Owner-Occupied Housing:

Life-cycle Implications for the Household Portfolio

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Until recently, the conventional wisdom in the portfolio choice literature held that anyone who simply added housing to the vector of assets and claimed to identify the optimal portfolio as the vector of asset holding that achieved mean-variance efficiency was "incorrect". As is often the case, the conventional wisdom is valid in a particular set of circumstances. In this case, the conventional wisdom -- that problems arise when applying a standard mean-variance optimization framework to a portfolio problem incorporating housing – is valid if the problem is set up as the choice over the quantities of all assets (the quantity of housing as well as the quantities of each financial asset) in a sequence of repeated mean-variance optimizations. The issue is that capital gains on housing are essentially different from capital gains on financial assets in the sense that an increase in the asset price of housing is strongly correlated with the price of a good which is quantitatively important in the household's future consumption bundle (future housing services), and therefore do not represent gains in wealth exactly comparable to the gains which come from increases in the price of a financial asset. From a personal perspective, I may enjoy reading the real estate section of the paper every Sunday to note how much the value of my San Diego home has increased from 2000 to 2005 (measured in dollar value). However, a $200,000 increase in the value of my home does not increase my command over goods and services (i.e., is not an increase in wealth) in the same sense that a $200,000 increase in the value of my stock holdings improves my command over goods and services, given that I now face a commensurately higher price for housing services, should I attempt to realize the capital gain.

The proscription against naively sticking housing into the vector of assets in a mean-variance framework holds when the quantity of housing is one of the choice variables in the optimization. In this paper, we consider the effect of housing in a portfolio allocation problem by conditioning on the current holding of housing, and finding the optimal holdings of financial assets, conditional on the current quantity of housing held. Since we are using the mean-variance framework to determine the optimal holding of financial assets, conditional on the current quantity of housing (rather than using the mean-variance framework to determine the holding of all assets including housing), in this context the conventional wisdom does not apply.

Taking the holding of housing as fixed, and determining the optimal portfolio of financial assets conditional on housing, is a well-defined subproblem within the household's overall optimization problem. That is, the overall problem of the household is to choose the optimal level of housing, holdings of financial assets, and the level of nondurable consumption in a continuous time setting. Adjustment of the quantity of housing requires the payment of a nonconvex adjustment cost, but nondurable consumption and financial assets can be adjusted frictionlessly. Because of the adjustment cost on housing, the solution to the general problem has a recursive structure: at each moment, the household is considering whether to sell the house, pay the adjustment cost, and choose a new house. Most of the time, it is not optimal to incur the adjustment cost, and having decided not to sell the house at that instant, the household then chooses the optimal level of nondurable consumption and the optimal holdings of financial assets conditional on the current level of housing. When, very infrequently, it is optimal to sell the house, the household optimally chooses the size of the new house. Thus, while the holding of housing is determined endogenously, it is not determined as part of the mean-variance optimization problem. Instead, the optimal holdings of financial assets, conditional on the current holdings of housing, is determined by the mean-variance framework.

The model is a variation of the housing model proposed in Flavin and Nakagawa (2008). Instead of assuming that the household can borrow or lend at the riskless rate, and take negative as well as positive positions in all financial assets (as in Flavin and Nakagawa (2008)), in this paper we consider the portfolio allocation problem when the household is constrained by nonnegativity constraints on financial assets. In particular, we assume that the only way the household can borrow is to borrow against a house in the form of a mortgage, the size of the mortgage is limited to 100% of the value of the house, and that financial assets other than the mortgage can be held only in nonnegative amounts. The constraint that the household can borrow only in the form of a mortgage is referred to as the borrowing, or collateral constraint.

Incorporating the collateral and nonnegativity constraints considerably complicates the problem, and requires computational rather than analytic solution of the optimal portfolios. If it turned out that for most households for most of the time the constraints were not binding (i.e., households' optimal portfolios occurred at an interior solution despite the presence of the constraints), we could jettison the constrained version of the problem and work with the considerably simpler unconstrained version of the problem. To determine whether (and under what circumstances) the collateral and nonnegativity constraints are likely to be binding, we calculate the optimal portfolios for a range of assumptions on the stochastic structure of asset returns.

