Money and Spending Multipliers with HA-IO

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Beyond representative agent, one sector

- Heterogeneous agents + input-output network
  - workers consume different bundles of goods
  - firms hire different bundles of workers (+ fixed factors)
- Heterogeneous nominal and real rigidities
  - sticky wage, work for sticky sector...
  - employer (or his customers...) relies on fixed factors
  - more or less elastic labor supply
- New questions:
  - how does policy redistribute across agents?
  - aggregate response to policy same as with rep agent?

Money multiplier

$$(\mathbb{L}_M)_h = \frac{\partial \log l_h}{\partial \log M}$$

- Cross section:
  - nominal rigidity $\uparrow$, real rigidity $\downarrow \iff$ price volatility $\downarrow$, employment volatility $\uparrow$
- Aggregate
  - substitute towards agents with more nominal rigidity / less real rigidity $\rightarrow$ more non-neutrality

Spending multiplier

$$(\mathbb{L}_G)_{hi} = \frac{\partial \log l_h}{\partial \log G_i}$$

- Spending affects relative demand for different workers
  - direct towards agents with more nominal rigidity / less real rigidity$\rightarrow$ larger multiplier
  - replicate aggregate consumption $\rightarrow$ "as if" rep agent
  - flex prices, no fixed factors, uniform labor supply elasticity $\rightarrow$ composition irrelevant for aggregate employment

Literature

HA-IO: Baqaee and Farhi (2018), Flynn, Patterson, Sturm (2020)

Monetary/fiscal policy with heterogeneous agents:

HANK: Werning (2015), Guerrieri and Lorenzoni (2017), Kaplan, Moll, Violante (2018), Auclert (2019), Auclert, Ronglie, Straub (2019); **open economy:** Benigno (2004), Gali and Monacelli (2008), Engel (2011), Huang and Liu (2005)

Monetary policy with input-output:

analytical: Basu (1995), Erceg et al (1999), Aoki (2001), Woodford (2003), Blanchard and Gali (2007); **quantitative:** Carvalho (2006), Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008, 2013), Carvalho and Nechio (2011), Bouakez, Cardia, and Ruge-Murcia (2014), Pasten, Schoenle and Weber (2016, 2017), Castro Cienfuegos (2019), Höynk (2019)

Spending multipliers: Bouakez, Rachedi, Santoro (2020), Cox, Muller, Pasten, Schoenle, Weber (2020)

Cross-sectional estimation: Nakamura and Steinnson (2014), Beraja, Hurst, Ospina (2016), Chodorow-Reich (2019), Auerbach, Gorodnichenko, Murphy (2019), Dupor, Karabarbounis, Kudlyak, Mehkari (2019), McLeay and Tenreyro (2018), Levy (2018), Hooper, Mishkin, Sufi (2019), Hazell, Herreno, Nakamura and Steinnson (2020)

Roadmap

- Setup
- Demand & supply blocks at high level
  - general expression for multipliers
  - "as if" results
- Specific structural model
  - break "as if" results
  - examples for intuition

Outline

Environment

- H worker types, K fixed factors, N production sectors
- Agents
  - consume different bundles of goods
  - own different shares of sectors and fixed factors
  - have different wage rigidity and labor supply elasticity
- Sectors
  - hire different bundles of workers and fixed factors
  - have different position in the input-output network
  - have different price rigidity and demand elasticity
- Log-linearized model
  - evolution described by measurable steady-state shares and elasticities

timing

Consumers

- Type-h preferences:

$$\frac{C_h\left(x_1, \ldots, x_N\right)^{1-\gamma_h}}{1-\gamma_h} - \frac{L_h^{1+\varphi_h}}{1+\varphi_h}$$

- Parameters:
  - wealth effects: $\Gamma \equiv diag\left(\gamma_1, \ldots, \gamma_H\right)$
  - Frish elasticities: $\Phi \equiv diag\left(\varphi_1, \ldots, \varphi_H\right)$
  - consumption shares $\beta = \left(\beta_{i,h}\right)$

