# The Fed Put and Monetary Policy: An Imperfect Knowledge Approach

Adrian Ifrim

UAB, Barcelona GSE & CREi [1]

This version: June 2021

**Abstract**

This paper argues that a central bank can increase macroeconomic stability by reacting explicitly to stock prices and therefore rationalizes the observed empirical evidence of the "Fed put". Waves of optimism/pessimism, unrelated to fundamental developments in the economy affect stock prices and aggregate demand through wealth effects which appear due to imperfect knowledge of the economy. Monetary policy can dampen these effects by influencing agents' expectations about stock prices and therefore eliminate the non-fundamental effects of booms and busts in stock prices. Reacting explicitly and transparently to stock prices increases welfare by 0.15% on average per period and brings efficiency gains compared to the standard Fed put policy.

*Keywords:* monetary policy, expectations, learning, stock prices, survey data

*JEL Classification:* D84, E32, E44, E52

[1] e-mail: adrian.ifrim@barcelonagse.eu. The author would like to thank Albert Marcet for his advice and support and Jordi Caballé for comments and feedback. The paper has also benefited from helpful discussions from participants at RES Junior Symposium 2021, UAB and UCL macro seminars. This research has received funding from the European Research Council (ERC) under the European Union's Seventh Framework Programme FP72007-2013 GA project number 324048 (APMPAL) and program GA project number 788547 (APMPAL-HET).

# I Introduction

How do asset prices affect the real economy and what is the proper response of central banks in the face of asset price cycles? In this paper I argue that stock prices affect aggregate demand and influence cyclical fluctuations through consumption wealth effects which appear due to agents' imperfect information about the structure of the economy. Consistent with recent evidence, agents extrapolate past returns and display slow and persistent movements in expectations. Booms and busts in asset prices driven by sentiment swings affect stock prices and the financial position of market participants which translate via consumption-wealth effects into changes in aggregate demand. In this environment monetary policy can increase macroeconomic stability and efficiency by managing long term expectations about capital gains by responding explicitly and transparently to asset prices. Crucial for this result is the assumption that agents understand and internalize into their expectations the response of central banks to asset prices. If on the contrary, the central bank acts in a discretionary manner and does not communicate the reaction to asset prices transparently, the gains are insignificant.

During the 1987 stock market crash, the aggressive easing of monetary policy that helped recovery of the economy and reflate asset prices has become to be known as the Greenspan put. The implied promise that the Fed will step in and help the financial markets, if needed, has continued over the years and has been relabelled as the Fed put. Recent evidence suggests that, although not explicitly, the Fed does indeed take into account the stance of the stock market when setting interest rates and moreover the main channel that they consider important is the consumption wealth effect: increases in stock prices make consumers feel wealthier and as a result adjust their consumption decisions accordingly.[2] The empirical evidence also shows that the marginal propensity to consume (MPC) out of (unrealized) capital gains can be as high as 20%.[3] Given the high volatility of stock prices these effects can have large impacts on aggregate demand. Nevertheless, most research concerning the optimal response of monetary policy to asset prices does not take into account the actual dynamics of stock prices or the consumption wealth effect as the main channel through which stock prices affect the real economy. The present paper tries to fill this gap.

[2]See Cieslak and Vissing-Jorgensen (2020) and the literature review section

[3]Di Maggio et al. (2020)

At the core of the paper is the realistic assumption that agents have imperfect knowledge about the determination of macroeconomic variables. Agents are internally rational, in the sense of Adam and Marcet (2011), maximizing their utility given their system of beliefs. Greenwood and Shleifer (2014), Adam et al. (2017) point out that survey measures of expectations are positively correlated with actual prices while actual returns tend to display a negative correlation. Rational expectations (RE) models have the opposite prediction, namely agents expect lower returns at the top of the cycle. In a lab experiment, Galí et al. (2020) show that agents' asset price beliefs are not consistent with rational expectations and propose that adaptive expectations fit better the experimental data. Moreover, the high volatility of stock prices relative to fundamentals, which has become known as the volatility puzzle, poses additional difficulty for RE macro-finance models.[4] I build on this evidence and specify the belief system of the agents as extrapolative, where agents use constant gain learning to update their beliefs about variables exogenous to their decision making. When stock prices depart from their fundamental value due to sentiment/expectation swings, consumption-wealth effects appear naturally in this framework since agents interpret their asset position as real wealth and modify the consumption decision accordingly. The proposed theory therefore links directly the volatility puzzle with stock price wealth effects.

The stock price consumption wealth effect is incorporated in a quantitative Two Agent New Keynesian (TANK) model where agents are heterogeneous with respect to their participation in the stock market and have homogeneous imperfect information about macroeconomic variables. The economy is hit by three shocks: supply (cost push), monetary policy and a sentiment shock which affects the beliefs of the agents on their expected capital gains. The latter will operate as a demand shock influencing stock prices and aggregate demand via the consumption wealth effect. The model is estimated on US data by targeting a standard set of business cycle and financial moments. Although not explicitly targeted, the model is able to capture remarkably well the dynamics of survey expectations regarding capital gains, inflation and interest rates and the joints dynamics of the real economy and financial markets.

Using the quantitative model estimated on US data I then compare the following two policies: responding to stock prices explicitly and transparently

[4]see Shiller (1981)

vs responding to stock prices without agents realising so. By transparency it is understood that agents take into account the reaction of policy to stock prices when forming expectations about future interest rates. The reverse is true under non-transparency. I consider two policies under the latter: the Fed put and the Fed put-call. The first one is a policy of taking into account stock prices in the Taylor rule only in bad times while the second in both good and bad times. I show that by reacting transparently to stock prices, monetary policy can increase welfare by 0.15% on average per period while if agents do not internalize the reaction of monetary policy to stock prices the gains are insignificant. This result emphasizes the key mechanism through which stock prices targeting influences the economy in this environment, namely through managing long-term expectations of capital gains.

The rest of the paper is organized as follows. Section II reviews the literature on monetary policy and stock price targeting. Section III presents a simple endowment economy to gain intuition into the origin of stock price wealth effects. Section IV incorporates the mechanism from the previous section in a quantitative TANK model with homogeneous imperfect information and estimates the model on US data. Section V studies the macroeconomic stability and welfare properties of stock prices targeting. Lastly, section VI concludes.

# II Related Literature

This paper contributes to the literature that analyses stock price targeting and monetary policy. The seminal papers of Bernanke and Gertler (2000, 2001) use a model with credit market frictions that features a financial accelerator effect in which exogenous shocks have an amplified effect on the economy. Using a calibrated version of the model they argue that targeting stock prices has no gain and that a central bank is better off, in terms of macroeconomic stability, by sticking to a flexible-inflation targeting regime. In reaching this conclusion their model does not take into account key financial facts like excess volatility of stock prices or market expectations. Carlstrom and Fuerst (2007) analyze the implications of stock price targeting on equilibrium determinacy and conclude that a central bank targeting explicitly stock prices raises the risk of inducing real indeterminacy in the system. Bullard et al. (2002) reach a similar conclusion. Cecchetti et al. (2000), using the same model as Bernanke and Gertler (2001), conclude that central banks can derive some benefit by reacting to stock prices. The main difference between Cecchetti et al. (2000) and Bernanke and Gertler (2000, 2001) is the assumption about the nature of the shock. In Cecchetti et al. (2000) the central bank knows that the swings in stock prices are non-fundamental and, with this knowledge, reacting to stock prices can increase economic performance. In the papers described above and in most of the literature, the effect of stock prices on the economy either come from the supply side, as in Bernanke and Gertler (2001), or from the central bank reacting explicitly to stock price deviation in the Taylor rule. Nisticò (2012) develops a NK model with OLG households that features a direct demand effect of stock prices on output in the IS equation.[5] The author concludes that targeting stock price growth increases macroeconomic stability. Bask (2012) also argues for stock price targeting in a model with both fundamental and technical traders.

In a follow up paper, Airaudo et al. (2015), using the same model as Nisticò (2012) analyze the stability and learnability of the model and conclude that, if the stock-wealth effect is sufficient strong, reacting to stock prices increases the policy space for which the equilibrium is both determinate and learnable.

Winkler (2019) introduces learning in a monetary model with financial frictions, similar to the one in Bernanke et al. (1999), and finds that the effects of shocks are amplified when agents learn about stock prices. The author also finds that by including a reaction to stock price growth in the Taylor rule improves macroeconomic stability. Adam et al. (2017) build a real model of the economy in which agents learn about stock price behaviour and which is quantitatively able to reproduce the joint behavior of stock prices and the business cycle. Airaudo (2016) studies asset prices in a monetary model in which agents have long-horizon learning and finds the existence of a wealth effect. The issue of stability is then analyzed in the context of the central bank responding to stock prices and finds that reacting to stock

[5] The effect appears due to the fact that in each period a fraction of households who own financial wealth die and are replaced by newcomers with zero stock holdings. Therefore, increases in stock prices in period $t$ (which forecast higher financial wealth next period) generate higher consumption due to the desire of households to intertemporally smooth consumption. Once next period arrives, some households will not be affected by the higher financial wealth (since they were replaced with newcomers who do not hold any) and therefore the increase in aggregate consumption seems higher than granted by the increase in financial wealth. In this sense stock prices affect consumption although this stock wealth effect is artificially generated by the assumption of households being replaced with 0 financial wealth ones.

prices increases the stability of the economy. Eusepi and Preston (2018a) show that imperfect knowledge about the structure of the economy generates wealth effects arising from long-term bond holdings. In their framework the steady state level of long-term bonds influence the magnitude of this effect and the effects of monetary policy. Agents have perfect knowledge of how prices of long-term bonds are determined but there exist a wedge between their forecast of the quantity of future bonds and the future level of taxes which make bonds net wealth giving rise to the wealth effect.

Cieslak and Vissing-Jorgensen (2020) analyze Federal Open Market Committee (FOMC) transcripts and conclude that the FED officials pay attention to asset prices and perceive the stock market as influencing the economy mainly through a consumption-wealth effect. They show that stock prices decreases between 2 consecutive FOMC meetings is one of the best predictors of subsequent federal rate cuts. Case et al. (2005, 2011) and more recently Chodorow-Reich et al. (2019) bring empirical evidence for the existence of this wealth effect. The magnitude of this effect is not insignificant either. Di Maggio et al. (2020) show that unrealised capital gains lead to MPC ranging from 20% for the bottom 50% of the wealth distribution to 3% for the top 30%.

# III Wealth effects in endowment economies

This section lays out a basic endowment economy where I show that incomplete information about stock prices fundamentally changes the equilibrium of the economy. In this environment stock prices affect the endogenous variables due to a wedge between actual stock prices and their expected discounted sum of dividends.

Consider a flexible price endowment economy populated by a continuum of households, indexed by $i$, who maximize their utility by choosing how much to consume, $C_t^i$, save in bonds, $B_t^i$ and invest in a risky asset, $S_t^i$. The risky asset is a claim to an exogenous stream of dividends, $D_t$. For simplicity assume that $D_t \sim \mathcal{N}(\mu, \sigma^2)$. Specifically, the problem of a typical household $i$ is

$$
\begin{aligned}
&\max_{C_t^i, B_t^i, S_t^i} E_0^{\mathscr{P}_i} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\sigma}}{1-\sigma} \\
&\quad s.t. \quad P_t C_t^i + B_t^i + S_t^i Q_t \leq B_{t-1}^i (1 + i_{t-1}) + S_{t-1}^i (Q_t + D_t) \\
&\qquad 0 \leq S_t^i \leq S^H, \forall\, t
\end{aligned} \tag{1}
$$

where $P_t$ is the aggregate price index, $i_t$ is the nominal interest rate (set exogeneously by the monetary authority) and $Q_t$ is the ex-dividend price of the risky asset. The expectation is taken over the subjective probability measure $\mathscr{P}_i$ which is household specific and different than the rational expectation hypothesis, denoted by $E$. Furthermore there is a central bank following a Taylor type rule $i_t = \phi_\pi \pi_t$ which is common knowledge among all the agents in the economy. The FOCs are

$$
\frac{1}{1+i_t} = \delta E_t^{\mathscr{P}_i} \Big\{ \Big( \frac{C_{t+1}^i}{C_t^i} \Big)^{-\sigma} \frac{1}{1+\pi_{t+1}} \Big\}, \tag{2}
$$

$$
Q_t = \delta E_t^{\mathscr{P}_i} \Big\{ \Big( \frac{C_{t+1}^i}{C_t^i} \Big)^{-\sigma} \frac{(Q_{t+1} + D_{t+1})}{1+\pi_{t+1}} \Big\}. \tag{3}
$$

Letting $\mathscr{W}_t^i = B_{t-1}^i(1 + i_{t-1}) + S_{t-1}^i(Q_t + D_t)$ and after imposing a transversality condition, the intertemporal BC becomes

$$
\frac{\mathscr{W}_t^i}{P_t} = E_t^{\mathscr{P}_i} \sum_{j=0}^{\infty} \delta^j \Big( \frac{C_{t+j}^i}{C_t^i} \Big)^{-\sigma} C_{t+j}^i. \tag{4}
$$

Equation 4 says that the discounted sum of future consumption equals real wealth.

I am going to analyze an equilibrium where all agents are identical although they do not know this to be true. This will prove to be essential to the pricing of the risky asset and for the existence of the wealth effect. Given that agents have the same preferences, constraints and beliefs they will make the same decisions. Equilibrium implies

$$\begin{aligned} \int_0^1 B_t^i \, di &= 0, \\ \int_0^1 C_t^i \, di &= C_t = d_t, \\ \int_0^1 S_t^i \, di &= 1. \end{aligned} \tag{5}$$

Aggregating equation (4), imposing $E^{\mathscr{P}_i} = E^{\mathscr{P}}$ and applying the equilibrium condition (5) yields

$$q_t + d_t = E_t^{\mathscr{P}} \sum_{j=0}^{\infty} \delta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} C_{t+j}. \tag{6}$$

where $q_t$ and $d_t$ are the risky asset real price and real dividends.

In order to make clear the different implications of the expectations of the agents I will first derive the optimal decision rule of the agents in the case of rational expectations. Having this benchmark, the imperfect information case will be analyzed next and compared to the RE benchmark.

## III.A Rational Expectations

First I will assume that agents have RE: $E^{\mathscr{P}} = E$. Given this, the FOC with respect to stock prices can be substituted forward to arrive at

$$q_t = E_t \sum_{j=1}^{\infty} \delta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} d_{t+j}. \tag{7}$$

Given this, equation (6) becomes

$$E_t \sum_{j=0}^{\infty} \delta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} d_{t+j} = E_t \sum_{j=0}^{\infty} \delta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} C_{t+j}. \tag{8}$$

Applying a first-order approximation around a non-stochastic steady state yields

$$E_t \sum_{j=0}^{\infty} \delta^j \tilde{d}_{t+j} = E_t \sum_{j=0}^{\infty} \delta^j \tilde{C}_{t+j}. \tag{9}$$

Using the fact that $E_t(\tilde{C}_{t+k}) = \tilde{C}_t + \frac{1}{\sigma} E_t \sum_{j=0}^{k-1} (i_{t+j} - \pi_{t+j+1})$ we arrive at the optimal consumption rule for the household.

**Lemma 1.** *Optimal consumption decision under RE*

$$\tilde{C}_t = (1-\delta) E_t \sum_{j=0}^{\infty} \delta^j \tilde{d}_{t+j} - \frac{1}{\sigma} \delta E_t \sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1}). \tag{10}$$

Equation (10) highlights the standard transmission mechanism of monetary policy which operates through the inter-temporal substitution of consumption which is influenced by the whole future path of real interest rates. Notice that I have not imposed yet the equilibrium condition for the goods market: $\tilde{C}_t = \tilde{d}_t$. Imposing this condition, using the interest rate rule and the process for dividends yields the unique RE equilibrium condition.

**Proposition 1.** *RE Equilibrium*

$$\pi_t = -\frac{\sigma}{\phi_\pi} \tilde{d}_t \tag{11}$$

Similarly to Eusepi and Preston (2018a) inflation is a linear function of the endowment process. Stock prices or beliefs about stock prices do not influence the real economy. Anticipating the next section, this will not be the case under imperfect knowledge and the reason will soon be clear.

## III.B Imperfect Knowledge: Learning

In deriving the optimal decision (10) we have used the fact that the price of the risky asset, $q_t$, can be written as the discounted sum of dividends, as in equation (7). Indeed, under RE this is true. Under imperfect knowledge, we cannot iterate (3) forward since this would imply that any agent would know that either he is the marginal agent forever or that all the other agents in the economy share his beliefs, preferences and constraints.

Since agents have imperfect knowledge about the economy, even if agents know that the other agents share their preferences and constraints but have different beliefs, agent $i$ would not be able to apply the Law of Iterated Expectations (LIE) to his FOC since

$$q_t = \delta E_t^{\mathscr{P}_i}\Big\{\Big(\frac{C_{t+1}^i}{C_t^i}\Big)^{-\sigma}(q_{t+1}+d_{t+1})\Big\}$$
$$= \delta E_t^{\mathscr{P}_i}\Big\{\Big(\frac{C_{t+1}^i}{C_t^i}\Big)^{-\sigma}\big(d_{t+1}+\delta E_{t+1}^{\mathscr{P}_{mg}}\Big\{\Big(\frac{C_{t+2}^{mg}}{C_{t+1}^{mg}}\Big)^{-\sigma}(d_{t+2}+q_{t+2})\Big\}\Big\} \tag{12}$$

and $E_t^{\mathscr{P}_i}E_{t+1}^{\mathscr{P}_{mg}} \neq E_t^{\mathscr{P}_i}$. Here $\mathscr{P}^{mg}$ is the subjective probability measure of the marginal agent which is not known by agent $i$ at time $t$. The marginal agent is the agent with the highest valuation of the asset which will determine the price of the asset in that period.[6] Therefore, in this environment, the optimality condition for stock prices is of the one-step ahead form which after log-linearization becomes

$$\tilde{q}_t = (1-\delta)E_t^{\mathscr{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathscr{P}}(\tilde{q}_{t+1}) + \sigma(\tilde{C}_t - E_t^{\mathscr{P}}\tilde{C}_{t+1}) \tag{13}$$

where the expectation regarding stock prices follows an updating equation

$$E_t^{\mathscr{P}}(\tilde{q}_{t+1}) = E_{t-1}^{\mathscr{P}}(\tilde{q}_t) + \lambda(\tilde{q}_{t-1} - E_{t-1}^{\mathscr{P}}(\tilde{q}_t)).^7 \tag{14}$$

Using the previous results, the assumption of the Average Marginal Agent described in Appendix A results in the optimal decision of the household under imperfect knowledge.

**Lemma 2.** *Optimal consumption decision under Imperfect Knowledge*

$$\tilde{C}_t \approx (1-\delta)E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j\tilde{d}_{t+j} - \frac{1}{\sigma}\delta E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j(i_{t+j}-\pi_{t+j+1})$$
$$+ \underbrace{\delta\tilde{q}_t - (1-\delta)\Big[E_t^{\mathscr{P}}\sum_{j=1}^{\infty}\delta^j\tilde{d}_{t+j} - \frac{\delta}{1-\delta}E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j(i_{t+j}-\pi_{t+j+1})\Big]}_{\textit{Wealth Effect=0 in RE}}. \tag{15}$$

The first line from (15) is the standard transmission mechanism as also found under RE (see equation (10)). The second line represents a new channel through which stock prices and beliefs about stock prices affect the consumption decision of the household. The second channel is the difference between

[6]see Adam and Marcet (2011) for more details

[7]See Appendix A for a detailed discussion of the consistency of this result in the context of long-horizon learning

actual stock prices and the discounted sum of future dividends. Under RE these terms would sum exactly to 0 since stock prices are exactly equal to the discounted sum of dividends. Under learning there is no reason for this to be the case. Since beliefs influence stock prices and vice verse, stock prices may drift away from their perceived fundamental value therefore causing agents to feel wealthier and increase consumption. In the current framework stock price wealth effects appear because agents do not have perfect knowledge about the economy and how stock price are actually determined. That people do not have perfect knowledge about how stock prices are determined should not surprise anyone. What is interesting is that this lack of knowledge is the principal determinant of stock price wealth effects.