Finally, we consider the implications of the model for the composition of the portfolio over the lifecycle. The model implies that, in the presence of the collateral and nonnegativity constraints, the optimal portfolio will depend on not only the household's degree of risk aversion, but also on the ratio of the house value to net worth. For a given degree of risk aversion, the percentage of the financial asset portfolio held in the form of stocks is a decreasing function of the ratio of house value to net worth over most of its range. Young homeowners typically have house values several times as large as their net worth; over the course of the lifecycle, the ratio of house value to net worth falls as the household accumulates wealth. Thus even if we consider two households with the same degree of risk aversion, the older household with a lower ratio of house value to net worth will generally hold a greater percentage of its portfolio of financial assets in the form of stocks than a younger household. While we do not attempt any formal statistic tests of the model, we conclude by examining the lifecycle patterns in the portfolio data from the repeated cross sections provided by the Survey of Consumer Finances.

Related literature

Incorporating housing into a standard portfolio model significantly alters a household's risk-return trade off and hence optimal holdings of financial portfolios (see, for example, Flavin and Yamashita (2002), Cocco (2005), Hu (2005), Yao and Zhang (2005), and Cauley, Pavlov and Schwartz (2007)). However, many studies adopt simplifying assumptions that may not capture certain aspects of housing investment. For example, Coco (2005) assumes that the value of home is perfectly correlated with aggregate labor income shocks, thus is non-stochastic with respect to permanent income. Yao and Zhang (2005), while endogenizing housing tenure decisions, assume unit elasticity between housing and non-durable consumption, which is much higher than the available estimates. They calibrate the optimal portfolio holdings using the risk premium on risky assets of 4 percent and the zero expected return on housing investment. Thus stocks are less attractive than the historical average and a household would purchase a home purely for consumption purposes.

Our model that relates housing investment to the composition of financial portfolios relates to the model of personal illiquid projects of Faig and Shum (2002). In their model, illiquidity in a personal project requires the investor to maintain relatively safe and liquid financial portfolios. Two models are similar as the assumed lack of correlation between personal projects and financial asset returns rules out the possibility of hedging the investment in illiquid assets with financial portfolios. However, their analogy of housing as an example of illiquid personal projects does not conform to the characteristics of the housing markets. Faig and Shum assume that investors would incur substantial losses when a house is sold before the final period. However, because the secondary markets for housing are well developed, their model's assumptions may pertain better to small businesses rather than housing. In addition, we allow housing to be used as collateral, which in turn could significantly alter financial portfolios.[1] Our model is more realistic in describing the role of housing in asset allocation as we model explicitly the housing price risk and lumpiness of housing investment. Our model directly relates the relative size of investment in housing to the composition of financial assets portfolios, rather than treating it as a background risk.

With respect to life-cycle patterns of asset allocation, Gomes and Michaelides (2005) incorporate fixed costs and investor heterogeneity to model optimal portfolio allocation with undiversifiable labor income risk. They establish that investor heterogeneity is the key to introducing life-cycle patterns of risky-asset holdings observed in data. Our model, on the other hand, implies that the size of housing investment relative to net worth dictates an investor's optimal portfolios. Since the relative size of housing varies across households, our model introduces heterogeneity of financial portfolio holdings even if all households have the identical preferences and face the same stochastic structure of the asset markets. Furthermore, Gomes and

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[1] Small business entrepreneurs often use housing as collateral to finance their businesses. See Hurst and Lusardi (2004).

Michaelides use the Epstein-Zin recursive utility function that separates the coefficient of risk aversion from the elasticity of intertemporal substitution (EIS).

Section 1: A model of asset allocation with housing as an asset

In analyzing the role of housing in the portfolio allocation problem, our objective is to model housing in a way that incorporates the important aspects in which housing differs from financial assets. To that end, we assume that, due to imperfections in the rental market for housing services, a homeowner makes a single choice regarding the quantity of residential real estate to acquire; this choice simultaneously determines both the investment in housing as an asset and the flow of housing services consumed by the household. As in Flavin and Nakagawa (2008), once the household purchases a particular house, no adjustments to the size (or any other attribute) can be made without selling the existing house and incurring an adjustment cost proportional to the value of the house sold, and purchasing a new house. Because the transactions cost is assumed proportional (with factor of proportionality λ) to the value of the house sold, the model incorporates a "nonconvex" adjustment cost, in the sense that the adjustment cost is not convex in the size of the adjustment.