Consumers

- Type-h budget constraint:

$$P_h C_h = \underbrace{W_h L_h}_{\text{labor}} + \underbrace{\sum_k \mathcal{Z}_{kh} R_k K_k}_{\text{fixed factors}} + \underbrace{\sum_i \Theta_{ih} \Pi_i}_{\text{profits}} - \underbrace{T_j}_{\text{lump-sum tax}}$$

- Factor income shares:

$$\varsigma_h \equiv \frac{W_h L_h}{GDP}, \quad \varsigma_k \equiv \frac{R_k K_k}{GDP}$$

- Agents' income shares:

$$s_h \equiv \frac{P_h C_h}{GDP} = \varsigma_h + \sum_j \mathcal{Z}_{kh} \varsigma_k$$

Producers

- CRS sectoral production functions:

$$Y_i = \overbrace{A_i}^{\text{Hicks-neutral shifter}} F_i(\underbrace{L_{ih}}_{\text{labor}}, \underbrace{K_{ik}}_{\text{fixed factors}} \{ \underbrace{x_{ij}}_{\text{intermediate inputs}} \})$$

- Factor shares $\alpha = \begin{pmatrix} \alpha_{ih} & \alpha_{ik} \end{pmatrix}$, input shares $\Omega = \Omega_{ij}$
- Domar weights: $\lambda^T \equiv \beta^T (I - \Omega)^{-1}$
- Elasticities of substitution

Producers

- Continuum of firms within sectors, CES bundle
  - fraction δ_i of producers adjust price after seeing A
  - notation: $\Delta = diag(\delta_1 ... \delta_N)$
- Sticky wages: add labor unions with sticky price
- Optimal input subsidies (τ_i), log-linearize around efficient equilibrium

Policy instruments

- Government spending: $G = (G_1 \dots G_N)^T$, normalize $G^* = \mathbf{0}$
- Money supply ($\longleftrightarrow$ nominal GDP), normalize $M^* = 1$

$$\sum_h P_h C_h + \sum_i G_i = M$$

- Budget constraint:

$$\sum_h T_h = \sum_i (G_i + \tau_i mc_i y_i)$$

- For this presentation:

$$T_h = \sum_i \left[\left[(I - \Omega)^{-1} \alpha \right]_{hi}^T G_i + \Theta_{ih} \tau_i mc_i y_i \right]$$

seignorage

Outline

Supply: $l = \mathcal{L}(w, G)$

- Prices and profits:

$$\pi = \Delta (I - \Omega\Delta)^{-1} \alpha w, \;\; \Pi = -(I - \Delta)(I - \Omega\Delta)^{-1} \alpha w$$

- Consumption:

$$c = rw + \underbrace{\mathcal{Z} w_K}_{\text{fixed factors}} + \underbrace{\hat{\Theta}^T \Pi}_{\text{profits}} - \underbrace{T(G)}_{\text{taxes}}, \;\; rw = w_L - \delta_\beta(\alpha) w$$

- Consumption-leisure tradeoff:

$$\Gamma c + \Phi l = rw \rightarrow l = \mathcal{L}(w, G)$$

Demand

- Aggregate GDP:

$$\delta_{\bar{\beta}}(\alpha)\, w + \varsigma_L^T l = d\log M$$

- Factor income shares:
  - direct effect ($w\uparrow$, $l\uparrow \Rightarrow \varsigma\uparrow$)
  - change in wages/prices $\to$ substitution$\to$ factor demand
  - change in private incomes, spending $\to$ factor demand

$$\mathbb{S}_w w + \mathbb{S}_l l = \mathbb{S}_G G$$

  - $\varsigma^T \mathbb{S} = \mathbf{0}$

Equilibrium

- Aggregate demand:

$$\underbrace{\left(\delta_{\bar{\beta}}\left(\alpha\right)+\varsigma_L^T\mathbb{S}_w\right)}_{\mathcal{E}^T}w = d\log M - \varsigma_L^T\mathcal{L}_g G$$