In order to determine the learning equilibrium I will assume the following:

1. similarly to RE, agents have perfect knowledge about the dividend process, therefore $E^{\mathscr{P}} d_{t+j} = \mu$

2. agents know the interest rate rule, therefore $E^{\mathscr{P}} i_{t+j} = \phi_\pi E^{\mathscr{P}} \pi_{t+j}$

3. agents think that inflation and stock prices follow an unobserved component model

$$\begin{aligned} x_t &= \beta_t^x + \epsilon_t \\ \beta_t^x &= \beta_{t-1}^x + \psi_t \end{aligned} \tag{16}$$

   where $x = (\tilde{q}, \pi)'$.

Denoting by $\hat{\beta}_{t-1} = (\hat{\beta}^\pi_{t-1}, \hat{\beta}^q_{t-1})$ period $t$ subjective expectations, agents use the following optimal recursive algorithm to update their beliefs

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \lambda(x_t - \hat{\beta}_{t-1}) \tag{17}$$

where $\lambda$ is the constant gain coefficient which governs the speed at which agents incorporate new information into current beliefs.[8] Given these assumptions the expectations of real interest rates from equation (15) can be evaluated as

[8] As it is usually done in the learning literature, in order to avoid the simultaneity formation of beliefs and equilibrium variables, agents form expectations at period $t$ using information from the previous period

$$E_t^{\mathscr{P}} \sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1}) = \phi_\pi \pi_t + \frac{\delta\phi_\pi - 1}{1-\delta} \beta_{t-1}^{\pi}. \tag{18}$$

Substituting the forecasts, (18), into the consumption equation (15), applying assumption 1 and market clearing in the goods market, $\tilde{C}_t = \tilde{d}_t$, gives the data-generating process or the actual law of motion for inflation.

**Proposition 2.** *Learning Equilibrium*

$$\pi_t = \frac{\delta\sigma}{\phi_\pi}\beta_{t-1}^{q} - \Big(\frac{\sigma}{\phi_\pi} - \frac{(1-\sigma)(\delta\phi_\pi - 1)}{(1-\delta)\phi_\pi}\Big)\beta_{t-1}^{\pi} - \frac{\sigma}{\phi_\pi}\tilde{d}_t. \tag{19}$$

The learning equilibrium is fundamentally different from the RE counterpart. The first term in the above equation is totally absent from the RE equilibrium relation. Beliefs about stock prices influence directly inflation in equilibrium through a stock price wealth effect. Eusepi and Preston (2018a) reach a similar conclusion for the case of long-term bonds, although in that case agents are assumed to know the pricing map and learn about taxes and long term bonds.

Having gained this intuition into the origin of the effect of stock prices on consumption, I now move to a more complete general equilibrium monetary model in which stock price wealth effects influence the aggregate economy.

# IV Monetary Policy and Stock Prices: Quantitative Evaluation

This section describes a heterogeneous agents New Keynesian model with learning in which agents hold subjective beliefs about the variables which are exogenous to their decision making (from the point of view of an individual agent). There are two types of consumers and the only source of heterogeneity between them is the fact that a constant fraction, $\mathscr{O}$, of the agents is assumed not to participate in the stock market. This assumption is in line with the empirical evidence on US stock market participation. Notice that while some the of agents are excluded from saving in stocks all agents have access to the bond market and can smooth consumption by investing in a riskless asset. In essence, the model is a two-agent New Keynesian model (TANK) with homogeneous imperfect information. The economy is comprised of households, final goods producers, intermediary goods producers, a mutual fund and a central bank conducting monetary policy.

## IV.A Households

The economy is populated by a continuum of infinitely lived consumers indexed by $i$ who choose consumption, $C_t^i$, labor, $N_t^i$, bond holdings, $B_t^i$, stock holdings in a mutual fund, $S_t^i$, and receive income in form of dividends, $D_t$ and wages, $W_t$. The mutual fund is introduced to abstract from the portfolio choice of the households and its problem will be described in a later section. Let $i = S^U, S^C$ denote the agents who do/do not participate in the stock market . The problem of the household is to maximize utility subject to a standard budget constraint

$$\begin{aligned} \max_{C_t^i, N_t^i, B_t^i, S_t^i} & E_0^{\mathscr{P}} \sum_{t=0}^{\infty} \delta^t \Big[ \frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{(N_t^i)^{1+\phi}}{1+\phi} \Big] \\ s.t. \quad & P_t C_t^i + B_t^i + S_t^i Q_t \leq B_{t-1}^i (1 + i_{t-1}) + W_t N_t^i + S_{t-1}^i (Q_t + D_t) \\ & 0 \leq S_t^i \leq S^H, \forall\, t \end{aligned} \tag{20}$$

where $P_t$ is the aggregate price index, $i_t$ is the nominal interest rate, $Q_t$ is the ex-dividend price of the mutual fund share, $W_t$ is the nominal wage and $D_t$ is the nominal dividend paid by the mutual fund . Short-selling is not allowed and there is an upper bound for stock holdings, $S^H$, which can be bigger than 1.[9]

The optimality conditions of the household problem are

$$\frac{(N_t^i)^{\phi}}{(C_t^i)^{-\sigma}} = w_t, \tag{21}$$

[9]I assume this upper bound on stock holdings is never reached.

$$\frac{1}{1+i_t} = \delta E_t^{\mathscr{P}}\Big\{\Big(\frac{C_{t+1}^i}{C_t^i}\Big)^{-\sigma}\frac{1}{1+\pi_{t+1}}\Big\}, \tag{22}$$

$$Q_t = \delta E_t^{\mathscr{P}}\Big\{\Big(\frac{C_{t+1}^i}{C_t^i}\Big)^{-\sigma}\frac{(Q_{t+1}+D_{t+1})}{1+\pi_{t+1}}\Big\}, \tag{23}$$

where $\pi_{t+1}$ is the inflation rate between $t$ and $t+1$ and $w_t = \frac{W_t}{P_t}$ is the real wage. Equation (21) determines the consumption and labor decision, equation (22) is the Euler equation and equation (23) is the asset pricing equation. Also notice that equation (23) holds with equality as long as $S_t^i \in [0, S^H)$.

The only difference from the standard household problem is the operator $E_t^{\mathscr{P}}$. The expectations of the households are determined using the subjective probability measure $\mathscr{P}$ that assigns probabilities to the variables the household is trying to forecast. I proceed in deriving the consumption decision of the household following the anticipated utility framework of Preston (2005).[10] The intertemporal budget constraint of the household reads

$$\frac{\mathscr{W}_t^i}{P_t} \approx E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j\Big(\frac{C_{t+j}^i}{C_t^i}\Big)^{-\sigma}\big[C_{t+j}^i - w_{t+j}^{\frac{1+\phi}{\phi}}(C_{t+j}^i)^{\frac{-\sigma}{\phi}}\big] \tag{24}$$

where $\mathscr{W}_t^i = B_{t-1}^i(1+i_{t-1})+S_{t-1}^i(Q_t+D_t)$ represents nominal wealth at time $t$.

Log-linearization of equation (57) around a steady state characterized by $\pi = 0$, $S = 1$, $C = Y$ yields

$$\tilde{\mathscr{w}}_t^i = (1-\delta)E_t^{\mathscr{P}}\Big\{\sum_{j=0}^{\infty}\delta^j\Big[-r_{t+j}^N + \sigma\tilde{c}_t^i + \Delta_r\,\tilde{c}_{t+j}^i - \frac{\Delta_i}{1-\delta}\tilde{w}_{t+j}\Big]\Big\}. \tag{25}$$

and $\mathscr{w}\ \tilde{\mathscr{w}}_t^i = (1+i)b_{t-1}^i + q(\tilde{S}_{t-1}^i + \tilde{q}_t) + d(\tilde{S}_{t-1}^i + \tilde{d}_t)$ where $\mathscr{w} = \frac{d}{1-\delta}$. In

[10] A large body of the literature uses the *Euler Equation* approach to introduce learning in DSGE models. This approach entails that after solving the model using the RE assumption, expectations are replaced mechanically with some subjective expectations. This approach implies that agents are mixing two probability measures, the RE measure, on the one hand, and the subjective one. Furthermore the stock market wealth effect is not present under this approach since agents implicitly know the mapping from dividends to prices. See Preston (2005) for a detailed discussion of this issue and Eusepi and Preston (2018b) for a comparison between these two approaches.

the above expression any variable $\tilde{x}$ denotes percentage deviation of real variables from their steady-state values $Y$, $q$, $d$ represent steady-state values of aggregate output, real stock price and real dividends.
Log-linearization of the Euler equation (22) yields

$$\tilde{c}_t^i = E_t^{\mathscr{P}} \tilde{c}_{t+1}^i - \frac{1}{\sigma}(i_t - E_t^{\mathscr{P}_i} \pi_{t+1}) \tag{26}$$

which can be rewritten as

$$E_t^{\mathscr{P}}(\tilde{c}_{t+k}^i) = \tilde{c}_t^i + \frac{1}{\sigma} E_t^{\mathscr{P}_i} \Big[ \sum_{j=0}^{k-1} i_{t+j} - \pi_{t+j+1} \Big] \tag{27}$$

Substitution of equation (27) in the linearized budget constraint (25) and rearranging results in the decision rule of the household[11]

$$\tilde{c}_t^i = \Delta_i \tilde{\omega}_t^i + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(i_{t+j} - \pi_{t+j+1}). \tag{28}$$

Equation (28) makes clear that the consumption decision of the household today depends not only on the next period output and interest rate (as dictated by the standard Euler equation) but on the whole future path of wages, inflation and interest rates, as well as on the current wealth. Therefore, the agent will need to form expectations/forecasts for all future $\pi$ and $\tilde{w}$ and $i$ using the subjective probability measure $\mathscr{P}$. The next proposition presents the optimal consumption decision for the two types of agents.

**Proposition 3.** *The log-linearized aggregate consumption decisions at time $t$ for households participating in the stock market ($U$) and excluded from trading stocks ($C$) are given by*

$$\begin{aligned} \tilde{c}_t^U = \Delta_i \Big[ \frac{(1+i)}{\omega} b_{t-1}^U + \tilde{S}_{t-1}^U + \delta \tilde{q}_t + (1-\delta)\tilde{d}_t \Big] + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) \\ - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(i_{t+j} - \pi_{t+j+1}). \end{aligned} \tag{29}$$

$$\tilde{c}_t^C = \Delta_i \Big[ (1+i) b_{t-1}^C \Big] + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(i_{t+j} - \pi_{t+j+1}). \tag{30}$$

[11] See appendix for expressions of the composite parameters and details about derivation of equation (28)

Notice that the only difference between the optimal consumption decisions of the two types of households is given by the first term from both equations, namely the asset position at time $t$.

## IV.B Firms

### Intermediate goods producers

There is a continuum of firms indexed by $j$ which produce differentiated goods using the Cobb-Douglas production function with labor input $N_t(j)$

$$Y_t(j) = N_t(j)^{1-\alpha}. \tag{31}$$

Firms are subject to nominal rigidities when setting prices. Following Calvo (1983), each firm cannot reset its price in a given period with probability $\theta$. The problem of the firm is to maximize profits subject to the demand function

$$\begin{aligned} &\max_{P_t^*} \quad \sum_{k=0}^{\infty} \theta^k E_t^{\mathscr{P}_j} \left\{ Q_{t,t+k} (P_t^* Y_{t+k/t} - \psi_{t+k}(Y_{t+k/t})) \right\} \\ &s.t. \quad Y_{t+k/t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \end{aligned} \tag{32}$$

where $Y_{t+k/k}$ denotes output in period $t+k$ for a firm that last reset price in period $t$, $\psi_t()$ is the cost function and $Q_{t,t+k} = \delta^k \left(\frac{Y_{t+k}}{Y_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor for nominal profits.[12] Notice that compared to the RE framework, the stochastic discount factor is a function of aggregate output and not of consumption. This is because firms do not know the problem of the households or of the mutual fund and therefore, it makes possible for firms to hold subjective beliefs about aggregate outcomes.

The solution to the profit maximization problem yields the optimal price setting decision of the firm

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}_j} \left[ Y_{t+k}^{1-\sigma} P_{t+k}^{\epsilon} MC_{t+k/k} \right]}{\sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}_j} \left[ Y_{t+k}^{1-\sigma} P_{t+k}^{\epsilon-1} \right]} \tag{33}$$

where $MC_{t+k/k}$ is the real marginal cost of a firm which last updated prices

[12] I assumed implicitly that households and firms share the same belief $\mathscr{P}$

in period $t$. After log-linearization the previous relation becomes

$$p_t^* = (1-\delta\theta)\sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}_j}\left\{\frac{\alpha}{1-\alpha+\epsilon\alpha}\tilde{y}_{t+k} + \frac{1-\alpha}{1-\alpha+\epsilon\alpha}(\tilde{w}_{t+k} + \epsilon_{t+k}^u) + p_{t+k}\right\} \tag{34}$$

where $\epsilon_{t+k}^u$ is an exogenous process interpreted as a cost-push shock.

**Final goods producers**

The consumption good in this economy is produced by perfectly competitive firms which use intermediary goods as inputs in their CES production function:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{1-\epsilon}} \tag{35}$$

Profit maximization yields the following demand for intermediary goods:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t \tag{36}$$

where $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index.

## IV.C Mutual Fund

For the sake of simplicity, I abstract from the portfolio choice of the households and instead I assume the existence of a mutual fund which holds all the intermediary firms in this economy and issues shares with nominal price $Q_t$ which are sold in a perfectly competitive market to the household sector. The asset pricing equation of the mutual fund is given by:

$$Q_t = \delta E_t^{\mathscr{P}}\left\{\left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma}\frac{(Q_{t+1} + D_{t+1})}{1+\pi_{t+1}}\right\} \tag{37}$$

which in equilibrium will be the same as the asset pricing equation of the households.

## IV.D Central Bank

The monetary authority sets the interest rate by following a Taylor rule

$$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \epsilon_t^i \tag{38}$$

where $\epsilon_t^i$ is a stochastic process with zero mean which can be interpreted as a monetary policy shock.

## IV.E Equilibrium

Defining aggregate consumption of the two types of households as

$$C_t^C = \int_0^1 C_t^{i,C} di, \; C_t^U = \int_0^1 C_t^{i,U} di,$$

and aggregate labour as

$$N_t^C = \int_0^1 N_t^{i,C} di, \; N_t^U = \int_0^1 N_t^{i,U} di,$$

the equilibrium conditions are

$$\begin{aligned}
&\int_0^1 B_t^i \, di = 0, \\
&\int_0^1 C_t^i \, di = C_t = \mathcal{O} C_t^C + (1-\mathcal{O}) C_t^U = Y_t, \\
&\int_0^1 N_t(j) \, dj = \int_0^1 N_t^i \, di = \mathcal{O} N_t^C + (1-\mathcal{O}) N_t^U, \\
&S_t^U = 1.
\end{aligned} \tag{39}$$

First equation is the bond market clearing condition which assumes that bonds are in 0 net supply. The next two equations are the good market and labor market clearing conditions and finally the last equation requires clearing in the equity market.

On the supply side, since producers of intermediate goods are identical, the fraction of firms that will re-optimize each period $(1-\theta)$ will choose the same price, $p_t^*$. This fact combined with the definition of the aggregate price level (see Final goods producers section) results in the following aggregate law of motion for inflation:

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{40}$$

Aggregating the household decision rule (28) and combining it with the market clearing condition (39) results in the demand block of the model, the IS equation

$$\tilde{y}_t = \Delta_i\, \mathscr{O}(\delta\tilde{q}_t+(1-\delta)\tilde{d}_t)+\Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j})-\frac{\delta}{\sigma}\Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(i_{t+j}-\pi_{t+j+1}). \tag{41}$$

Equation (41) implies that not only current and future wages and real interest rates affect output today but also current stock prices. Agents do not internalize the fact that their pricing equation is determining stock prices today but instead they hold subjective beliefs about its evolution, therefore creating an equity channel effect: an increase in the equity prices today makes the consumers feel wealthier which affects aggregate consumption and output. As discussed in the previous section this stock price wealth effect appears because of the difference between actual stock prices and their fundamental value, determined by the discounted sum of dividends. If the same economy would be studied under the Euler Equation approach, then there would be no equity channel effect.

Combining the law of motion of inflation (40) with the pricing equation of the firms (34) results in the supply block of the model, the Phillips Curve equation

$$\begin{aligned}\pi_t =&\Theta_y \sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}} \tilde{y}_{t+k} + \Theta_w \sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}} \tilde{w}_{t+k} \\ &+ \sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}} u_{t+k} + (1-\theta)\delta \sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}} \pi_{t+k+1}.\end{aligned} \tag{42}$$

Log-linearization of equation (23) yields the low of motion of stock prices:

$$\tilde{q}_t = (1-\delta)E_t^{\mathscr{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathscr{P}}(\tilde{q}_{t+1}) - (i_t - E_t^{\mathscr{P}}(\pi_{t+1})). \tag{43}$$

Given optimal prices, firms supply the desired output which determines the amount of labour

$$N_t = Y_t^{1/(1-\alpha)} e^{d_t} \tag{44}$$

which is obtained by aggregating the individual production technologies. The last term captures price dispersion and is given by $d_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)$. Wages are determined by the optimality condition of the households

$$w_t = \frac{N_t^{\phi}}{Y_t^{\sigma}}. \tag{45}$$

Finally, real dividends are given by the profits of the firms

$$D_t = Y_t - w_t N_t. \tag{46}$$

## IV.F Agents' model of learning

The subjective belief system of the agents can be characterized by the probability space $(\Omega, \mathscr{P})$ with a typical element $\omega \in \Omega, \omega = \{Y_t, P_t, Q_t, D_t, W_t, u_t, \epsilon^i\}$. As in Eusepi and Preston (2018a) the belief model includes the variables (exogenous from the point of view of the individual agents) which agents need to forecast in order to make optimal consumption decision today. These are output, inflation, stock prices dividends and wages.

I assume agents believe that output, inflation, wages, dividends and equity prices follow an unobserved component model

$$\begin{aligned} z_t &= \beta_t + \zeta_t \\ \beta_t &= \rho \beta_{t-1} + \vartheta_t \end{aligned} \tag{47}$$

where $z_t = (\tilde{y}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_t, \tilde{w}_t)'$, $\rho \in (0, 1]$, $\zeta_t \sim N(0, \sigma_\zeta^2 I_5)$ and $\vartheta_t \sim N(0, \sigma_\vartheta^2 I_5)$. Agents have also knowledge of the Taylor rule that the central bank is following and uses it to forecast interest rates. As usually done in the learning literature, agents have full knowledge of the exogenous shocks.[13] The optimal filter for $E^{\mathscr{P}}(\beta_t / g^{t-1}) = \hat{\beta}_t$ is the Kalman filter and optimal updating implies the following recursion of beliefs

$$\hat{\beta}_t = \rho\, \hat{\beta}_{t-1} + \lambda(z_t - \hat{\beta}_{t-1}) + e_3\, \epsilon_t^{\beta} \tag{48}$$

where $\hat{\beta}_t = [\hat{\beta}_t^y, \hat{\beta}_t^\pi, \hat{\beta}_t^q, \hat{\beta}_t^d, \hat{\beta}_t^w]'$, $g^{t-1} = \{g_{t-1}, g_{t-2} \dots g_1\}$ denotes information up to time $t$, $e_3 = (0, 0, 1, 0, 0)'$, $\epsilon_t^{\beta}$, is a shock to stock price beliefs (sentiment shock) and $\lambda$ is the steady state Kalman gain which controls the

[13] There is no reason for agents to know these shocks and how they affect the other variables. In fact if we assume that agents do not have perfect knowledge about the exogenous shocks the dynamics of the model would be quite different. For example, if agents do not understand how monetary policy affects the economy and just observe an increase/decrease in interest rates then the IRF of output would exhibit the same hump shape response that we observe in the empirical VARs, e.g. Christiano et al. (2005). This is not related to the wealth effect so I prefer to stick to the status-quo in the literature on this issue.

speed of learning.[14] Adam et al. (2017) show that survey data regarding price expectations are captured well by an extrapolative updating equation of the form (48). Nagel and Xu (2019) call this "learning with fading memory" and links it to the theoretical biology literature which models memory decay in organisms.

