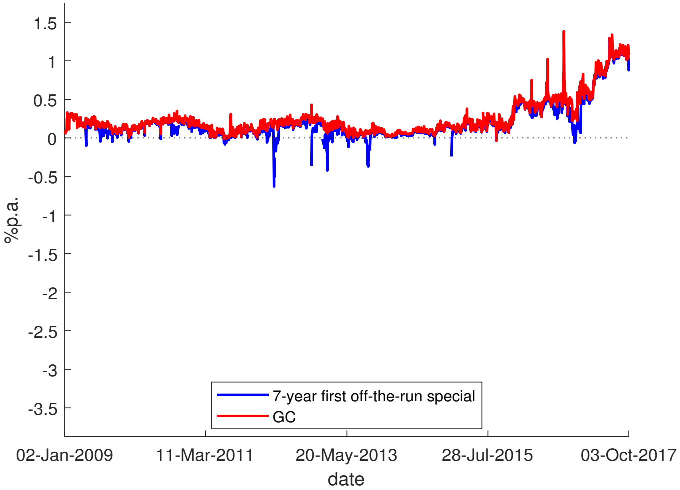
7-Year First Off-the-Run Special Rates



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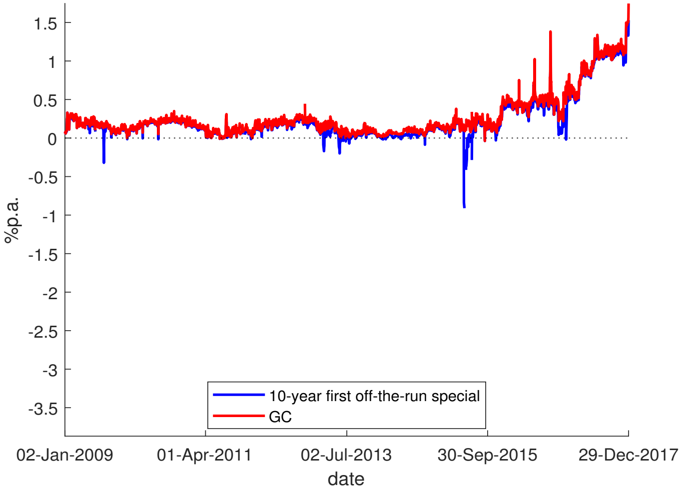
10-Year First Off-the-Run Special Rates



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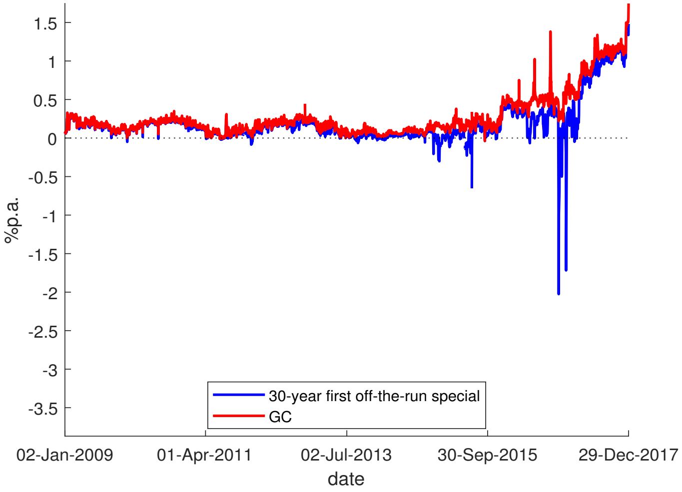
30-Year First Off-the-Run Special Rates



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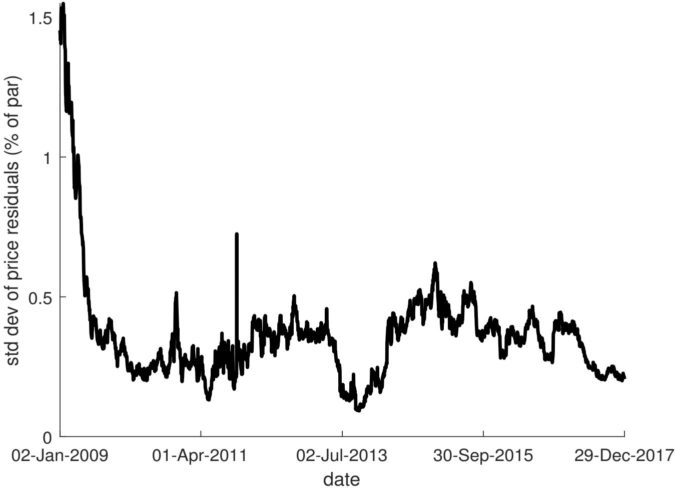
Fit over Time ("normal" bonds)