- Relative demand:

$$-\underbrace{\left(\mathbb{S}_w+\mathbb{S}_I\mathcal{L}_w\right)}_{\equiv\mathcal{S}_w}w = \left(\mathbb{S}_G+\mathbb{S}_I\mathcal{L}_g\right)G$$

Equilibrium

- Aggregate demand:

$$\underbrace{\left(\delta_{\bar{\beta}}\left(\alpha\right)+\varsigma_L^T\mathcal{L}_w\right)}_{\mathcal{E}^T}w=d\log M$$

- Relative demand:

$$-\underbrace{\left(\mathbb{S}_w+\mathbb{S}_I\mathcal{L}_w\right)}_{\equiv\mathcal{S}_w}w=\mathbb{S}_G G$$

- Decomposition:

$$\mathcal{S}_w=\mathcal{S}^{XS}\left(I-\mathbf{1}\varsigma^T\right)-\bar{\mathcal{S}}\varsigma^T$$

Money multiplier

- Full symmetry, no fixed factors $\Longrightarrow \bar{\mathcal{S}} = \mathbf{0}$

$$\mathbb{W}_m = \frac{\mathbf{1}}{\delta_{\bar{\beta}}(\bar{\alpha}) + \frac{1}{\gamma+\varphi}\left(1 - \delta_{\bar{\beta}}(\bar{\alpha})\right)} d\log M,\ \mathbb{L}_m = \frac{\frac{1}{\gamma+\varphi}\left(1 - \delta_{\bar{\beta}}(\bar{\alpha})\right)}{\delta_{\bar{\beta}}(\bar{\alpha}) + \frac{1}{\gamma+\varphi}\left(1 - \delta_{\bar{\beta}}(\bar{\alpha})\right)}$$

- Proportional increase
- Satisfy CIA constraint
- Balance excess demand

$$\mathbb{W}_m = \frac{\mathbf{1} + \mathcal{S}^{XS-1}\bar{\mathcal{S}}}{\mathcal{E}^T\left[\mathbf{1} + \mathcal{S}^{XS-1}\bar{\mathcal{S}}\right]} d\log M,\ \ \mathbb{L}_m = \mathcal{L}_w \mathbb{W}_m$$

Spending neutrality

- $\Gamma = \mathbb{O}$ OR uniform γ, φ and no fixed factors
- No effect on relative demand $\iff$ replicate aggregate consumption basket

$$\mathbb{S}_G G = 0 \iff G \propto \bar{\beta}$$

- Multiplier $\approx$ one sector, representative agent:

$$\mathbb{L}_g \bar{\beta} = \mathbb{L}_m + (\mathbf{1} - \mathbb{L}_m) \frac{\gamma}{\gamma + \varphi}$$

Spending multiplier

$$\mathbb{L}_g \bar{\beta} = \mathbb{L}_m + (\mathbf{1} - \mathbb{L}_m) \frac{\gamma}{\gamma + \varphi}$$

- Wealth effect in labor supply
- Satisfy CIA constraint
- Balance excess demand

$$\mathbb{L}_g = \mathbb{L}_m \mathbf{1}^T + \left(I - \mathbb{L}_m \varsigma_L^T \right) \mathcal{L}_g + \left[\mathcal{L}_w - \mathbb{L}_m \mathcal{E}^T \right] \mathcal{S}^{XS\,-1} \mathbb{S}_G$$

Irrelevance of composition

- Flex prices, no fixed factors, uniform γ and φ

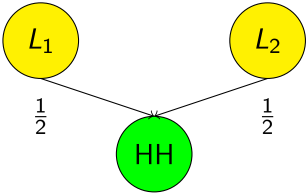
$$l = \mathcal{L}(w, G) = \frac{1-\gamma}{\gamma+\varphi} \underbrace{\left(I - \lambda^T \alpha\right) w}_{\text{real wage}} + \frac{\gamma}{\gamma+\varphi} \underbrace{v \sum_i G_i}_{\text{tax}}$$

$$\Longrightarrow \bar{\mathbb{L}}_G = \frac{\gamma}{\gamma+\varphi} \sum_i G_i$$