It follows from (47) that the agents forecasts/beliefs about output, inflation, equity prices, dividends and wages are given by

$$E_t^{\mathscr{P}} z_{t+k} = \rho^{k-1} \hat{\beta}_{t-1} \tag{49}$$

where beliefs, $\hat{\beta}_t$, are updated each period according to (48).

Belief system (47) together with the optimal filtering rule imply that agents learn about the long-run conditional means of the variables in the economy. As argued in Eusepi et al. (2018) the belief system proposed is less restrictive than might be thought since usually the drift term drives the largest deviations from rational expectations predictions.

## IV.G Full linearized model and learning dynamics

Equilibrium equations (41), (42), (43), (38), (45) and (46) together with the the learning scheme represented by equations (48) and (49) fully characterize the dynamics of this economy. Substituting agents' subjective forecasts (49) into equilibrium conditions, results in the following system of equations

$$\begin{aligned} A\, Z_t &= B\, (\hat{\beta}_t^Z + e_3 \epsilon_t^{\beta}) + C\, \epsilon_t \\ \hat{\beta}_t^Z &= \rho\, \hat{\beta}_{t-1}^Z + \lambda (Z_{t-1} - \hat{\beta}_{t-1}^Z) \end{aligned} \tag{50}$$

where

$$Z_t = (\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t, \tilde{w}_t)',$$

$$\hat{\beta}_{t-1}^Z = (\hat{\beta}_{t-1}^y, \hat{\beta}_{t-1}^\pi, \hat{\beta}_{t-1}^q, \hat{\beta}_{t-1}^i, \hat{\beta}_{t-1}^d, \hat{\beta}_{t-1}^w)',$$

$$\epsilon_t = (u_t, \epsilon_t^i)',$$

[14] The resulting equation of belief updating is optimal given the assumption of agents observing the transitory component with a lag. In that case $\epsilon_t^q$ represents the new information about the transitory component. For further details and derivation see Appendix 6 from Adam et al. (2017)

$$e_3 = (0, 0, 1, 0, 0, 0)'.$$[15]

The stock market and the real output gap are determined simultaneously in equilibrium. Suppose that in period $t$ agents are hit by a wave of optimism which causes stock prices to increase in the same period. This in turn triggers the stock price wealth effect and increases output contemporaneously via the IS equation. Central bank reacts to this increase in output by increasing interest rates. The increase in interest rate has two effects. Firstly by the intertemporal substitution channel of monetary policy it lowers consumption and output today. Secondly it affects negatively stock prices which through the stock price wealth effect might further decrease output. If monetary policy does not react strongly enough, the increase in interest rates might not be sufficient to counteract the initial increase in stock prices which will trigger a positive revision in stock price beliefs which reinforces further the raise in stock prices. The system is self-referential in the sense of Marcet and Sargent (1989): prices affect beliefs which influence prices therefore resulting in a positive feedback loop. Policy can play an important role in breaking or further accommodating this positive feedback loop.

## IV.H Estimation of the model

I start by calibrating/estimating the parameters of the model on US quarterly data. I calibrate some of the parameters of the model using standard values found in the literature. I set the elasticity of substitution among goods, $\epsilon$, to 6 and the Frisch elasticity of labor-supply, $\phi$, to 0.75 following the recommendation from Chetty et al. (2011). From the supply side, the share of labor, $\alpha$, equals 1/3 and the probability of not being able to adjust prices, $\theta$, is set to 2/3 implying an average duration of keeping prices fixed of 3 quarters. The Taylor rule response to output-gap is set to 0.5/4 and the one for inflation to 1.5. The response of the central bank to the stock-price gap is set to 0 for now but its effect on financial stability will be discussed in a later section. As a benchmark I use three exogenous shocks that will drive the dynamics of the model: cost push shocks, $u_t$ and equity belief shocks, $\epsilon_t^{\beta}$ and monetary policy shocks, $\epsilon^i$. These shocks follow AR(1) processes:

[15] see Appendix B for details of matrices $A$, $B$ and $C$.

$$x_t = \rho_x x_{t-1} + \xi_t^x \tag{51}$$

where $x \in \{u, \epsilon^\beta, \epsilon^i\}$ and $\xi_t^x \sim \mathcal{N}(0, \sigma^x)$. Sentiment shocks are assumed to be $i.i.d, \rho_\beta = 0$.

The risk aversion parameter, $\sigma$ is set to 1 and the discount factor, $\delta$ to 0.9928. The stock ownership is set to 0.47 which represents the average stock ownership over the period 1989-2019 according to the Survey of Consumer Finances. The calibration is summarized in the following table.

| **Calibrated** | Symbol | Value |
|---|---|---|
| Discount factor | $\delta$ | 0.9928 |
| Risk aversion coef. | $\sigma$ | 1 |
| Frisch labor supply elasticity | $\frac{1}{\phi}$ | 0.75 |
| Elasticity of substitution | $\epsilon$ | 6 |
| Prob. of not adjusting price | $\theta$ | 2/3 |
| Share of labor | $\alpha$ | 0.25 |
| Taylor-rule coef. of inflation | $\phi_\pi$ | 1.5 |
| Taylor-rule coef. of output | $\phi_y$ | 0.5/4 |
| Equity Share Ownership | $1 - \mathcal{O}$ | 0.47 |

**Table I:** Calibrated Parameters

The rest of the parameters: standard deviation of cost push shock, $\sigma^u$, standard deviation of belief shock, $\sigma^\beta$, standard deviation of monetary policy shock, $\sigma^{\epsilon_i}$, persistence of cost-push shock, $\rho_u$, persistence of monetary policy shock, $\rho_{\epsilon_i}$, kalman gain coefficient, $\lambda$ and autoregressive coefficient of beliefs, $\rho$, are jointly estimated using the method of simulated moments (MSM) to match a set of eight business cycle and financial moments.

Defining $\theta = (\sigma^u, \sigma^\beta, \sigma^{\epsilon_i}, \rho_u, , \rho_{\epsilon_i}, \lambda, \rho)$ as the vector of parameters to be estimated, the MSM estimator is given by

$$\hat{\theta} = \arg\min_{\theta} \; [\hat{S} - S(\theta)]' \, \hat{\Sigma} \, [\hat{S} - S(\theta)] \tag{52}$$

where $\hat{S}$ is the vector of empirical moments to be matched, $S(\theta)$ is the model moments counterpart and $\hat{\Sigma}$ is a weighting matrix.[16] The vector of empirical

[16] I use the inverse of the estimated variance-covariance matrix of the data moments, $\hat{S}$. The latter is obtained using a Newey-West estimator and the delta method as in Adam

moments is given by

$$\hat{S} = \left[\, \hat{\sigma}(y), \hat{\sigma}(\pi), \hat{\rho}_{y,\pi}, \hat{E}(P/D), \hat{\sigma}(P/D), \hat{\rho}(P/D), \hat{\sigma}(r^e), \hat{\sigma}(r^f) \,\right]' \tag{53}$$

where

$\hat{\sigma}(y)$ : standard deviation of the business cycle component of real output,

$\hat{\sigma}(\pi)$ : standard deviation of business cycle component of inflation rate,

$\hat{\rho}_{y,\pi}$ : correlation between inflation and business cycle component of output,

$\hat{E}(P/D)$ : average of the Price Dividend ratio of stock market index,

$\hat{\sigma}(P/D)$ : standard deviation of Price Dividend ratio,

$\hat{\rho}(P/D)$: persistence of the PD ratio,

$\hat{\sigma}(r^e)$ : standard deviation of rate of return of the stock market index,

$\hat{E}(r^f)$: average real short term interest rate,

$\hat{\sigma}(r^f)$: standard deviation of real short term interest rate. [17]

The model is estimated on quarterly US data for the post-war period 1955Q1-2018Q4. The data for the business cycle statistics are obtained from the FRED database: the inflation rate is measured as the % change in the CPI for all urban consumers [CPIAUCSL], output as real GDP [GDPC1] and the fed funds rate [FEDFUNDS] is used for the short term nominal interest rate. The real interest rate is obtained by subtracting the ex-post inflation rate from the nominal short term interest rate. Data on real stock market prices and dividends are obtained from Robert Shiller webpage. Since data is monthly, quarterly variables have been obtained by selecting end of period values.[18]

Table II summarizes the estimated parameters while Table III shows the data moments and the model implied counterparts.

et al. (2016). For further details on the estimation of $\hat{\Sigma}$ please refer to online appendix of that paper.

[17] To extract the business cycle component I use the Hodrick-Prescott filter with a smoothing parameter of 1600 for quarterly data

[18] Averaging monthly variables instead of taking end of period does not change the results; the correlation between the two series is 0.997

| **Estimated** | Symbol | Value | *S.E* |
|---|---|---|---|
| Std. cost push shock | $\sigma^u$ | 0.0013 | 0.0001 |
| Std. equity belief shocks | $\sigma^{\beta_q}$ | 0.0623 | 0.0001 |
| Std. MP shocks | $\sigma^{\epsilon_i}$ | 0.0007 | 0.0001 |
| Autoregressive coef. cost push shock | $\rho_u$ | 0.9539 | 0.0285 |
| Autoregressive coef. MP shocks | $\rho_{\beta_q}$ | 0.9685 | 0.0031 |
| Kalman gain | $\lambda$ | 0.0011 | 0.0002 |
| Autoregressive coef. beliefs | $\rho$ | 0.99 | 0.0004 |

**Table II:** Estimated Parameters
*S.E are obtained by Monte Carlo simulations from 100 repetitions*

| | Symbol | Data Moment | Learning Model Moment | Learning Model $t$-ratio | RE Model |
|---|---|---|---|---|---|
| **Business Cycle** | | | | | |
| Std. dev. of output | $\sigma(y)$ | 1.45 | 1.47 | -0.39 | 0.27 |
| Std. dev. of inflation | $\sigma(\pi)$ | 0.54 | 0.45 | 1 | 0.29 |
| Correlation output/inflation | $\rho_{y,\pi}$ | 0.29 | 0.26 | 0.36 | -1 |
| **Financial Moments** | | | | | |
| Average PD ratio | E (P/ D) | 154 | 154 | -0.38 | 138 |
| Std. dev. of PD ratio | $\sigma(P/D)$ | 63 | 65 | -0.34 | 9 |
| Auto-correlation of PD ratio | $\rho(P/D)$ | 0.99 | 0.96 | 0.57 | 0.05 |
| Std. dev. of equity return (%) | $\sigma(r^e)$ | 6.02 | 6.05 | 0.04 | 9 |
| Std. dev. real risk free rate (%) | $\sigma(r^f)$ | 0.72 | 0.8 | 0.59 | 0.0017 |
| **Non Targeted moments** | | | | | |
| Average Equity Return (%) | $E(r^e)$ | 1.78 | 0.9 | 1.92 | 0.73 |
| Average real risk free rate (%) | $E(r^f)$ | 0.32 | 0.78 | -3.5 | 0.72 |
| volatility ratio stock prices/output | $\sigma(Q)/\sigma(y)$ | 6.7 | 5.2 | 2 | 23 |
| corr. Stock Prices/ output | $\rho(Q, y)$ | 0.5 | 0.45 | 0.53 | 1 |
| Consumption Wealth Effect | $dy/dQ$ | [0.03-0.2] | 0.09 | | 0 |
| corr. Survey Expect./ PD ratio | $\rho(PD_t, E_t(r^e_{t,t+4}))$ | 0.74 | 0.45 | | -1 |
| Std. dev. Expected Returns(%) | $\sigma(E_t(r^e_{t,t+4}))$ | 2.56 | 1.8 | | |

**Table III:** Model implied moments
*Data moments are computed over the period 1955Q1: 2018Q3. Moments have been computed as averages over 1000 simulations, each of 260 time periods. Subjective expectations are measured by the UBS Gallup survey for own portfolio returns for the period 1998Q2-2007Q3. t-ratios are defined as (data moment-model moment)/ S.E of data moment. The RE model moments are computed using the parameters estimated using the learning model*

The consumption wealth effect in the model economy is 0.09 meaning that for every 1% increase in stock market wealth consumption responds by

0.09%. The magnitude delivered by the estimation is well within the bounds usually found in the empirical literature. As mentioned in the introduction, Di Maggio et al. (2020) finds that consumption wealth effect from unrealized capital gains ranges between 3-20%. The model matches well business cycle moments, the volatility of financial variables and the persistence of the PD ratio. Although not targeted in the estimation, the model delivers a stock market which is 5.2 times more volatile than the real economy at the business cycle frequency and which has a 0.45 correlation coefficient with the output-gap. Figure 21 from appendix D shows the business cycle component of real GDP and the US stock market represented by the S&P 500 index. The stock market is 6.7 times more volatile than the real economy.

The model implied beliefs about stock prices are positively correlated with the PD ratio and are several orders of magnitude less volatile than actual realised returns, replicating the survey evidence. Similarly to other findings from the learning literature, the model is not able to match the equity premium since although prices are very volatile this volatility is not priced since it comes from subjective beliefs.

Using the estimated and calibrated parameters, Figure 1 presents one simulation of 260 periods from the learning model for the actual stock price and the ex-post rational price. The figure shows that while the rational price ($P^*$) does not fluctuate much, actual stock prices experience booms and busts of magnitudes of up to 100 % in absolute values. The figure can be directly compared to Figure 1 in Shiller (1981) which is the evidence of excess volatility compared to fundamentals. Figure 2 shows one random simulation for the PD ratio.

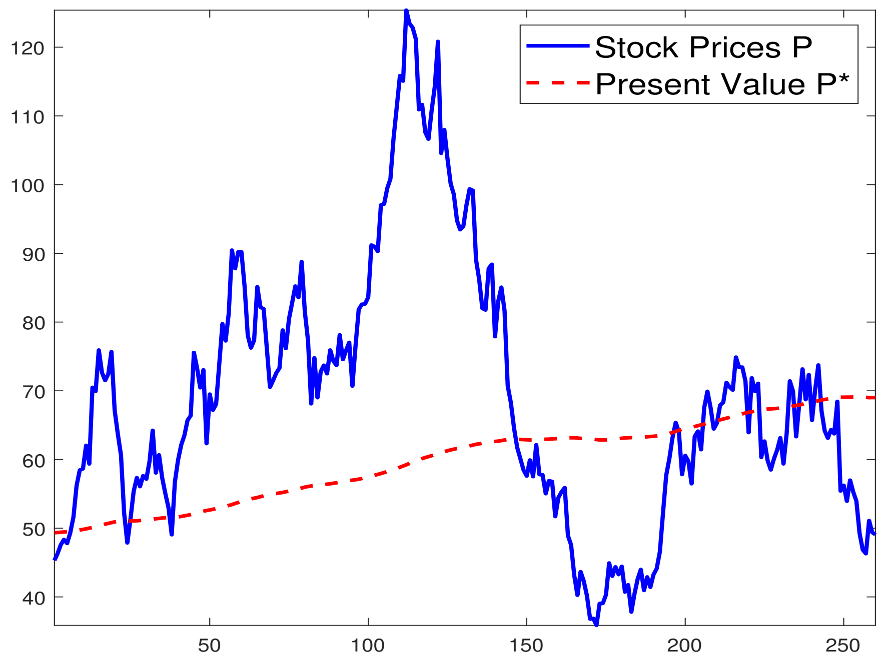



**Figure 1:** Stock Prices and Discounted Dividends
*The figure presents one simulation of 260 periods for the time series of stock prices and the corresponding present value of discounted dividends or ex-post rational price in the language of Shiller (1981). Similar to that study, it has been assumed that the end value for the rational price, $P^*$, is the sample average of the real stock price. Given that, the rest of the time series for the rational price can be backed out by the following recursion* $P_t^* = \delta P_{t+1}^* + D_t$ *where* $D_t$ *is the real dividend at time* $t$.

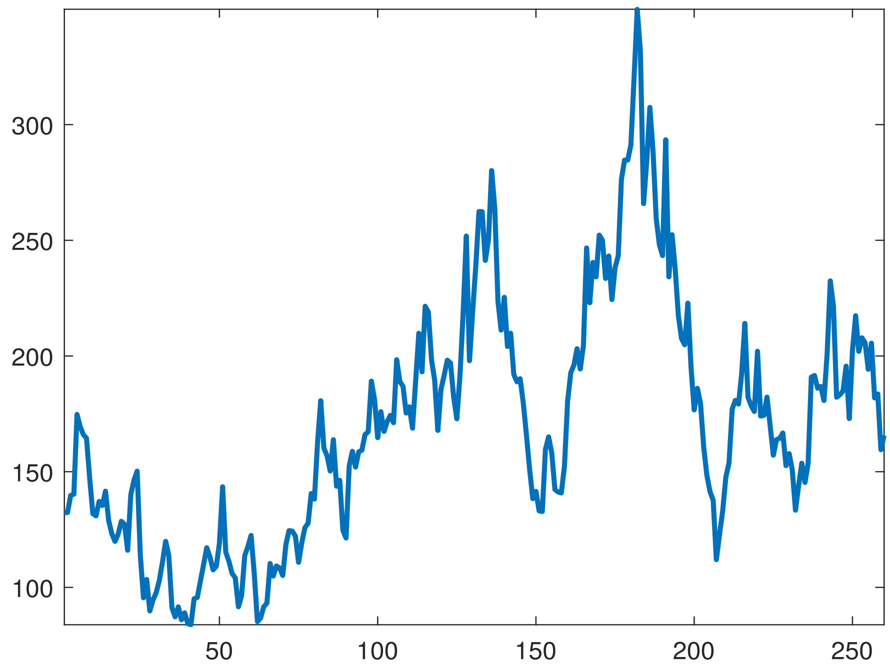



**Figure 2:** One simulation of the PD ratio

## IV.I Do Sentiment Shocks matter?

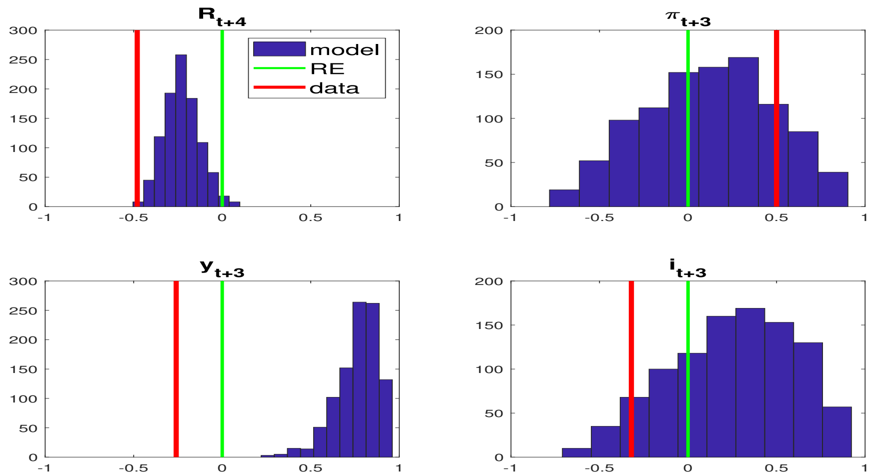
The model estimated in the previous sections matches remarkably well the dynamics (especially volatility) of the stock market and its joint behaviour with survey expectations. Responsible for this success is the combination of learning and sentiment shocks. Imperfect information has the role of creating a direct effect of stock prices on output via the consumption wealth channel while sentiment shocks have the objective of creating realistic dynamics of stock price expectations which affect affect stock prices and via the before mentioned channel, the real economy. Table IV re-estimates the model without sentiment shocks while keeping all the other ingredients as before. The model fails in matching financial and expectation moments, fact attested by the large $t$-ratios of the moments. This shows that in the present framework, sentiment shocks are crucial for replicating the dynamics of the financial market.