early sample fit   back

Model: Special Spreads

- Non-negative special spread on bond i at time t:

$$y_t^i = \Bigg[\underbrace{y_{t,\tau_i}^D}_{\text{deterministic component}} + \underbrace{y_t^S + x_t^i}_{\text{stochastic component}} \Bigg]^2$$

where

- y_{t,τ_i}^D is the deterministic component of the square root of the special spread that depends on time since issuance
- y_t^S is a stochastic aggregate repo factor
- x_t^i is a stochastic bond-level residual that follows

$$x_{t+1}^i = \rho x_t^i + \sigma_x \varepsilon_{t+1}^i$$

Observed Repo Factor

- Repo factor: average deviation from auction cycle:

$$y_t^S = \frac{1}{n_t} \sum_{i=1}^{n_t} \left[\sqrt{y_t^i} - y_{t,\tau_i}^D \right]$$

where we include on-the-run and first off-the-run for the 10-year note.

- Idiosyncratic factor is the residual:

$$x_t^i = \sqrt{y_t^i} - y_t^S - y_{t,\tau_i}^D$$

- x_t^i and y_{t,τ_i}^D appended to the state vector X_t, bond by bond.

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Model: State Variables

- We assume 3 latent bond-pricing factors (level slope curvature)
- State vector X_t: $[L, S, C, y^S, y^D_{\tau_i}, x^i]$
  1. Level picture
  2. Slope picture
  3. Curvature picture
  4. Aggregate repo factor
  5. Deterministic repo factor
  6. Idiosyncratic repo factor
- On-the-run bonds and first-off-the-runs load on all six factors. All other bonds load only on L, S, C.

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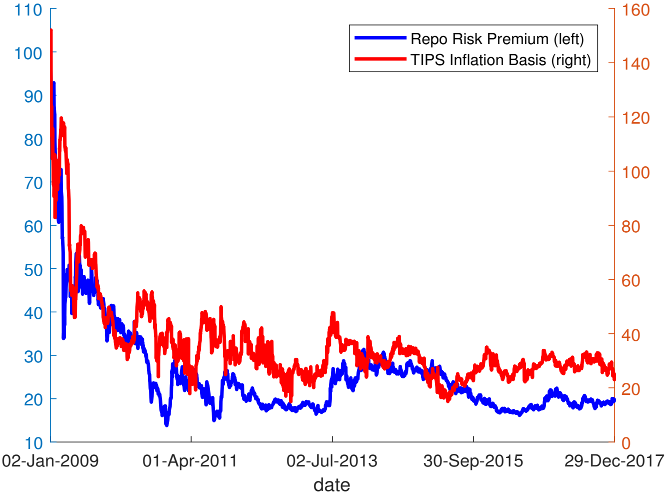
TIPS-Treasury Bond Puzzle



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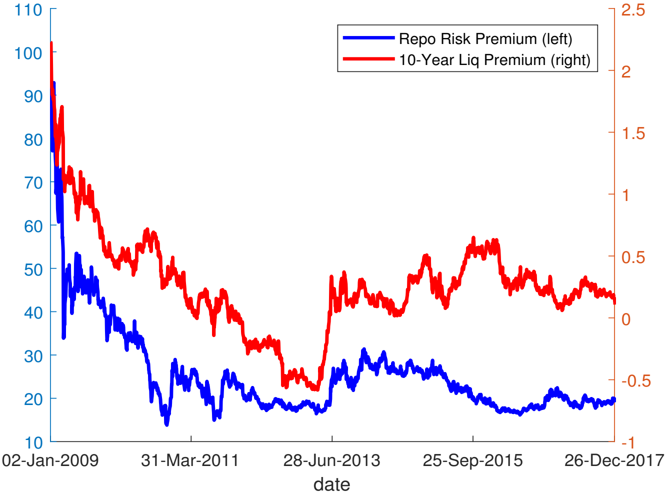
TIPS Relative Illiquidity



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