- Aggregate real wages unaffected by spending
- Same labor supply elasticity for all agents

Outline

Italy vs Germany



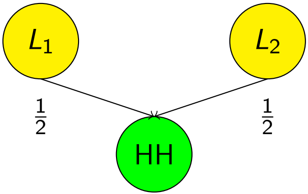
- Cross-section: $I \downarrow$ for sticky workers in a contraction

$$I_{sticky} - I_{flex} \propto \frac{\varphi\theta\bar{\delta}}{1 + \varphi\theta\bar{\delta}}(\delta_{flex} - \delta_{sticky})d \log M$$

- Substitution $\rightarrow$ more non-neutrality:

$$\bar{\mathbb{L}}_m = \frac{1 + \frac{\left(\delta_{flex} - \delta_{sticky}\right)^2}{1-\bar{\delta}} \frac{\varphi\theta}{1+\varphi\theta\bar{\delta}}}{1 + \varphi\frac{\bar{\delta}}{1-\bar{\delta}} - (\varphi - 1)\frac{\left(\delta_{flex} - \delta_{sticky}\right)^2}{1-\bar{\delta}} \frac{\varphi\theta}{1+\varphi\theta\bar{\delta}}}$$

Italy vs Germany

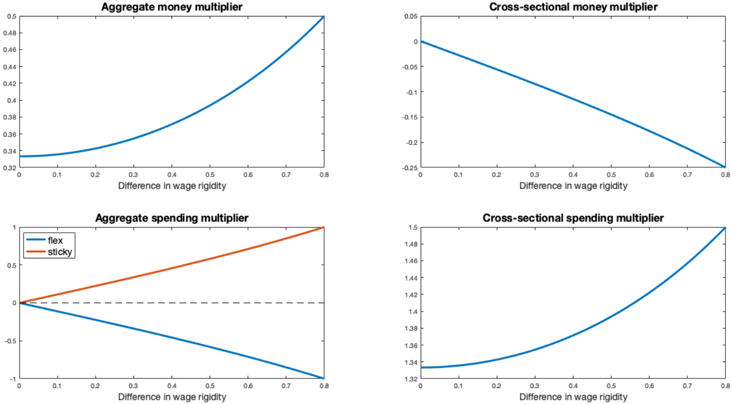


- Spending increases agg employment iff directed to sticky sector:

$$\bar{\mathbb{L}}_g = \frac{\delta_{flex} - \delta_{sticky}}{1 + \varphi\theta\bar{\delta}} \left(G_{sticky} - G_{flex}\right)$$

- Substitution $\rightarrow$ smaller XS multiplier

$$l_1 - l_2 = \left[1 - \frac{\varphi\theta\bar{\delta}}{1 + \varphi\theta\bar{\delta}}\right] \left(G_1 - G_2\right)$$



Aggregate money multiplier
Difference in wage rigidity
Cross-sectional money multiplier
Difference in wage rigidity
Aggregate spending multiplier
flex
sticky
Difference in wage rigidity
Cross-sectional spending multiplier
Difference in wage rigidity

Labor supply elasticity

- Expansion benefits elastic workers ($\varphi_E < \varphi_I$):

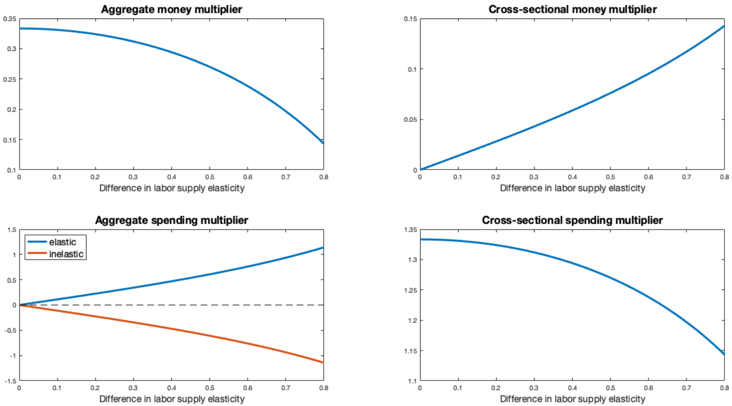
$$l_E - l_I = (\varphi_I - \varphi_E)\frac{\theta\delta}{1+\bar{\varphi}\theta\delta}\bar{\mathbb{L}}_m$$