| **Business Cycle** | Symbol | Data Moment | Learning Model Moment | Learning Model $t$-ratio | RE Model |
|---|---|---|---|---|---|
| Std. dev. of output | $\sigma(y)$ | 1.45 | 0.62 | 5.5 | 0.27 |
| Std. dev. of inflation | $\sigma(\pi)$ | 0.54 | 0.29 | 3.4 | 0.29 |
| Correlation output/inflation | $\rho_{y,\pi}$ | 0.29 | 8.6 | -3.2 | -1 |
| **Financial Moments** | | | | | |
| Average PD ratio | E (P/ D) | 154 | 134 | 1.33 | 133 |
| Std. dev. of PD ratio | $\sigma(P/D)$ | 63 | 11 | 4.8 | 9 |
| Auto-correlation of PD ratio | $\rho(P/D)$ | 0.99 | 0.84 | 3.2 | 0.05 |
| Std. dev. of equity return (%) | $\sigma(r^e)$ | 6.02 | 0.79 | 12 | 9 |
| Std. dev. real risk free rate (%) | $\sigma(r^f)$ | 0.72 | 0.78 | 0.7 | 0.0017 |
| **Non Targeted Moments** | | | | | |
| volatility ratio stock prices/output | $\sigma(Q)/\sigma(y)$ | 6.7 | 1.05 | 7.2 | 23 |
| corr. Stock Prices/ output | $\rho(Q,y)$ | 0.5 | 0.99 | 3.74 | 1 |
| Average Equity Return (%) | $E(r^e)$ | 1.78 | 0.76 | 2.23 | 0.73 |
| Average real risk free rate (%) | $E(r^f)$ | 0.32 | 0.75 | 3.4 | 0.72 |
| Consumption Wealth Effect | $dy/dQ$ | [0.03-0.2] | 0.09 | | 0 |
| Std. dev. Expected Returns(%) | $\sigma(E_t(r^e_{t,t+4}))$ | 2.56 | 1.72 | | |
| corr. Survey Expect./ PD ratio | $\rho(PD_t, E_t(r^e_{t,t+4}))$ | 0.74 | -0.99 | | -1 |

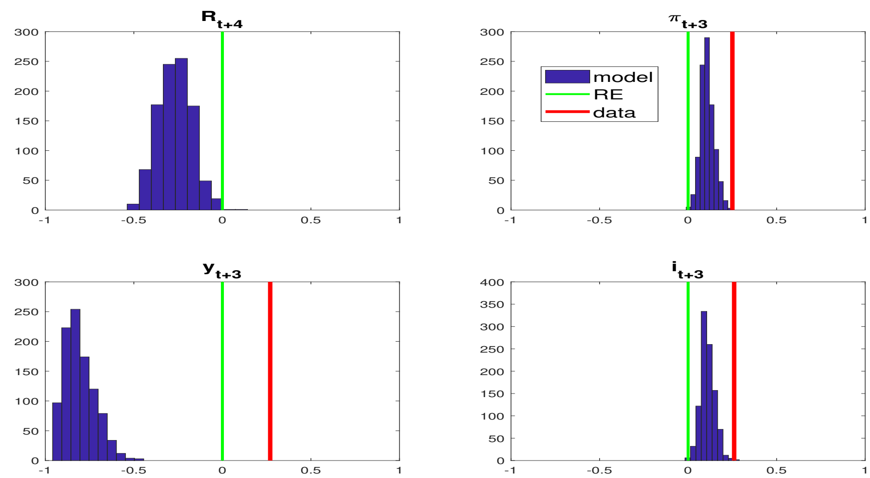
**Table IV:** Model implied moments excluding Sentiment Shocks.
*Moments have been computed as averages over 10.000 simulations, each of 260 time periods. Subjective expectations are measured by the UBS Gallup survey for own portfolio returns. Survey data covers the period 1998Q1-2007Q3. Re-estimated parameter vector:* $\hat{\theta} = (\sigma^u, \sigma^{\epsilon_i}, \rho_u, \rho_{\epsilon_i}, \lambda, \rho) = (0.0009, 0.0076, 0.9864, 0.0684, 0.9976, 0.0154)$

## IV.J Survey Expectations vs Model Beliefs

Coibion and Gorodnichenko (2012, 2015) bring evidence in favour of information rigidity in expectation formation and show that aggregate forecasts of inflation and other macroeconomic variables exhibit under-reaction described by a positive relation between forecast revisions and forecast errors. Let $FR^x_{t,h} = x_{t+h/t} - x_{t+h/t-1}$ and $FE^x_{t,h} = x_{t+h} - x_{t+h/t}$ denote the forecast revision and forecast error for variable $x$ at time $t$ and horizon $h$.




**(a)** $\rho(PD_t, FE^x_{t,h})$




**(b)** $\rho(FE^x_{t,h}, FR^x_{t,h})$

**Figure 3:** Forecast Error Predictability
*The figure shows the correlation coefficient for forecast errors with the PD ratio and the revision in beliefs for 1 year ahead expected capital gains and three quarters ahead inflation, output growth and interest rates. Expected capital gains are measured by the US Gallup survey (own portfolio) which covers the period 1998Q1-2007Q3. Similar to Coibion and Gorodnichenko (2015) the 3 quarters ahead also includes the nowcast. The survey data for the macroeconomic variables comes from SPF and covers the period 1981Q1-2016Q4. The series for the revision of beliefs is not available for the US Gallup survey. The model statistics are computed over 1000 simulations each of 260 time periods*

Using the survey of professional forecasters (SPF) for inflation, output growth and interest rates and the US Gallup survey for expected capital gains, Figure 3 presents the correlation coefficients between the forecast error and PD ratio and between forecast errors and revisions in beliefs for both model and data. Under RE both of these coefficients would be 0.

Panel (a) shows the correlation coefficient between the PD ratio and the forecast errors for each of the four variables considered. When stock prices are high agents tend to systematically over-predict future capital gains, fact reproduced by the model (top-left figure). For the forecast errors of inflation and interest rates the model also produces reasonable ranges for the correlation coefficients. Nevertheless, for output growth the model generates a positive relation while the data suggests the opposite. This result is due to the fact that in the model the agents learn about the output-gap while in survey data agents predict directly output growth.

Following Coibion and Gorodnichenko (2015) panel (b) presents the correlation between the forecast errors and revision in beliefs. The model replicates the positive (under reaction) correlation for inflation and interest rates although the magnitude is smaller in the model. For output, the model delivers again a wrong sign of the coefficient for the reason outlined above. It is useful to compare these results to the ones in Winkler (2019). The model presented there is able to reproduce the patterns of forecast errors for output and other variables but fails in the inflation one. The model outlined here delivers the opposite result: it matches well inflation and other variables and fails regarding the subjective output forecast error dynamics. The mechanism through which stock prices affect the real economy is nevertheless different (supply vs demand) and that could explain at least partially the difference in results.

# V Stock Price Targeting and Macroeconomic Stability

Stock price booms and busts driven by market sentiment affect the real economy through a consumption wealth effect. Since these wealth effects are reflected in output and inflation dynamics, monetary policy could in principle, by responding to just two macroeconomic variables influence or eliminate the non-fundamental effect of stock prices on the real economy. Compared to the RE assumption, in the current economic environment, agents have imperfect information about the structural relations between the real economy and stock prices. To this end, the Taylor rule is augmented with a lagged response to stock prices: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$.[19] To investigate the effect of monetary policy on the wealth effect, figure V plots the magnitude of the wealth effect when the central bank targets only one variable at the time: inflation, output-gap or stock prices.

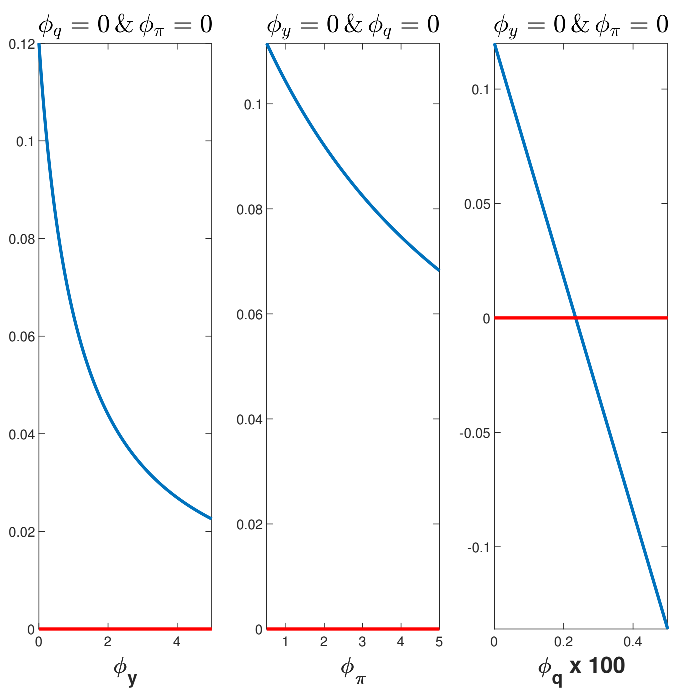



**Figure 4:** Stock Price Wealth Effects and Monetary Policy
*Each panel presents the magnitude of the wealth effects as a function of the central bank response to output, inflation and stock prices while keeping the other coefficients fixed at 0. The Taylor rule is of the following type:* $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$

To the extent that the central bank would want to eliminate the effects of the fluctuations of stock prices on output then the only possibility under this simple Taylor rule would be to include an explicit response to stock prices into the monetary policy reaction function. Figure V shows that responding

[19] Appendix D presents the results with a contemporaneous response to stock prices and none of the qualitative conclusions change.

stronger to inflation or output has a smaller effect on the stock price wealth effect than by responding directly to asset prices. In fact no matter how strong the central bank responds to inflation or output it would not manage to totally neutralize the effects of stock prices on output. The reason for this dynamics lies on the fact that agents do not internalize the relation between stock prices and output and as a consequence, the extra-volatility of stock prices with respect to the real economy would not be internalized if the central bank responds just to output and inflation.

Responding to stock prices might on the other hand introduce additional volatility in the economy which might destabilize the system. Figure V shows that this is not necessarily the case for small enough [20] reactions to stock prices and even more when there is in place a reaction to output [21].

[20] The maximum considered stock price reaction coefficient is 0.05 implying an increase of 1% in the interest rate as a result of a 20% increase in stock prices

[21] This is in line with the findings of Nisticò (2012); Airaudo et al. (2015) and Airaudo (2016)

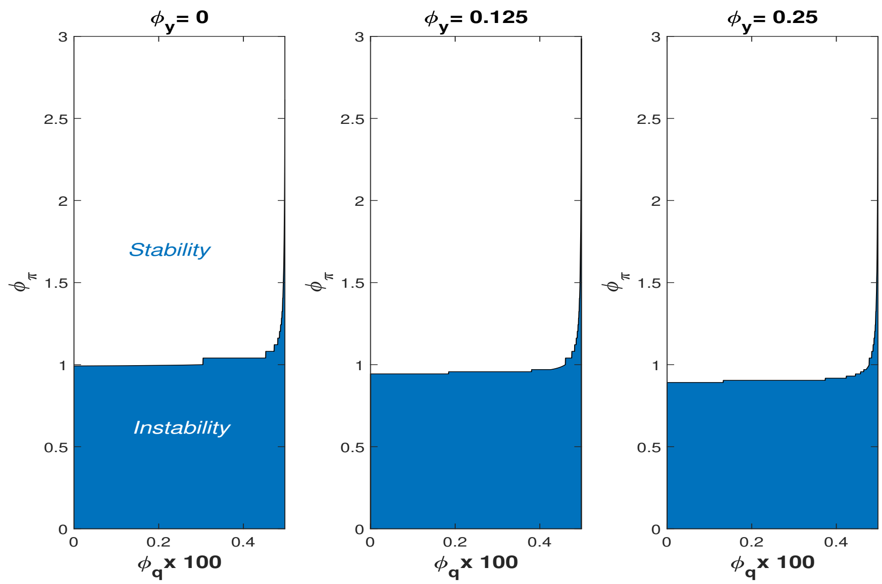



**Figure 5:** E-Stability and Monetary Policy
*The figure present the stability (white) and instability (blue) regions for different combinations of Taylor rule coefficients. Each panel plots the e-stability regions for different combinations of inflation (Y axis) and stock price (X axis) reaction coefficients while keeping the output reaction fixed. The Taylor rule is of the following type:* $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$. *The stability of the system is given by the eigenvalues of the matrix* $A^{-1}B$. *Following Evans and Honkapohja (2012), the dynamical system is e-stable if the largest eigenvalue of the previous matrix has the real part smaller than 1.*

To better understand the dynamics of the stock market and its effects on the economy, figure 6 reports the IRFs from a 1% shock in stock prices in the RE model and a 1% shock in the beliefs of agents about the stock prices in the imperfect information model. These two shocks would have a similar effect in a RE model but in the learning model, where agents hold subjective beliefs about the stock market and where these beliefs have a high degree of persistence, the effects of these shocks have very different implications for the dynamics of the stock-market and the real economy.

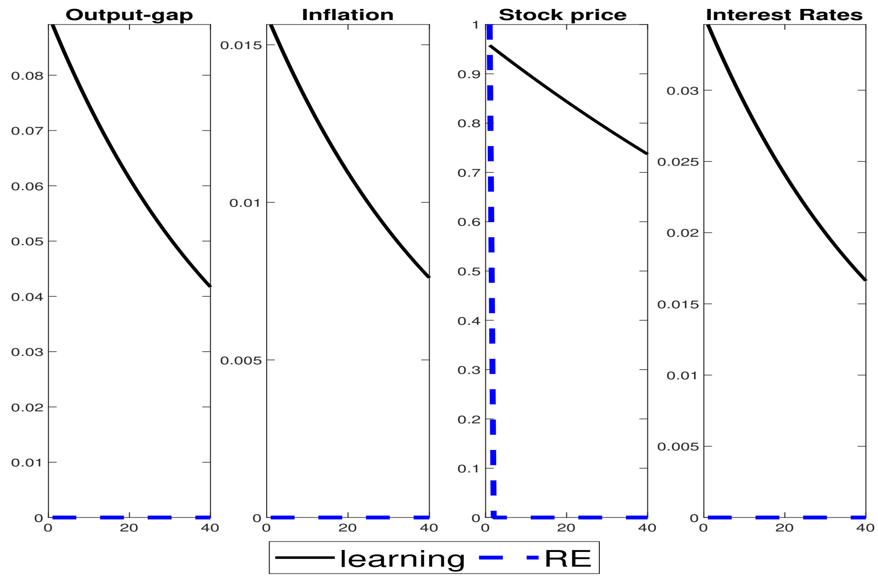



**Figure 6:** Stock Price vs Belief Shocks
*The figure presents the IRF of selected endogenous variables with respect to stock price shocks in RE and shocks to beliefs under the learning framework. Both shocks have an impact magnitude of 1% and persistence 0.*

The equity shock increases stock prices contemporaneously and then returns to 0 without affecting any other variable. Belief or sentiment shocks, on the other hand, although being *i.i.d*, have a persistent effect on stock prices. This is because of the persistence of beliefs which translates into further increases in prices, therefore justifying the initial beliefs. This rise in stock prices is then transmitted through wealth effects on the rest of the economy. Output, inflation and interest rates closely follow the dynamics of the stock market. Although the central bank does not target stock-prices directly, the interest rates rise as a response of increases in inflation and output-gap. Therefore, waves of optimism/ pessimism can affect the real economy without any fundamental change in the economy.

Until now it has been assumed that monetary policy does not respond explicitly to asset prices. Since sentiment shocks can impact the real economy through wealth effects and since monetary policy is rather effective in influencing the magnitude of this effect (see figure V) it is natural to ask how would the response of the economy, in the face of sentiment shocks, change if monetary policy would include a dedicated response to asset prices. Figure 7 answers this question by plotting the IRFs to sentiment shocks for different stock price reaction coefficients.

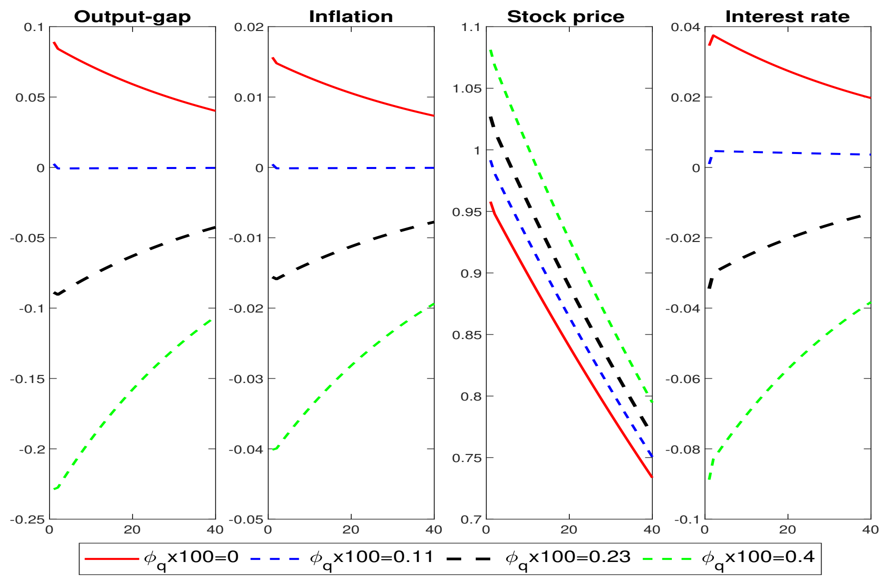



**Figure 7:** IRFs to Sentiment Shocks
*The figure presents the IRF to a 1 % i.i.d shock in equity price beliefs for different reaction coefficients to stock prices. The Taylor rule is of the following form:* $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$

The red line denotes the response in the case in which the central bank does not target explicitly stock prices. As the response of the monetary authority to stock prices increases, the effect of sentiment shocks on the economy decreases and is even reversed for large enough coefficients. The black dotted line shows that the central bank can approximately neutralize the effects of sentiment shocks on the economy by picking a reaction coefficient to stock prices around 0.0023. This response implies that the central bank commits to raise interest rates by 23 b.p. for every 100% increase in stock prices from their steady state value. Reacting too strongly to stock prices (green line) has the effect of reversing the effects of sentiment shocks and causing a recession.

A central bank reacting to stock prices and *communicating clearly this policy* can disrupt the effects of sentiment shocks on the economy by influencing agents expectations on stock prices and interest rates: agents take into account a possible rate increase in the case of positive stock price beliefs, which given the persistence of beliefs they internalize as a persistent interest rate increase, therefore adjusting the intertemporal consumption decision which counteracts the positive effect on the real economy of the initial optimism wave. The inclusion of stock price targeting in the Taylor rule does not, necessarily, create additional interest volatility in the economy since the sole fact that the central bank *threatens* to not tolerate large stock price swings is enough to influence agents sentiment and consequently the booms and busts would not materialize in the first place.

## V.A Monetary Policy and Welfare

What is the appropriate response of monetary policy in the face of real effects of swings in the stock market and does an explicit response to stock-prices improve macroeconomic stability or welfare? The current section tries to answer these questions using the model developed in this paper.

To analyze what are the implications of stock-price targeting on macroeconomic stability and welfare one would need to specify a welfare criterion under which different monetary policy rules can be examined. The literature on monetary theory[22] uses a second order approximation of the lifetime utility of the representative agent as a criterion of welfare. The resulting criterion, average welfare loss per period, is an increasing function of the volatility of output and inflation. In the current framework stock prices play an important role as a source of output and inflation variation and therefore the standard welfare function does not apply here and I proceed in deriving the analytical form of the welfare function describing the economy presented in section IV.

I assume the central bank maximizes welfare under the equilibrium probability measure and not under the subjective one held by the agents. Therefore the central bank (social planner) assumes a paternalistic objective for the agents.

**Lemma 3.** *Up to a second order approximation and ignoring terms independent of policy the expected utility in the TANK model with homogeneous*

[22]see Rotemberg and Woodford (1999) and Galí (2015)

*imperfect information is proportional to* $\sum_{t=0}^{\infty} -\mathscr{L}$ *where*

$$\mathscr{L} = \frac{\epsilon}{\psi} var(\pi_t) + \Upsilon_1 \, var(\tilde{y}_t) + \Upsilon_2 \, var(\tilde{q}_t) + \Upsilon_3 \, E(\tilde{y}_t \tilde{q}_t) \tag{54}$$

*is the average expected welfare loss per period measured as a fraction of steady state consumption.*

*Proof.* See Appendix 3 □

Lemma 3 shows that in the current framework the welfare of the agents depends on top of the variances of inflation and output also on the variability of stock prices and the correlation of stock prices with output.

The learning literature has also adopted this approach[23] although, compared to RE, here we are dealing with two types of beliefs: subjective vs model or objective beliefs. Adopting a criterion of the form (54) implies that we are assuming that the model or objective beliefs are what matter for the overall welfare of the agents. This mustn't necessarily be the case[24] but for now I will also adopt this assumption which is standard in the literature.

Figure 8 presents the implied welfare loss for different policy parameters in the case the economy is hit by cost-push, monetary policy and sentiment shocks. Each line in the figure corresponds to a different output coefficient in the Taylor rule for different values of the stock price reaction coefficients. Notice first that no matter the response to output, including a reaction to stock prices is always optimal but the benefit decreases the higher the reaction to output. Nevertheless, reacting too strongly to output variations decreases welfare since it worsens the output-inflation trade-off in the case of cost-push shocks. [25]

[23]see Eusepi and Preston (2018b)

[24]See Kahneman et al. (1997). Models with subjective beliefs potentially open the door to the interesting exploration of the importance of subjective beliefs to welfare and on the role of *remembered* utility. This line of research has been almost exclusively left to psychology or to the applied and experimental economists. One notable exception is Caines and Winkler (2018) which evaluates the welfare implications of monetary policy rules both using objective and subjective expectations and find that the implications for optimal monetary policy differ depending on which measure of expectations one chooses.

[25]A cost-push shock has the effect of increasing inflation contemporaneously. If the central bank responds strongly enough to inflation deviations, interest rate rise and output gap decreases which counteracts the initial increase in inflation. If at the same time the monetary authority reacts to output gap deviations then the interest rate will not increase as much and the initial impact of the cost push shock will dominate. The overall impact

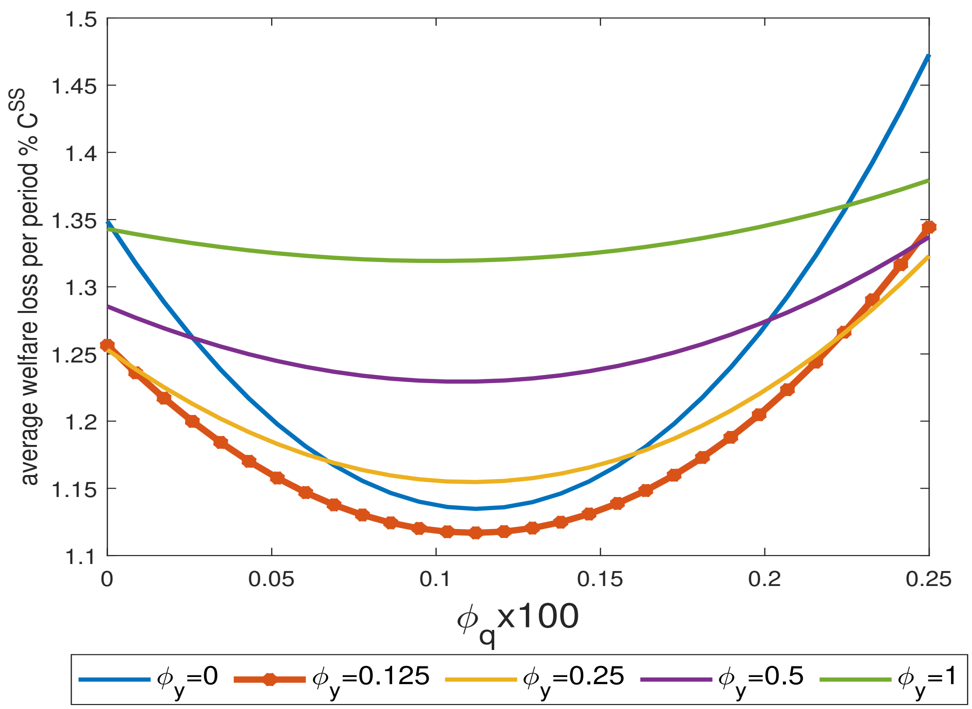



**Figure 8:** Welfare Maps
*The figure shows the average welfare loss per period as defined in equation (54) for different combinations of Taylor rule coefficients for output and stock prices while keeping the inflation reaction coefficient fixed at 1.5. Welfare losses have been computed as averages over 200 independent simulations, each one including 260 time periods using the estimated parameters from section IV.H*

The shape of the welfare loss as a function of the stock price targeting parameter has a $U$ shape: reacting too strongly to stock prices can in fact decrease welfare by introducing additional volatility in the economy. For the baseline parametrization of the Taylor rule ($\phi_y = 0.125$, red line in figure 8) including a dedicated coefficient to stock prices of 0.12% in the Taylor rule increases welfare by 0.15% on average per period. Figure 8 from appendix D, repeats this exercise for the case in which the economy is solely hit by sentiment shocks and shows that the 0.15% welfare gain comes from counter-

would be a less negative output gap and higher inflation. The optimal policy in standard New-Keynesian models (see for example Galí (2015) chapter 5) is to accommodate the cost-push shock by allowing a negative output-gap.

acting the inefficiencies arising from the waves of optimism/pessimism about capital gains.

Figure V.A decomposes the sources of the welfare gains by plotting the standard deviations of output, inflation and stock prices together with the co-movement between stock prices and output. Including a dedicated reaction to stock prices reduces both the volatility of inflation and output up to a certain point displaying the same $U$ shape dynamics as the welfare losses. Stock price volatility increases monotonically but the magnitude is relatively small. The last panel shows that responding to stock prices breaks the link between stock prices and output by reducing their co-movement generated by the stock price wealth effect.

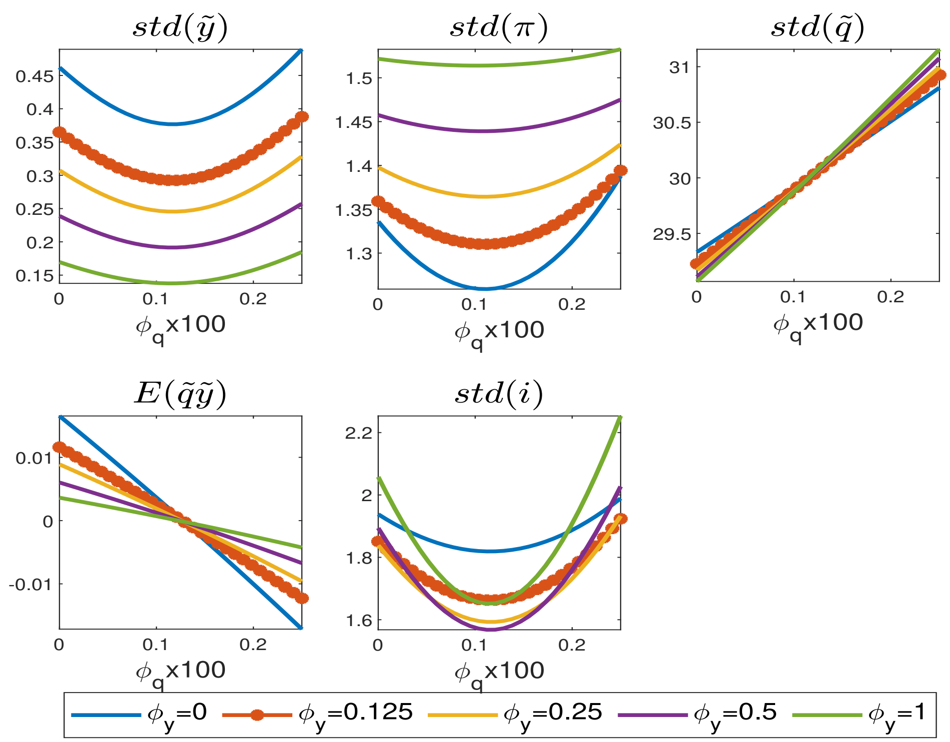



**Figure 9:** Influence of Monetary policy on Macroeconomic Volatility
*Implied volatility of output, inflation, stock prices, co-movement of output with stock prices and interest rates for different combinations of policy parameters. The Taylor rule is specified as* $i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + \phi_q \tilde{q}_{t-1}$.

## V.B The Case for Transparency

The analysis from the previous section regarding the welfare effects of responding explicitly to stock prices has been performed under the assumption that agents fully understand that monetary policy is responding to stock prices. Therefore, when forming expectations of interest rates agents would have internalized that given the current level of the stock market and their beliefs, the central bank would adjust accordingly the level of interest rates accordingly to the Taylor rule followed. In reality, the Fed does not react explicitly to stock prices although there is ample evidence that it does intervene when needed. As a result it is reasonable to assume that agents might not internalize this reaction of the monetary authority to stock prices. To investigate how would the previous results change I do the following exercise: assume (as before) that agents fully understand that the central bank responds to inflation and output but do not take into account any other reaction. In reality the central bank is responding to stock prices deviations even though agents do not internalize this fact. First I will consider the symmetric response to contemporaneous stock prices. For clarity, the following table summarizes the information set of the central bank and agents under transparency vs non-transparency.

| | Transparency | Non-Transparency |
|---|---|---|
| Central Bank | $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q \tilde{q}_{t-1}$ | $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q \tilde{q}_{t-1}$ |
| Agents | $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q \tilde{q}_{t-1}$ | $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t$ |

**Table V:** Information set under Transparency vs Non-Transparency
*The first line of the table shows the actions of the central bank while the second line shows how agents understand the central bank is responding to macroeconomic variables which is used to forecast future interest rates.*

Figure 10 shows the welfare losses arising in the case of transparency, where agents fully understand the whole monetary policy reaction function including the response to stock prices and the non-transparency case in which agents do not take into account the last policy response. The benefits of responding to stock prices when agents do not internalize the reaction (red line) are at most limited. This confirms that the management of agents expectations about capital gains is crucial for successfully counteracting the inefficiencies arising from the booms and busts of market sentiment.

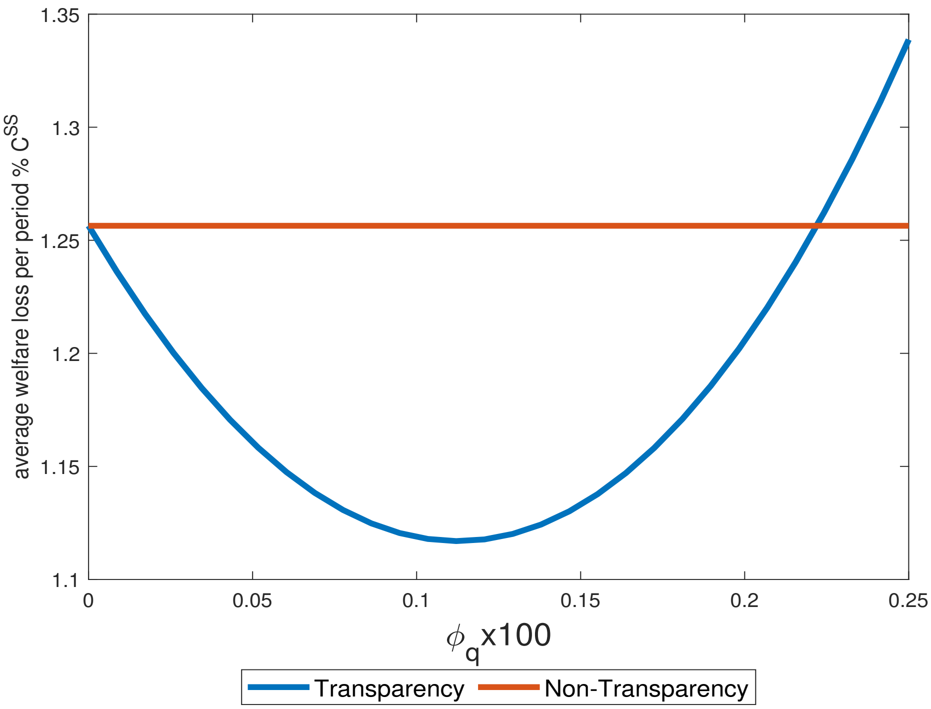



**Figure 10:** (Non) Transparency of stock price targeting
*Transparency implies that agents internalize the reaction to stock prices while in the non-transparency scenario agents only take into account the response to output and inflation in the Taylor Rule. The latter is specified as* $i_t = 1.5\,\pi_t + 0.125\,\tilde{y}_t + \phi_q \tilde{q}_{t-1}$.

Until now it has been assumed that under non-transparency the central bank reacts symmetrically to asset prices. Nevertheless, the empirical evidence shows that the Fed intervenes mostly in bad times while booms in stock prices are left to their own. Since stock price wealth effects appear both in booms and busts I will analyse the following two monetary policy rules. The first one implies that the central bank reacts only when lagged stock price deviations drop under a certain threshold, $Q^-$, which is the standard Fed put documented in the literature. In the second rule, in addition to reacting in bad times the central bank also reacts when the stock market increases above a specified threshold, $Q^+$, which I label the put-call rule. Specifically

$$i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}\mathbb{1}_{\tilde{q}_{t-1}<Q^-} \qquad \text{(Fed put)}$$

$$i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}(\mathbb{1}_{\tilde{q}_{t-1}<Q^-} + \mathbb{1}_{\tilde{q}_t>Q^+}) \qquad \text{(Fed put-call)}$$

where $\mathbb{1}_{\tilde{q}_t<Q^-}$ is an indicator functions taking a value of one if the condition $\tilde{q}_t < Q^-$ is satisfied and $Q^- < 0$ and $Q^+ > 0$.

Figure 11 shows that reacting both in bad and good times (Fed put-call) is more efficient that reacting just in bad times (Fed put) for all combinations of threshold variables.

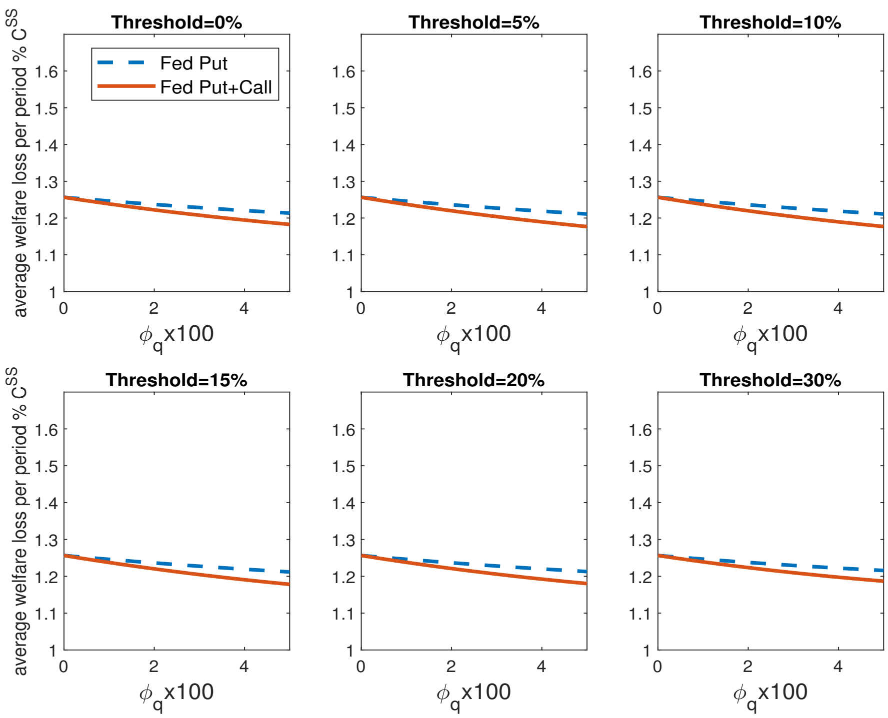



**Figure 11:** Welfare Implications of Fed put and call under Non-Transparency

*It has been assumed that $Q^- = -Q^+$ for the Fed put-call rule. Non-transparency implies that although the central bank is reacting to stock prices using either of the two nonlinear rules considered, agents do not internalize this fact and form beliefs regarding interest rates using the systematic component of the Taylor rule concerning only output and inflation: $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t$. The Fed put is specified as $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}\mathbb{1}_{\tilde{q}_{t-1}<Q^-}$ while the Fed put-call is $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}(\mathbb{1}_{\tilde{q}_{t-1}<Q^-} + \mathbb{1}_{\tilde{q}_t>Q^+})$.*

Notice that the x-axis for the stock price reaction coefficient is ten-times larger than Figure 10 which shows that if agents do not incorporate into their beliefs the reaction of monetary policy to stock prices the central bank has to respond more strongly overall in order to reduce the effects of wealth effects on aggregate demand. Moreover, the welfare implications between the two rules is minimal for small enough stock price reaction coefficients (of the order 0.005 or smaller). Furthermore, Figure 11 reveals that a threshold of around 10% attains the highest efficiency gains for both policies considered.[26]

Figure 12 plots the welfare losses implied by the policy of responding symmetrically and transparently to stock prices and the two nonlinear policies, namely the Fed put and the Fed put-call, both with a threshold of 10%. Two conclusions are apparent from this figure. Firstly, if the central bank does not want to announce explicitly and transparently that it takes into account stock prices when setting interest rates then reacting both is booms and busts is welfare improving compared to the policy of reacting only in bad times (Fed put). Secondly, announcing transparently the reaction to stock prices in such a way that agents internalize into their expectations this policy is several orders of magnitude better than the first policy.

[26] See also Figure 20 from appendix D

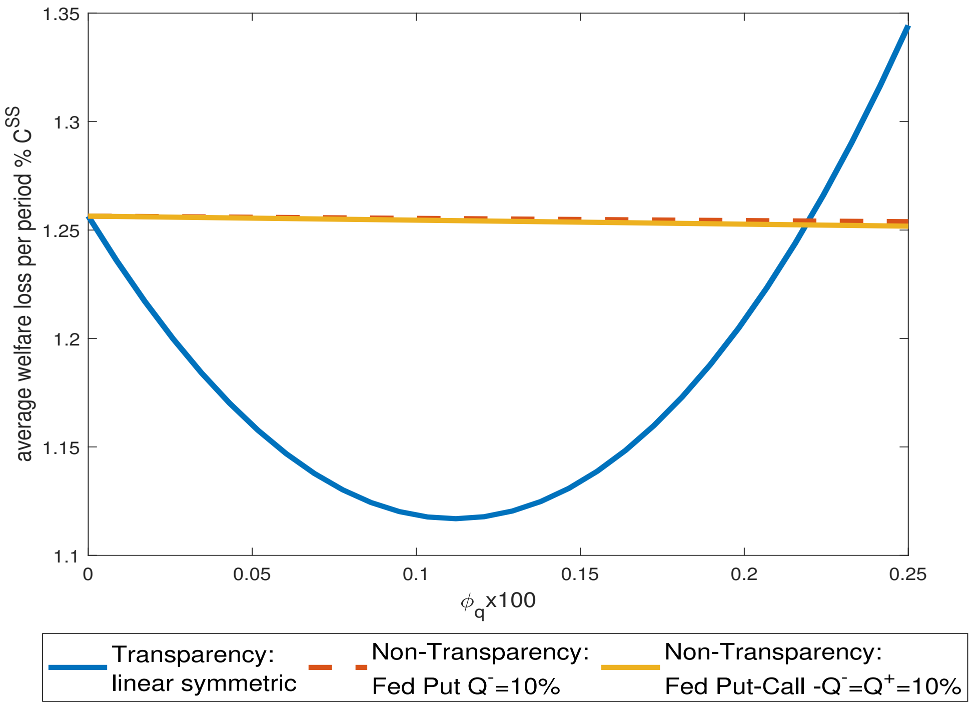



**Figure 12:** Welfare Implications of Transparency vs Fed put-call under Non-Transparency
*See the details from Figure 12 and Table V.*

# VI Conclusions

The interaction between monetary policy and stock prices has been a long standing subject both among academic economists and market professionals. Recent evidence suggests that the Fed is responding to stock prices and the main channel that it considers as important as a link between financial markets and the real economy is the consumption wealth effect. The empirical literature has also found that this effect can have magnitudes ranging from 3 to 20%. Given this evidence I first show in a simple endowment economy show how stock prices can influence the consumption decision of the agents when the information they posses about the structure of the economy is imperfect. Departures of stock prices from the expected discounted sum of dividends give rise to a consumption wealth effect through which stock prices influence aggregate demand. The result links directly the volatility puzzle with stock price wealth effects.