- Substitution $\rightarrow$ larger aggregate multiplier:

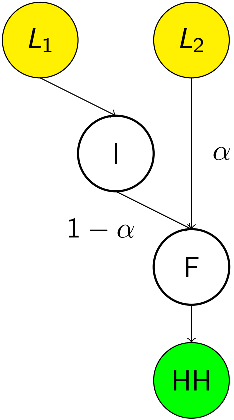
$$\bar{\mathbb{L}}_m = \frac{1}{1+\bar{\varphi}\frac{\delta}{1-\bar{\delta}} - \frac{\delta}{1-\bar{\delta}}\frac{\theta\delta}{1+\bar{\varphi}\theta\delta}(\varphi_I - \varphi_E)^2}$$

- Spending increases $\bar{l}$ iff directed to elastic workers:

$$\bar{\mathbb{L}}_g \propto \frac{\varphi_I - \varphi_E}{\bar{\varphi} + \varphi_E\varphi_I\theta\delta}(G_E - G_I)$$


Aggregate money multiplier
Cross-sectional money multiplier
Aggregate spending multiplier
elastic
inelastic
Cross-sectional spending multiplier
Difference in labor supply elasticity
Difference in labor supply elasticity
Difference in labor supply elasticity
Difference in labor supply elasticity

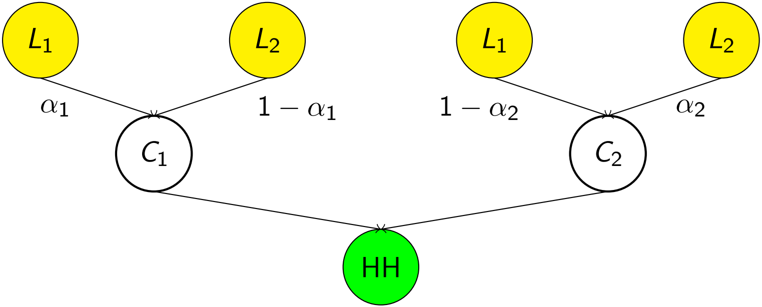
Input-output linkages



- Longer chain $\sim$ stickier wage
- Replace

$$\delta_F - \delta_I = \delta - \delta^2$$

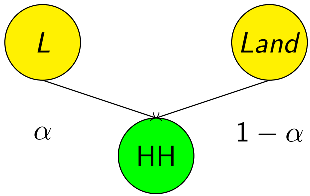
Chain-weighted ES



- XS spending multiplier:

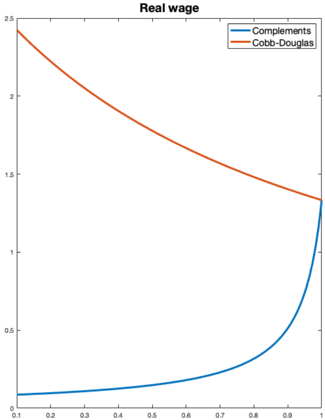
$$l_2 - l_1 = \frac{\varphi\beta_1(1-\beta_1)(\alpha_1-\alpha_2)\left(\frac{G_1}{\beta_1}-\frac{G_2}{1-\beta_1}\right)}{1+\varphi\left[\frac{\beta_1(1-\beta_1)(\alpha_1-\alpha_2)^2}{s_1(1-s_1)}\sigma\delta+\left(1-\frac{\beta_1(1-\beta_1)(\alpha_1-\alpha_2)^2}{s_1(1-s_1)}\right)\theta\right]}$$