The mechanism is embedded in a TANK model with homogeneous imperfect information where agents differ only regarding their participation in the equity market. I estimate the model on US data using two standard shocks, cost push and monetary policy and a sentiment shock which affects agents' beliefs about future capital gains. Quantitatively, the model does remarkably well in matching the financial market and the dynamics of survey expectations while producing a smooth business cycle.

Given the the estimated model I ask weather responding to stock price can improve macroeconomic stability and welfare. By targeting stock prices the monetary authority does not introduce additional volatility in the economy and furthermore is especially efficient in counteracting the effects that sentiment swings have on the real economy via the consumption wealth effect. I show that if the central bank announces explicitly and transparently a 12bp increase in interest rates for every 100% increase in the stock market from the long run average welfare improves by 0.15% on average per quarter. If on the contrary, the central bank reacts to stock prices in a non-transparent manner, the gains are limited.

Central banks can increase macroeconomic stability and welfare by responding explicitly to stock prices and can counteract the effects of asset price movements on the real economy by shutting down the wealth effect channel of stock prices. The analysis is limited to standard monetary policy instruments, e.g interest rate target and further research is needed in order to understand the effects and interaction of non-conventional instruments like forward guidance and quantitative easing which have become the norm in the last years.

# References


Adam, K. and Marcet, A. (2011). Internal rationality, imperfect market knowledge and asset prices. *Journal of Economic Theory*, 146(3):1224–1252.

Adam, K., Marcet, A., and Beutel, J. (2017). Stock price booms and expected capital gains. *American Economic Review*, 107(8):2352–2408.

Adam, K., Marcet, A., and Nicolini, J. P. (2016). Stock market volatility and learning. *The Journal of Finance*, 71(1):33–82.

Airaudo, M. (2016). Monetary policy and asset prices with infinite horizon learning. Technical report, Tech. rep.

Airaudo, M., Nisticò, S., and Zanna, L.-f. (2015). Learning, monetary policy, and asset prices. *Journal of Money, Credit and Banking*, 47(7):1273–1307.

Bask, M. (2012). Asset price misalignments and monetary policy. *International Journal of Finance & Economics*, 17(3):221–241.

Bernanke, B. and Gertler, M. (2000). Monetary policy and asset price volatility. Technical report, National bureau of economic research.

Bernanke, B. S. and Gertler, M. (2001). Should central banks respond to movements in asset prices? *american economic review*, 91(2):253–257.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393.

Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of economic theory*, 140(1):162–196.

Bullard, J. B., Schaling, E., et al. (2002). Why the fed should ignore the stock market. *Review-Federal Reserve Bank of Saint Louis*, 84(2):35–42.

Caines, C. and Winkler, F. (2018). Asset price learning and optimal monetary policy. *Available at SSRN 3070761.*

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398.

Carlstrom, C. T. and Fuerst, T. S. (2007). Asset prices, nominal rigidities, and monetary policy. *Review of Economic Dynamics*, 10(2):256–275.

Case, K. E., Quigley, J. M., and Shiller, R. J. (2005). Comparing wealth effects: the stock market versus the housing market. *Advances in macroeconomics*, 5(1).

Case, K. E., Quigley, J. M., and Shiller, R. J. (2011). Wealth effects revisited 1978-2009. Technical report, National Bureau of Economic Research.

Cecchetti, S. G., Genberg, H., Lipsky, J., and Wadhwani, S. (2000). *Asset prices and central bank policy*. Centre for Economic Policy Research.

Chetty, R., Guren, A., Manoli, D., and Weber, A. (2011). Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review*, 101(3):471–75.

Chodorow-Reich, G., Nenov, P. T., and Simsek, A. (2019). Stock market wealth and the real economy: A local labor market approach. Technical report, National Bureau of Economic Research.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45.

Cieslak, A. and Vissing-Jorgensen, A. (2020). The economics of the fed put. *Unpublished Manuscript, University of California at Berkeley.*

Coibion, O. and Gorodnichenko, Y. (2012). What can survey forecasts tell us about information rigidities? *Journal of Political Economy*, 120(1):116–159.

Coibion, O. and Gorodnichenko, Y. (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, 105(8):2644–78.

Di Maggio, M., Kermani, A., and Majlesi, K. (2020). Stock market returns and consumption. *The Journal of Finance*, 75(6):3175–3219.

Eusepi, S., Giannoni, M. P., and Preston, B. (2018). Some implications of learning for price stability. *European Economic Review*, 106:1 – 20.

Eusepi, S. and Preston, B. (2018a). Fiscal foundations of inflation: imperfect knowledge. *American Economic Review*, 108(9):2551–89.

Eusepi, S. and Preston, B. (2018b). The science of monetary policy: An imperfect knowledge perspective. *Journal of Economic Literature*, 56(1):3–59.

Evans, G. W. and Honkapohja, S. (2012). *Learning and expectations in macroeconomics*. Princeton University Press.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.

Galí, J., Giusti, G., Noussair, C. N., et al. (2020). Monetary policy and asset price bubbles: a laboratory experiment. Technical report.

Greenwood, R. and Shleifer, A. (2014). Expectations of returns and expected returns. *The Review of Financial Studies*, 27(3):714–746.

Kahneman, D., Wakker, P. P., and Sarin, R. (1997). Back to bentham? explorations of experienced utility. *The quarterly journal of economics*, 112(2):375–406.

Marcet, A. and Sargent, T. J. (1989). Convergence of least squares learning mechanisms in self-referential linear stochastic models. *Journal of Economic theory*, 48(2):337–368.

Nagel, S. and Xu, Z. (2019). Asset pricing with fading memory. Technical report, National Bureau of Economic Research.

Nisticò, S. (2012). Monetary policy and stock-price dynamics in a dsge framework. *Journal of Macroeconomics*, 34(1):126–146.

Preston, B. (2005). Learning about monetary policy rules when long-horizon expectations matter. *International Journal of Central Banking*.

Rotemberg, J. J. and Woodford, M. (1999). Interest rate rules in an estimated sticky price model. In *Monetary policy rules*, pages 57–126. University of Chicago Press.

Shiller, R. (1981). Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review*, 71(3):421–36.

Winkler, F. (2019). The role of learning for asset prices and business cycles. *Journal of Monetary Economics*.

# Appendices

## Appendix A Consistency of one-step ahead forecasts

Equation (4) from section III holds with equality from the perspective of the agent only under the Rational Expectation assumption. Under imperfect information agent will not have knowledge of the fact that he will be the marginal agent forever and therefore cannot substitute with equality the FOC 2 in the budget constraint to obtain equation 4. Letting, $\lambda_t$ denote the lagrange multiplier associeted with FOC with respect to stock prices and assuming the agent knows that he will be the marginal agent in the bond market (equation (1) holding with equality) the intertemporal budget constraint becomes

$$\frac{\mathscr{W}_t^i}{P_t} = E_t^{\mathscr{P}_i} \sum_{j=0}^{\infty} \delta^j \left( \frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} C_{t+j}^i + A_t. \tag{55}$$

where

$$A_t = \sum_{j=1}^{\infty} \delta^j E_t^{\mathscr{P}} E_{t+1}^{\mathscr{P}} \dots E_{t+j-1}^{\mathscr{P}} \left( \frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} \frac{\lambda_{t+j}}{\prod_{s=0}^{j} (1 + \pi_{t+s})} \tag{56}$$

is the term collecting all the Lagrange multipliers $\lambda$ which take into account that the agent does not know that he will be marginal in all future periods. In steady state $\lambda = 0$ and $A = 0$. Specifically

$$\lambda_{t+1} = \delta \left[ E_t^{\mathscr{P}_{mg}} \left( \left( \frac{C_{t+2}^{mg}}{C_{t+1}^{mg}} \right)^{-\sigma} (P_{t+2} + D_{t+2}) \right) - E_t^{\mathscr{P}_i} \left( \left( \frac{C_{t+2}^{i}}{C_{t+1}^{i}} \right)^{-\sigma} (P_{t+2} + D_{t+2}) \right) \right] S_{t+1}$$

is the perceived error of agent $i$ with respect to the marginal agent valuation. If $A_t$ is sufficiently small up to a first order approximation then we can describe accurately the optimal consumption decision of the agent by equation 15 as if the agent knows he is the marginal agent. I call this the *Average Marginal Agent* assumption.

**Average Marginal Agent Assumption:** *up to a first order approximation $A_t \approx 0$*

Notice that Assumption 1 is in line with the equilibrium actual law of motion since all agents who have access to the equity market are identical. In this environment the agent cannot apply the Law of Iterated Expectations when forming beliefs and therefore the linearized FOC with respect to stock prices is of one-step ahead form:

$$\tilde{q}_t = (1-\delta) E_t^{\mathscr{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathscr{P}}(\tilde{q}_{t+1}) + \sigma(\tilde{C}_t^i - E_t^{\mathscr{P}} \tilde{C}_{t+1}^i) \tag{57}$$

.

This result is a mix between the *long-horizon learning* approach of Preston (2005) and the *Euler Equation* approach. Under the Average Marginal agent assumption the optimal consumption decision under long-horizon learning given by equation 15 is consistent under Internal Rationality with the one step ahead pricing equation 57.

# Appendix B Model Derivation Details

**Demand Side**

Replacing $Q_t$ in the budget constraint with equation (23) and rearranging, I obtain

$$\mathscr{W}_t^i = (P_t C_t^i - W_t N_t^i) + B_t^i + \delta S_t^i E_t^{\mathscr{P}} \left\{ \frac{A_{t+1}}{A_t} \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{(Q_t + D_t)}{1+\pi_{t+1}} \right\} \tag{58}$$

where $\mathscr{W}_t^i = B_{t-1}^i(1+i_{t-1}) + S_{t-1}^i(Q_t + D_t)$ represents wealth at time $t$. Adding and subtracting $\delta E_t^{\mathscr{P}}\left\{\frac{A_{t+1}}{A_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{(B_t^i(1+i_t))}{1+\pi_{t+1}}\right\}$ from the RHS, equation (58) becomes

$$\begin{aligned}\mathscr{W}_t^i &= (P_tC_t^i - W_tN_t^i) + \delta E_t^{\mathscr{P}}\left\{\frac{A_{t+1}}{A_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{\mathscr{W}_{t+1}^i}{1+\pi_{t+1}}\right\} \\ &+ B_t^i\left(1 - \delta E_t^{\mathscr{P}}\left\{\frac{A_{t+1}}{A_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{1+i_t}{1+\pi_{t+1}}\right\}\right) \\ &= (P_tC_t^i - W_tN_t^i) + \delta E_t^{\mathscr{P}}\left\{\frac{A_{t+1}}{A_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{\mathscr{W}_{t+1}^i}{1+\pi_{t+1}}\right\}\end{aligned} \tag{59}$$

where the second equality follows from the Euler equation of the household. Substituting forward for $\mathscr{W}_{t+1}$ I obtain

$$\mathscr{W}_t^i = E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j\frac{A_{t+j}}{A_t}\left(\frac{C_{t+j}^i}{C_t^i}\right)^{-\sigma}\frac{(P_{t+j}C_{t+j}^i - W_{t+j}N_{t+j}^i)}{\prod_{s=0}^{j}(1+\pi_{t+s})} \tag{60}$$

where I have imposed the following transversality condition

$$\lim_{j\to\infty} E_t^{\mathscr{P}}\frac{A_{t+j}}{A_t}\left(\frac{C_{t+j}^i}{C_t^i}\right)^{-\sigma}\frac{\mathscr{W}_{t+j}^i}{\prod_{s=0}^{j}(1+\pi_{t+s})} = 0. \tag{61}$$

Dividing equation (60) by $P_t$ leads to the following expression for the real wealth

$$\frac{\mathscr{W}_t^i}{P_t} = E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j\frac{A_{t+j}}{A_t}\left(\frac{C_{t+j}^i}{C_t^i}\right)^{-\sigma}\left[C_{t+j}^i - w_{t+j}^{\frac{1+\phi}{\phi}}(C_{t+j}^i)^{\frac{-\sigma}{\phi}}\right]. \tag{62}$$

The steady state (SS) of the model corresponds to the RE SS and is given by

$$\begin{aligned}Y &= \left((1-\alpha)\frac{\epsilon-1}{\epsilon}\right)^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\phi}} \\ w &= Y^{\sigma+\frac{\phi}{1-\alpha}} \\ d &= Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}} \\ q &= \frac{\delta}{1-\delta}d.\end{aligned} \tag{63}$$

At the SS equation 62 becomes

$$\begin{aligned} q + d &= \sum_{j=0}^{\infty} \delta^j (Y - w^{\frac{1+\phi}{\phi}} Y^{\frac{-\sigma}{\phi}}) \\ \frac{q}{\delta} &= \frac{Y^{\sigma + \frac{1+\phi}{1-\alpha}}}{1-\delta}. \end{aligned} \tag{64}$$

Applying a first order Taylor approximation to the IBC around a non-stochastic steady state yields

$$\begin{aligned} \tilde{w}_t^i =& \frac{\delta}{q} E_t^{\mathscr{P}} \Big\{ \sum_{j=0}^{\infty} \delta^j \Big[ -(Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}}) r_{t+j}^N - \sigma (Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}}) (\tilde{c}_{t+j}^i - \tilde{c}_t{}^i) \\ & + (Y + \frac{\sigma}{\phi} Y^{\sigma+\frac{1+\phi}{1-\alpha}}) \tilde{c}_{t+j}^i - \frac{1+\phi}{\phi} Y^{\sigma+\frac{1+\phi}{1-\alpha}} \tilde{w}_{t+j} \Big] \Big\} \\ =& (1-\delta) E_t^{\mathscr{P}} \Big\{ \sum_{j=0}^{\infty} \delta^j \Big[ - r_{t+j}^N + \sigma \tilde{c}_t^i + ( \frac{Y + \frac{\sigma}{\phi} Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}}} - \sigma) \tilde{c}_{t+j}^i - \frac{1+\phi}{\phi} \frac{Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}}} \tilde{w}_{t+j} \Big] \Big\}. \end{aligned} \tag{65}$$

where $r_{t+j}^N = a_t - a_{t+j}$. Moving from the first line to the third made use of (63).

Log-linearization of the Euler equation (22) yields

$$\tilde{c}_t^i = E_t^{\mathscr{P}} \tilde{c}_{t+1}^i - \frac{1}{\sigma} (i_t - E_t^{\mathscr{P}} \pi_{t+1} - r_{t+1}^N) \tag{66}$$

which can be rewritten as

$$E_t^{\mathscr{P}} (\tilde{c}_{t+k}^i) = \tilde{c}_t^i + \frac{1}{\sigma} E_t^{\mathscr{P}} \Big[ \sum_{j=0}^{k-1} i_{t+j} - \pi_{t+j+1} - r_{t+j}^N \Big]. \tag{67}$$

Substituting equation 67 in 65, rearranging and using the fact that

$$\sum_{j=0}^{\infty} \delta^j \sum_{k=0}^{j-1} R_t = \frac{\delta}{1-\delta} \sum_{j=0}^{\infty} \delta^j R_t$$

for any variable $R_t$, yields

$$\tilde{c}_t^i = \Delta_i \tilde{w}_t^i + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}} (\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}} (i_{t+j} - \pi_{t+j+1} - \Gamma^r r_{t+j}^N). \tag{68}$$

where

$$\begin{aligned}
\Delta_i &= \frac{Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}} \\
\Delta_w &= (1-\delta)\frac{1+\phi}{\phi}\frac{Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}}, \\
\Delta_r &= \frac{(1-\sigma)Y + \sigma\frac{1+\phi}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}}, \\
\Gamma^r &= 1 + \frac{1-\delta}{\delta}\sigma\frac{Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{(1-\sigma)Y + \sigma\frac{1+\phi}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}}.
\end{aligned} \tag{69}$$

Evaluating expectations in equation 68 using the PLM of agents and applying the equilibrium conditions results in the demand side of the model

$$\begin{aligned}
\tilde{y}_t - \Delta_i\delta\tilde{q}_t - \Delta_i(1-\delta)\tilde{d}_t - \Delta_w\tilde{w}_t + \frac{\delta}{\sigma}\Delta_r i_t &= \frac{\delta}{1-\rho\delta}\Delta_w\hat{\beta}^w_{t-1} - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r\phi_y\hat{\beta}^y_{t-1} \\
&- \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r\phi_q\hat{\beta}^q_{t-1} - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r(\phi_\pi - \frac{1}{\delta})\hat{\beta}^\pi_{t-1} \\
&- \frac{\delta}{\sigma}\frac{\delta\rho_i}{1-\delta\rho_i}\delta_r\epsilon^i_t + \frac{\delta}{\sigma}\Delta_r\Gamma_r(1-\rho_a)\frac{\delta\rho_a}{1-\delta\rho_a}a_t.
\end{aligned} \tag{70}$$

**Supply Side**

The solution to the profit maximization problem yields the optimal price setting decision of the firm

$$P^*_t = \frac{\epsilon}{\epsilon-1}\frac{\sum_{k=0}^{\infty}(\theta\delta)^k E^{\mathscr{P}}_t\left[\frac{A_{t+k}}{A_t}Y^{1-\sigma}_{t+k}P^{\epsilon}_{t+k}MC_{t+k/k}\right]}{\sum_{k=0}^{\infty}(\theta\delta)^k E^{\mathscr{P}}_t\left[\frac{A_{t+k}}{A_t}Y^{1-\sigma}_{t+k}P^{\epsilon-1}_{t+k}\right]} \tag{71}$$

where

$$MC_{t+k/k} = \frac{1}{1-\alpha}\frac{W_{t+k}}{P_{t+k}}\left(\frac{P^*_t}{P_{t+k}}\right)^{\frac{-\epsilon\alpha}{1-\alpha}}Y^{\frac{\alpha}{1-\alpha}}_{t+k}e^{\epsilon^u_{t+k}}. \tag{72}$$

In the above equation $\epsilon^u_{t+k}$ is a shock to the the marginal costs of the firm and will be interpreted as a cost-push shock.

Log-linearization around a 0 inflation steady state and noting that at

SS $\frac{\epsilon-1}{\epsilon} = \frac{1}{1-\alpha} Y^{\sigma+\frac{\phi+\alpha}{1-\alpha}}$ yields the pricing decision rule of the firms

$$p_t^* = (1-\delta\theta) \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}} \left\{ \frac{\alpha}{1-\alpha+\epsilon\alpha} \tilde{y}_{t+k} + \frac{1-\alpha}{1-\alpha+\epsilon\alpha} (\tilde{w}_{t+k} + \epsilon_{t+k}^u) + p_{t+k} \right\}. \tag{73}$$

Substracting $p_{t-1}$ from both sides and taking into account that in equilibrium $\pi_t = (1-\theta)(p_t^* - p_{t-1})$ results in the equation for inflation

$$\pi_t = \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{\alpha}{1-\alpha+\epsilon\alpha} \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}} \tilde{y}_{t+k} + \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}} \tilde{w}_{t+k}$$
$$+ \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}} u_{t+k} + (1-\theta)\delta \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}} \pi_{t+k+1} \tag{74}$$

where $u_{t+k} = \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\epsilon} \epsilon_{t+k}^u$ is an exogeneous AR(1) process with persistence $\rho_u$ and zero mean.

Evaluating the expectations using agents PLM results in the demand block of the model, the Phillips curve

$$\pi_t - \Theta_y \tilde{y}_t - \Theta_w \tilde{w}_t = \Theta_{\beta^y} \hat{\beta}_{t-1}^y + \Theta_{\beta^w} \hat{\beta}_{t-1}^w + \Theta_{\beta^\pi} \hat{\beta}_{t-1}^\pi + \Theta_u u_t \tag{75}$$

where

$$\begin{aligned}
\Theta_y &= \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{\alpha}{1-\alpha+\epsilon\alpha} \\
\Theta_w &= \frac{(1-\theta\delta)(\theta)}{1-\theta} \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \\
\Theta_{\beta^y} &= \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{\alpha}{1-\alpha+\epsilon\alpha} \frac{\theta\delta}{1-\theta\delta\rho} \\
\Theta_{\beta^w} &= \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \frac{\theta\delta}{1-\theta\delta\rho} \\
\Theta_{\beta^\pi} &= \frac{(1-\theta)\delta}{1-\theta\delta\rho} \\
\Theta_u &= \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \frac{1}{1-\theta\delta\rho_u}
\end{aligned} \tag{76}$$

**Asset Prices**

Log-linearization of the FOC wrt to stock holding yields the asset pricing equation

$$\tilde{q}_t = (1-\delta)\hat{\beta}^d_{t-1} + \delta\hat{\beta}^q_{t-1} - (i_t - \hat{\beta}^\pi_{t-1}) \tag{77}$$

where $\epsilon^q_t$ is a stochastic process with persistence $\rho_q$ and can be interpreted as a equity market fad.

**Equilibrium**

Labor is demand determined and is obtained by log-linearization of the production function

$$\tilde{n}_t = \frac{\tilde{y}_t}{1-\alpha}. \tag{78}$$

Wages come from the FOC wrt to labor from the households problem which after loglinearization becomes

$$\tilde{w}_t = \phi\tilde{n}_t + \sigma\tilde{y}_t \tag{79}$$

Dividends are given are given by the profits of the firms

$$D_t = Y_t - W_t N_t \tag{80}$$

which after log-linearization becomes:

$$\tilde{d}_t = \frac{Y}{d}\tilde{y}_t - \frac{WN}{d}(\tilde{n}_t + \tilde{w}_t). \tag{81}$$

using the expressions for labor and wages, the above equation can be rewritten only as a function of $\tilde{y}_t$

$$\tilde{d}_t = \psi_d\, \tilde{y}_t. \tag{82}$$

where $\phi_d = \frac{Y}{d} - \frac{WN}{d}(\sigma + \frac{1+\phi}{1-\alpha})$

**Belief System**

Let $z_t = (\tilde{y}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_t, \tilde{w}_t)'$. Agents think that $z_t$ follows an unobserved component model

$$
\begin{aligned}
z_t &= \beta_t + \zeta_t \\
\beta_t &= \rho \beta_{t-1} + \vartheta_t
\end{aligned}
\tag{83}
$$

where $\beta_t$ is the permanent component. Agents have perfect knowledge about interest rates and about the shock process. Agents form expectations at time $t$ using information up to $t-1$. Denoting these time $t$ expectations by $\beta_{t-1}$, agents update their beliefs following the recursion

$$
\hat{\beta}_t = \rho\, \hat{\beta}_{t-1} + \lambda(z_t - \hat{\beta}_{t-1}). \tag{84}
$$

Given that agents forecast $E_t^{\mathscr{P}} z_{t+k} = \rho^{k-1}\hat{\beta}_t$ we can evaluate the subjective expectations necessary to compute the Actual Law of Motion (ALM) as

$$
\begin{aligned}
\sum_{j=1}^{\infty} \delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) &= \sum_{j=1}^{\infty} \delta^j \rho^{j-1} \hat{\beta}^w_{t-1} = \frac{\delta}{1-\rho\delta} \hat{\beta}^w_{t-1}, \\
\sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(i_{t+j}) &= \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\phi_{pi}\pi_{t+j} + \phi_y \tilde{y}_{t+j} + \phi_q \tilde{q}_{t+j} + \epsilon^i_{t+j} \\
&= i_t + \frac{\delta}{1-\delta\rho}(\phi_{pi}\hat{\beta}^\pi_{t-1} + \phi_y \hat{\beta}^y_{t-1} + \phi_q \hat{\beta}^q_{t-1}) + \frac{\delta\rho_i}{1-\delta\rho_i}\epsilon^i_t \\
\sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\pi_{t+j+1}) &= \frac{1}{1-\delta\rho}\hat{\beta}^\pi_{t-1} \\
\sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(r^N_{t+j}) &= \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(a_t - a_{t+j}) \\
&= \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}((1-\rho_a)\rho_a^{j-1} a_t) = \frac{(1-\rho_a)\delta}{1-\delta\rho_a} a_t \\
\sum_{j=0}^{\infty} (\delta\theta)^j E_t^{\mathscr{P}}(\tilde{y}_{t+j}) &= \tilde{y}_t + \frac{\theta\delta}{1-\theta\delta\rho}\hat{\beta}^y_{t-1} \\
\sum_{j=0}^{\infty} (\delta\theta)^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) &= \tilde{w}_t + \frac{\theta\delta}{1-\theta\delta\rho}\hat{\beta}^w_{t-1} \\
\sum_{j=0}^{\infty} (\delta\theta)^j E_t^{\mathscr{P}}(\tilde{u}_{t+j}) &= \frac{\theta\delta}{1-\theta\delta\rho}\tilde{u}_t \\
\sum_{j=0}^{\infty} \delta\theta)^j E_t^{\mathscr{P}}(\pi_{t+j+1}) &= \frac{1}{1-\delta\rho\theta}\hat{\beta}^\pi_{t-1}
\end{aligned} \tag{85}
$$

**System in State-Space form**

The system of equations determining $\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t$ and $\tilde{w}_t$ can be written in a compact form

$$
A\, Z_t = B\, \hat{\beta}^Z_{t-1} + C\, \epsilon_t \tag{86}
$$

where

$$Z_t = (\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t, \tilde{w}_t)',$$

$$\hat{\beta}^Z_{t-1} = (\hat{\beta}^y_{t-1}, \hat{\beta}^\pi_{t-1}, \hat{\beta}^q_{t-1}, \hat{\beta}^i_{t-1}, \hat{\beta}^d_{t-1}, \hat{\beta}^w_{t-1})',$$

$$\epsilon_t = (\tilde{a}_t, \tilde{u}_t, \epsilon^q_t, \epsilon^i_t)',$$

$$A = \begin{pmatrix} 1 & 0 & -\Delta_i\delta & \frac{\Delta_r\delta}{\sigma} & -\Delta_i(1-\delta) & -\Delta_w \\ -\Theta_y & 1 & 0 & 0 & 0 & -\Theta_w \\ 0 & 0 & 1 & 1 & 0 & 0 \\ -\phi_y & -\phi_\pi & -\phi_q & 1 & 0 & 0 \\ \psi_d & 0 & 0 & 0 & 1 & 0 \\ -(\sigma + \frac{\phi}{1-\alpha}) & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} -\frac{\delta^2\Delta_r\phi_y}{\sigma(1-\delta\rho)} & -\frac{\delta^2\Delta_r}{\sigma(1-\delta\rho)}(\phi_\pi - \frac{1}{\delta}) & -\frac{\delta^2\Delta_r\phi_q}{\sigma(1-\delta\rho)} & 0 & 0 & \frac{\Delta_w\delta}{1-\rho\delta} \\ \Theta_{\beta^y} & \Theta_{\beta^\pi} & 0 & 0 & 0 & \Theta_{\beta^w} \\ 0 & 1 & \delta & 0 & 1-\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} \frac{\delta\Gamma^R\Delta_r}{\sigma}(1-\rho_a)\frac{\delta}{1-\delta\rho_a} & 0 & 0 & -\frac{\delta^2\Delta_r}{\sigma(1-\delta\rho_i)} \\ 0 & \Theta_u & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

## B.1 Lagged response to stock prices

The interest rule is

$$i_t = \phi_\pi\pi_t + \phi_y\tilde{y}_t + \phi_q\tilde{q}_{t-1} + \epsilon^i_t. \tag{87}$$

Given this response of monetary policy the forecast of interest rates is given by

$$\sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(i_{t+j}) = \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\phi_\pi \pi_{t+j} + \phi_y \tilde{y}_{t+j} + \phi_q \tilde{q}_{t+j-1}) + \epsilon_{t+j}^i$$
$$= i_t + \frac{\delta}{1-\delta\rho}(\phi_\pi \hat{\beta}_{t-1}^\pi + \phi_y \hat{\beta}_{t-1}^y) + \delta\phi_q \tilde{q}_t + \frac{\delta^2\phi_q}{1-\delta\rho}\hat{\beta}_{t-1}^q + \frac{\delta\rho_i}{1-\delta\rho_i}\epsilon_t^i \tag{88}$$

The IS equation becomes

$$\tilde{y}_t - (\Delta_i\delta - \frac{\delta^2\phi_q}{\sigma}\Delta_r)\tilde{q}_t - \Delta_i(1-\delta)\tilde{d}_t - \Delta_w \tilde{w}_t + \frac{\delta}{\sigma}\Delta_r i_t = \frac{\delta}{1-\rho\delta}\Delta_w \hat{\beta}_{t-1}^w - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r \phi_y \hat{\beta}_{t-1}^y$$
$$- \frac{\delta^3}{\sigma(1-\delta\rho)}\Delta_r \phi_q \hat{\beta}_{t-1}^q - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r(\phi_\pi - \frac{1}{\delta})\hat{\beta}_{t-1}^\pi - \frac{\delta}{\sigma}\frac{\delta\rho_i}{1-\delta\rho_i}\Delta_r \epsilon_t^i. \tag{89}$$

The system of equations determining $\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t$ and $\tilde{w}_t$ can be written in a compact form

$$A\, Z_t = B\, \hat{\beta}_{t-1}^Z + D\, Z_{t-1} + C\, \epsilon_t \tag{90}$$

where

$$Z_t = (\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t, \tilde{w}_t)',$$

$$\hat{\beta}_{t-1}^Z = (\hat{\beta}_{t-1}^y, \hat{\beta}_{t-1}^\pi, \hat{\beta}_{t-1}^q, \hat{\beta}_{t-1}^i, \hat{\beta}_{t-1}^d, \hat{\beta}_{t-1}^w)',$$

$$\epsilon_t = (\tilde{u}_t, \epsilon_t^i)',$$

$$A = \begin{pmatrix} 1 & 0 & -(\Delta_i\delta - \frac{\delta^2\phi_q}{\sigma}\Delta_r)\delta & \frac{\Delta_r\delta}{\sigma} & -\Delta_i(1-\delta) & -\Delta_w \\ -\Theta_y & 1 & 0 & 0 & 0 & -\Theta_w \\ 0 & 0 & 1 & 1 & 0 & 0 \\ -\phi_y & -\phi_\pi & 0 & 1 & 0 & 0 \\ \psi_d & 0 & 0 & 0 & 1 & 0 \\ -(\sigma + \frac{\phi}{1-\alpha}) & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} -\frac{\delta^2\Delta_r\phi_y}{\sigma(1-\delta\rho)} & -\frac{\delta^2\Delta_r}{\sigma(1-\delta\rho)}(\phi_\pi - \frac{1}{\delta}) & -\frac{\delta^3\Delta_r\phi_q}{\sigma(1-\delta\rho)} & 0 & 0 & \frac{\Delta_w\delta}{1-\rho\delta} \\ \Theta_{\beta^y} & \Theta_{\beta^\pi} & 0 & 0 & 0 & \Theta_{\beta^w} \\ 0 & 1 & \delta & 0 & 1-\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & -\frac{\delta^2\Delta_r}{\sigma(1-\delta\rho_i)} \\ \Theta_u & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$D = \begin{pmatrix} & & \underset{2\text{x}6}{0} & & & \\ 0 & 0 & \phi_q & 0 & 0 & 0 \\ & & \underset{3\text{x}6}{0} & & & \end{pmatrix},$$

where $\underset{a\text{x}b}{0}$ denotes a matrix of zeros of dimension $a\,\text{x}\,b$.

# Appendix C Welfare Approximation

Assuming the steady state is efficient under RE equalizes the consumption and labor decision of the two agents. This is ensured by a tax subsidy on sales by the fiscal authority which is rebated back to firms as a lump sum transfer conditional on a balanced budget. This ensures that profits are zero at the steady state but not otherwise since markups will vary over time. At steady steady

$$\begin{gathered} C^C = C^U = C \\ N^C = N^U = N \\ Y = N^{1-\alpha} \\ w = N^\phi Y^\sigma \\ \frac{V'(N)}{U'(C)} = w = (1-\alpha)\frac{Y}{N} \end{gathered} \tag{91}$$

Following Bilbiie (2008) assume the social planner is maximizing a weighted average of the utility of the agents $U_t(\cdot) = \mathscr{O}\ U^C(C_t^C, N_t^C) + (1-\mathscr{O})\ U^U(C_t^U, N_t^U)$. Up to a second order approximation the utility of type $j$ can be written as

$$\begin{aligned}\hat{U}_t^j(\cdot) &= U^j(C_t^j, N_t^j) - U(C, N)\\ &\approx U_C C\ \left(\hat{c}_t^j + \frac{1-\sigma}{2}(\hat{c}_t^j)^2\right) - V_N N\ \left(\hat{n}_t^j + \frac{1+\phi}{2}(\hat{n}_t^j)^2\right) + t.i.p + H.O.T\end{aligned} \tag{92}$$

where the hat variables denote log deviation from the flexible price RE equilibrium which given the absence of fluctuations in the natural output (e.g. TFP) coincides with the steady state of the model. Explicitly, $\hat{c}_t = log(C_t) - log(C)$, $t.i.p$ denotes terms independent of policy and $H.O.T$ higher order terms (greater than 2). In equilibrium $\hat{c}_t = \hat{y}_t$ and $\hat{n}_t = \frac{1}{1-\alpha}\hat{y}_t + d_t$ where $d_t = log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di$[27]. Given this and aggregating across agents

$$\begin{aligned}\hat{U}_t(\cdot) \approx U_C C\ &\left[\hat{c}_t + \frac{1-\sigma}{2}\left(\mathscr{O}\ (\hat{c}_t^C)^2 + (1-\mathscr{O})(\hat{c}_t^U)^2\right)\right]\\ - V_N N &\left[\hat{n}_t + \frac{1+\phi}{2}\left(\mathscr{O}\ (\hat{n}_t^C)^2 + (1-\mathscr{O})(\hat{n}_t^U)^2\right)\right] + H.O.T\end{aligned} \tag{93}$$

Using the last equation from 91 we can write $\frac{V_N N}{U_C C} = (1-\alpha)$. The linear terms from the utility approximation boil down to

$$U_C C\ (\hat{c}_t) - V_N N\,(\hat{n}_t) = -U_C C\ \left[(1-\alpha)d_t\right] + H.O.T \tag{94}$$

Regarding the quadratic terms we can also rewrite them in terms of output-gaps and stock prices. First notice that from Proposition 1 we have

$$\hat{c}_t^U - \hat{c}_t^C = \Delta_i\left[\delta\hat{q}_t + (1-\delta)\hat{d}_t\right] = \Delta_i\left[\delta\hat{q}_t + (1-\delta)\psi_d\,\tilde{y}_t\right].$$

Using the previous relation together with goods market clearing and FOC with respect to labor for the two types of agents we obtain the following

[27] see Galí (2015) pag 87

$$
\begin{aligned}
\tilde{y}_t &= \mathscr{O}\tilde{c}_t^C + (1-\mathscr{O})\tilde{c}_t^U \\
\tilde{y}_t &= \mathscr{O}\tilde{c}_t^C + (1-\mathscr{O})\left\{\Delta_i\left[\delta\tilde{q}_t + (1-\delta)\tilde{d}_t\right] + \tilde{c}_t^C\right\} \\
\tilde{c}_t^C &= \tilde{y}_t - (1-\mathscr{O})\Delta_i\left[\delta\tilde{q}_t + (1-\delta)\psi_d\,\tilde{y}_t\right] \\
\tilde{c}_t^C &= [1-(1-\mathscr{O})\Delta_i(1-\delta)\psi_d]\tilde{y}_t - (1-\mathscr{O})\Delta_i\delta\tilde{q}_t \\
&= \Upsilon_{cy}^C\,\tilde{y}_t - \Upsilon_{cq}^C\,\tilde{q}_t \\
\tilde{n}_t^C &= \frac{\tilde{w}_t - \sigma\,\tilde{c}_t^C}{\phi} = \frac{\left(\frac{\phi}{1-\alpha}+\sigma\right)\tilde{y}_t + \phi d_t - \sigma\,\tilde{c}_t^C}{\phi} \\
&= \left(\frac{1}{1-\alpha}+\frac{\sigma}{\phi}\right)\tilde{y}_t - \frac{\sigma}{\phi}\left(\Upsilon_{cy}^C\,\tilde{y}_t - \Upsilon_{cq}^C\tilde{q}_t\right) + d_t \\
&= \left(\frac{1}{1-\alpha}+\frac{\sigma}{\phi}(1-\Upsilon_{cy}^C)\right)\tilde{y}_t + \frac{\sigma}{\phi}\Upsilon_{cq}^C\tilde{q}_t + d_t \\
&= \Upsilon_{ny}^C\,\tilde{y}_t + \Upsilon_{nq}^C\,\tilde{q}_t + d_t \qquad (95) \\
\tilde{c}_t^U &= \Delta_i\left[\delta\tilde{q}_t + (1-\delta)\psi_d\,\tilde{y}_t\right] + \Upsilon_y^C\,\tilde{y}_t - \Upsilon_q^C\tilde{q}_t \\
&= []1 + \mathscr{O}\Delta_i(1-\delta)\psi_d]\,\tilde{y}_t + \mathscr{O}\Delta_i\delta\tilde{q}_t \\
&= \left[\Delta_i(1-\delta)\psi_d + \Upsilon_{cy}^C\right]\tilde{y}_t + (\Delta_i\delta - \Upsilon_{cq}^C)\tilde{q}_t \\
&= \Upsilon_{cy}^U\,\tilde{y}_t + \Upsilon_{cq}^U\,\tilde{q}_t \\
\tilde{n}_t^U &= \frac{\tilde{w}_t - \sigma\,\tilde{c}_t^U}{\phi} = \frac{\left(\frac{\phi}{1-\alpha}+\sigma\right)\tilde{y}_t + \phi d_t - \sigma\,\tilde{c}_t^U}{\phi} \\
&= \left(\frac{1}{1-\alpha}+\frac{\sigma}{\phi}\right)\tilde{y}_t - \frac{\sigma}{\phi}\left(\Upsilon_{cy}^U\,\tilde{y}_t + \Upsilon_{cq}^U\tilde{q}_t\right) + d_t \\
&= \left[\frac{1}{1-\alpha}+\frac{\sigma}{\phi}(1-\Upsilon_{cy}^U)\right]\tilde{y}_t - \frac{\sigma}{\phi}\Upsilon_{cq}^U\tilde{q}_t + d_t \\
&= \Upsilon_{ny}^U\,\tilde{y}_t - \Upsilon_{nq}^U\,\tilde{q}_t + d_t.
\end{aligned}
$$