Real Estate

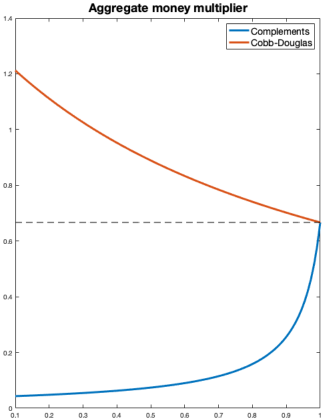


- Price stickiness vs labor share
  - $\theta < \frac{\delta}{1-\delta} \rightarrow$ real wage $\uparrow$ less $\rightarrow$ smaller multiplier
  - $\theta > \frac{\delta}{1-\delta} \rightarrow$ real wage $\uparrow$ more $\rightarrow$ larger multiplier

$$\bar{\mathbb{L}}_m = \frac{1 - \frac{1-\alpha}{1-\alpha+\varphi\theta}}{1 + \varphi\frac{\delta}{1-\delta} - (1+\varphi\theta)\frac{1-\alpha}{1-\alpha+\varphi\theta}}$$

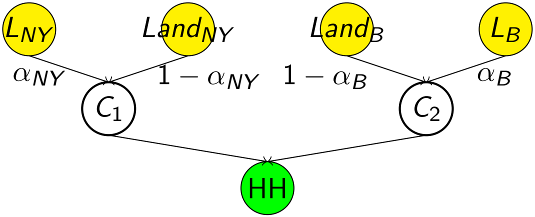


Real wage
Complements
Cobb-Douglas
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Aggregate money multiplier
Complements
Cobb-Douglas
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

NY or Boise?



- Locate construction projects in Boise $\iff \theta < \frac{\delta}{1-\delta}$

$$\bar{\mathbb{L}}_G \propto \varphi\theta \left(\frac{\delta}{1-\delta} - \theta \right) (\alpha_B - \alpha_{NY}) (G_B - G_{NY})$$

- Geographic mobility:
  - $\sigma\delta < \theta$: must live where you work $\rightarrow$ construction $\uparrow$ in NY
  - $\sigma\delta > \theta$: work from home $\rightarrow$ construction $\uparrow$ in Boise

$$I_B - I_{NY} \propto \theta (\sigma\delta - \theta) (\alpha_B - \alpha_{NY}) d \log M$$

Outline

Data

- I'm looking into:
  - ACS $\rightarrow$ employment shares
  - CEX $\rightarrow$ consumption bundles
  - BEA $\rightarrow$ capital shares
  - ADP $\rightarrow$ wage rigidity
- Suggestions?

Outline

Conclusion

- Monetary expansion:
  - cross-section: nominal rigidity $\uparrow$, real rigidity $\downarrow \iff$ price volatility $\downarrow$, employment volatility $\uparrow$
  - aggregate: substitution $\rightarrow$ more non-neutrality
- Government spending changes demand composition
  - larger multiplier iff target workers with more nominal / less real rigidity
  - "as if" representative agent $\iff$ replicate private consumption basket
- Spending vs transfers: TBD

Timing

One-period model

- Period 0: prices are pre-set
- Period 1: money supply and spending shock
    - only a fraction of producers can adjust prices
    - production and consumption take place
    - the world ends

back

Seignorage

- Consumers need to purchase new money issuances
  - agent h buys share v_h
- Revenues are fully rebated through lump-sum transfers
- Budget constraint:

$$P_h C_h + \underbrace{v_h dM}_{\text{money purchase}} = \text{income}_h - T_h + \underbrace{v_h dM}_{\text{seignorage rebate}}$$

back

Shares

- Change in shares

$$\left(I - \frac{\partial \log \text{demand}}{\partial \log \text{income}}\right) \partial \log \varsigma = \left(\frac{\partial \log \text{demand}}{\partial \log w} + \frac{\partial \log \text{profits}}{\partial \log w}\right) w + \frac{\partial}{}$$

- Definition of factor shares

$$\left(I - \frac{\partial \log \text{demand}}{\partial \log \text{income}}\right) \partial \log \varsigma = \left(I - \frac{\partial \log \text{demand}}{\partial \log \text{income}}\right) (w + I)$$

back