Using these last results we can derive the quadratic terms for consumption and labor in terms of output gaps and stock prices

$$(\tilde{c}_t^C)^2 = (\Upsilon_{cy}^C)^2\tilde{y}_t^2 + (\Upsilon_{cq}^C)^2\tilde{q}_t^2 - 2\,\Upsilon_{cy}^C\Upsilon_{cq}^C\tilde{y}_t\,\tilde{q}_t + H.O.T \qquad (96)$$

$$(\tilde{c}_t^U)^2 = (\Upsilon_{cy}^U)^2\tilde{y}_t^2 + (\Upsilon_{cq}^U)^2\tilde{q}_t^2 + 2\,\Upsilon_{cy}^U\Upsilon_{cq}^U\tilde{y}_t\,\tilde{q}_t + H.O.T \qquad (97)$$

$$(\tilde{n}_t^C)^2 = (\Upsilon_{ny}^C)^2\tilde{y}_t^2 + (\Upsilon_{nq}^C)^2\tilde{q}_t^2 + 2\,\Upsilon_{ny}^C\Upsilon_{nq}^C\tilde{y}_t\,\tilde{q}_t + H.O.T \qquad (98)$$

$$(\tilde{n}_t^U)^2 = (\Upsilon_{ny}^U)^2\tilde{y}_t^2 + (\Upsilon_{nq}^U)^2\tilde{q}_t^2 - 2\,\Upsilon_{ny}^U\Upsilon_{nq}^U\tilde{y}_t\,\tilde{q}_t + H.O.T. \qquad (99)$$

The aggregate *per-period* approximation of the welfare function is then, up to a second order approximation

$$\hat{U}_t(\cdot) \approx -U_C C \left[(1-\alpha)d_t + \Upsilon_1\ \tilde{y}_t^2 + \Upsilon_2\ \tilde{q}_t^2 + \Upsilon_3\ \tilde{q}_t\tilde{y}_t\right] \tag{100}$$

where

$$\begin{aligned}
\Upsilon_1 &= \left[\frac{1+\phi}{2}(1-\alpha)\left(\mathscr{O}(\Upsilon^C_{ny})^2 + (1-\mathscr{O})(\Upsilon^U_{ny})^2\right) - \frac{1-\sigma}{2}\left(\mathscr{O}(\Upsilon^C_{cy})^2 + (1-\mathscr{O})(\Upsilon^U_{cy})^2\right)\right] \\
\Upsilon_2 &= \left[\frac{1+\phi}{2}(1-\alpha)\left(\mathscr{O}(\Upsilon^C_{nq})^2 + (1-\mathscr{O})(\Upsilon^U_{nq})^2\right) - \frac{1-\sigma}{2}\left(\mathscr{O}(\Upsilon^C_{cq})^2 + (1-\mathscr{O})(\Upsilon^U_{cq})^2\right)\right] \\
\Upsilon_3 &= \ \left[(1+\phi)(1-\alpha)\left(\mathscr{O}\Upsilon^C_{nq}\Upsilon^C_{ny} - (1-\mathscr{O})\Upsilon^U_{nq}\Upsilon^U_{ny}\right) + (1-\sigma)\left(\mathscr{O}\Upsilon^C_{cq}\Upsilon^C_{cy} - (1-\mathscr{O})\Upsilon^U_{cq}\Upsilon^U_{cy}\right)\right].
\end{aligned} \tag{101}$$

The price dispersion term, $(1-\alpha)\ d_t$, can be rewritten using the arguments from Galí (2015) as $(1-\alpha)\ d_t \approx \frac{\epsilon}{\psi}\ \pi_t^2$ where $\psi = \frac{(1-\theta)(1-\delta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

The average welfare loss per period in terms of steady steady consumption is

$$\mathscr{L} = \frac{\epsilon}{\psi} var(\pi_t) + \Upsilon_1\ var(\tilde{y}_t) + \Upsilon_2\ var(\tilde{q}_t) + \Upsilon_3\ E(\tilde{y}_t\tilde{q}_t) \tag{102}$$

# Appendix D Contemporaneous Response to stock prices

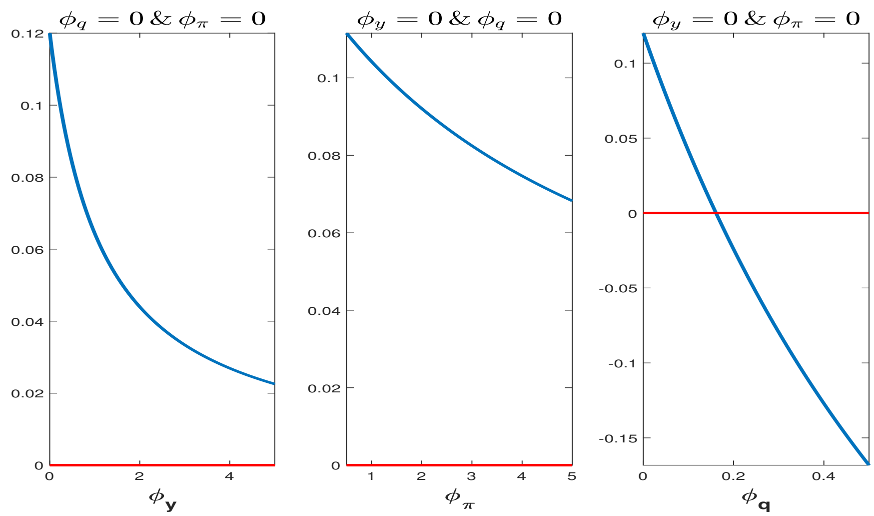



**Figure 13:** Stock Price Wealth Effects and Monetary Policy

*Each panel presents the magnitude of the wealth effects as a function of the central bank response to output, inflation and stock prices while keeping the other coefficients fixed at 0. The Taylor rule is of the following type:* $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$

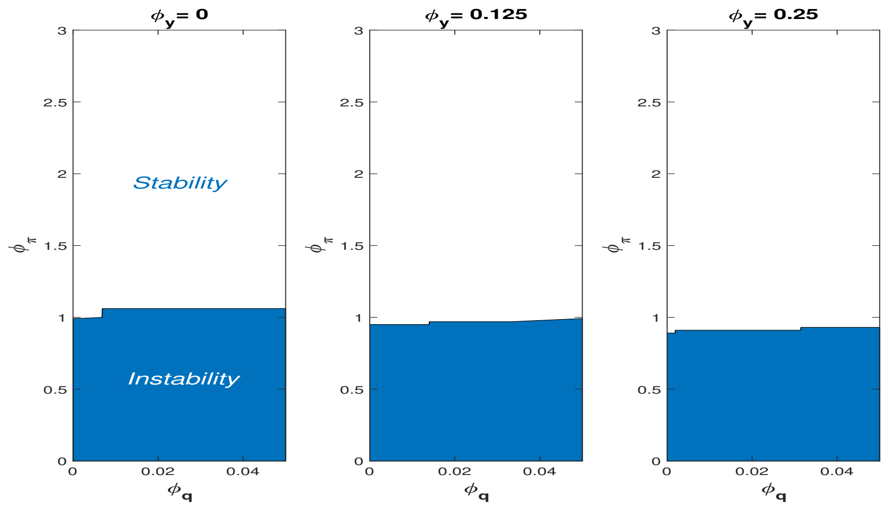



**Figure 14:** E-Stability and Monetary Policy
*The figure present the stability (white) and instability (blue) regions for different combinations of Taylor rule coefficients. Each panel plots the e-stability regions for different combinations of inflation (Y axis) and stock price (X axis) reaction coefficients while keeping the output reaction fixed. The Taylor rule is of the following type:* $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$. *The stability of the system is given by the eigenvalues of the matrix* $A^{-1}B$. *Following Evans and Honkapohja (2012), the dynamical system is e-stable if the largest eigenvalue of the previous matrix has the real part smaller than 1.*

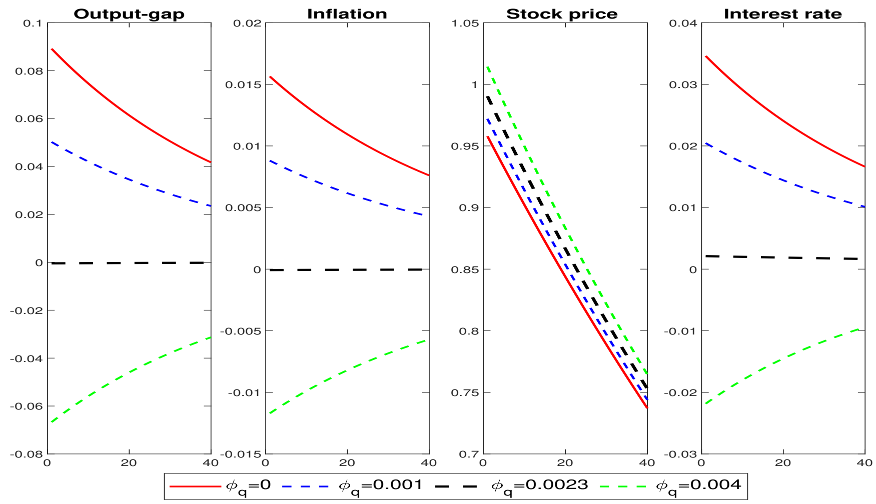



**Figure 15:** IRFs to Sentiment Shocks
*The figure presents the IRF to a 1 % i.i.d shock in equity price beliefs for different reaction coefficients to stock prices. The Taylor rule is of the following form:* $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$

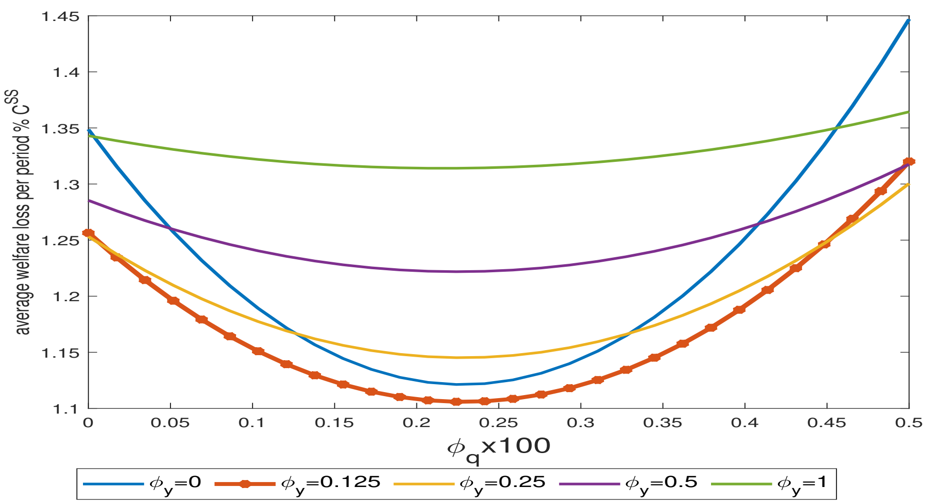



**Figure 16:** Welfare Maps
*The figure shows the average welfare loss per period as defined in equation (54) for different combinations of Taylor rule coefficients for output and stock prices while keeping the inflation reaction coefficient fixed at 1.5. Welfare losses have been computed as averages over 200 independent simulations, each one including 260 time periods using the estimated parameters from section IV.H*

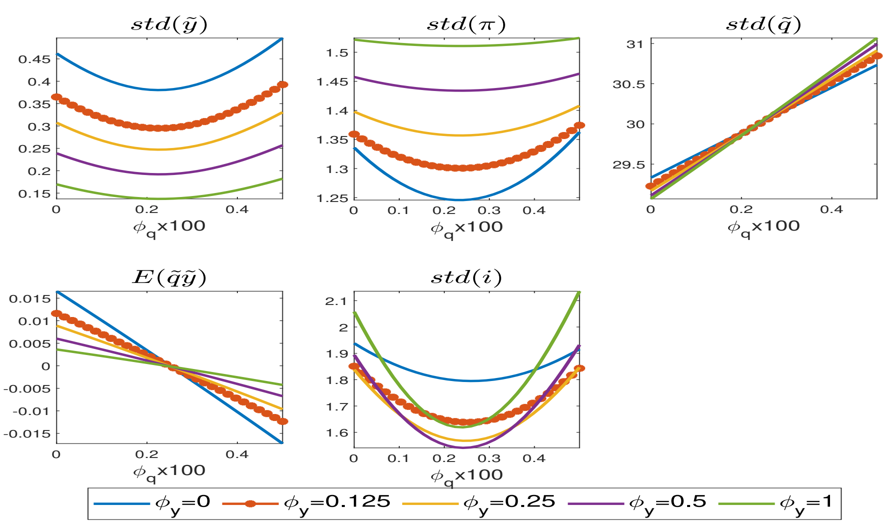



**Figure 17:** Influence of Monetary policy on Macroeconomic Volatility
*Implied volatility of output, inflation, stock prices, co-movement of output with stock prices and interest rates for different combinations of policy parameters. The Taylor rule is specified as* $i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + \phi_q \tilde{q}_t$.

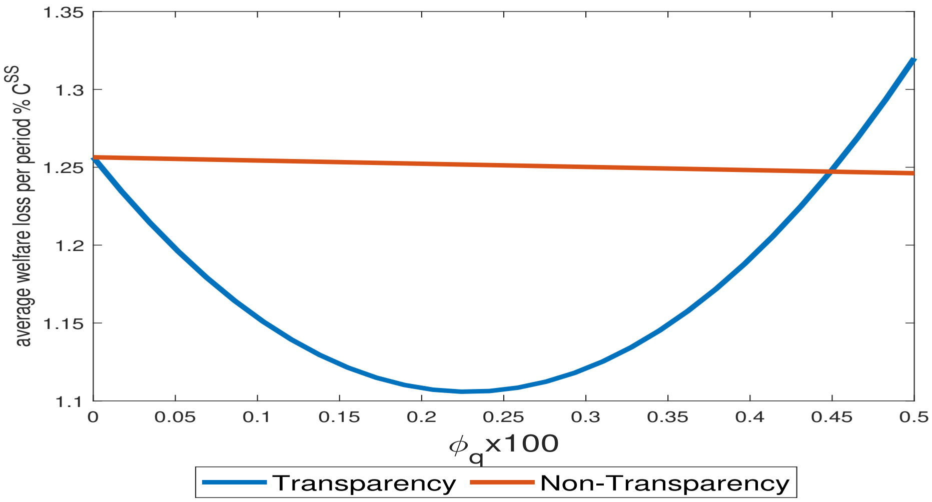



**Figure 18:** (Non) Transparency of stock price targeting
*Transparency implies that agents internalize the reaction to stock prices while in the non-transparency scenario agents only take into account the response to output and inflation in the Taylor Rule. The latter is specified as* $i_t = 1.5\,\pi_t + 0.125\,\tilde{y}_t + \phi_q \tilde{q}_t$.

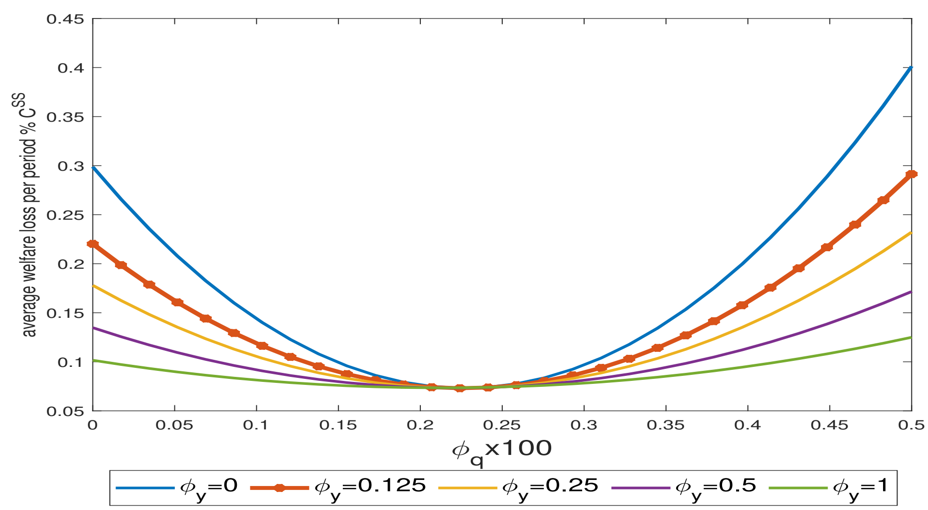



**Figure 19:** Welfare maps when the economy is hit only by Sentiment Shocks
*The figure shows the average welfare loss per period for different policy parameters for output and stock prices in the case the only source of variation in the economy is given by Sentiment Shocks. The volatility of sentiment shocks is the one estimated in section IV.*

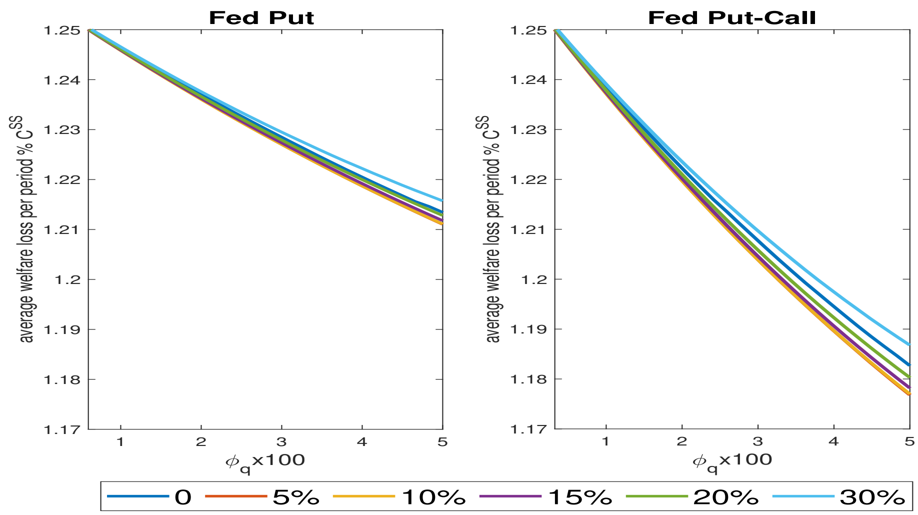



**Figure 20:** Welfare Implications of Fed Put and Call under Non-Transparency
$Q^{-} = -Q^{+}$

# Appendix E Additional Figures

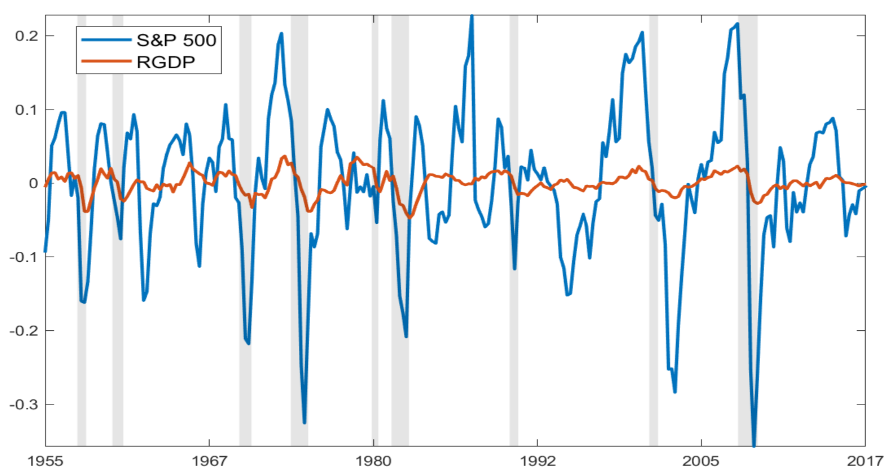



**Figure 21:** Real and Financial Volatility at business cycle frequency
*HP-filtered quarterly data; shaded bands denote NBER recessions*