The instantaneous utility function depends (nonseparably) on housing services and on a second good, referred to as the "nondurable consumption good" and denoted C_t, which is costlessly adjustable. The household's expected lifetime utility is given by:

(1) $$U = E\int_0^\infty e^{-\delta t} u(H_t, C_t)dt$$

The notation H_t represents a physical measure of the quantity of housing; in the simplest specification H_t can be thought of as a scalar measure of the square footage of the home, but in a more elaborate specification H_t could be interpreted as a vector of physical characteristics (square footage, number of fireplaces, quality of finish materials, etc). The crucial point is that H_t reflects some physical measure of the quantity of housing rather than the market value of the house. For ease of exposition, we interpret H_t as a scalar reflecting the square footage of the house. Using the nondurable good as numeraire, define:

(2) P_t = house price (per square foot) in the household's current market
P_t' = house price (per square foot) in the region to which the household relocates in the next move

Housing is subject to capital gains and losses, in the sense that the price of housing relative to the second consumption good is assumed to vary over time. Further, the model allows for cross sectional variation in the price of housing; housing prices in two regional markets will presumably be correlated but are not necessarily perfectly correlated.

In addition to housing, the household can invest in any of n risky financial assets, including T-bills, bonds, and stocks. There is no riskless asset, although the risk involved in holding T-bills is very small. Households can borrow, in the form of mortgages, an amount up to the value of their home. Unlike housing, financial assets (including mortgages) can be bought and sold with zero transaction cost. We abstract from labor income or human wealth, and assume that wealth is held only in the form of financial assets and the durable good. Thus wealth is given by:

(3) $$W_t = P_t H_t + \underline{X}_t \underline{\ell}$$

Where $\underline{X}_t$ = (1xn) vector of amounts (expressed in terms of the nondurable good) of the risky assets held, and $\underline{\ell}$ = (nx1) vector of ones. Using the first element of $\underline{X}_t$ to represent the mortgage, the corner constraints on the vector of financial assets are given by:

(4a) $-P_tH_t \le X_{1t} \le 0$ (collateral constraint on mortgage borrowing)
$X_{1t} \le W_t - P_tH_t$

(4b) $0 \le X_{it} \quad i = 2 \text{ to } n$ (nonnegativity constraints on other financial assets)

Equations (4a) and (4b) reflect our assumption that the household can borrow against the house but cannot borrow against financial assets or sell financial assets short. By imposing the collateral constraint on mortgage borrowing (4a), and the nonnegativity constraints on other financial assets (4b), we depart from the housing model considered in Flavin and Nakagawa (2008), which assumed that households could borrow or lend at the riskless rate. In this paper, because we are interested in characterizing the portfolio behavior of the typical, or median household, we impose the collateral constraint and nonnegativity constraints in order to model the realistic market constraints faced by a typical household.

Assuming that interest and dividend payments are reinvested so that the total return is received in the form of appreciation of the value of the asset, let $b_{i,t}$ denote the value of the ith risky asset. The vector of prices of the risky financial assets follows an n-dimensional Brownian motion process:

(5) $db_{i,t} = b_{i,t}((\mu_i + r_f)dt + d\omega_{i,t})$

Define the vector $\underline{\omega}_{F,t} = (\omega_{1,t}, \omega_{2,t}, \dots \omega_{n,t})$ as an n-dimensional Brownian motion with zero drift and with instantaneous covariance matrix Σ. Also define the corresponding vector of expected returns on financial assets as $\underline{\mu} = (\mu_1, \mu_2, \dots \mu_n)$. House prices also follow a Brownian motion:

(6)
$$dP_t = P_t(\mu_H dt + d\omega_{Ht})$$
$$dP'_t = P'_t(\mu_{H'} dt + d\omega_{H't})$$

where ω_{Ht} and $\omega_{H't}$ are Brownian motions with zero drift, instantaneous variance σ_P^2 and $\sigma_{P'}^2$, respectively, and instantaneous covariance σ_H.

Combining equations (5) and (6), define the ((n+2)x1) vector

$$(7) \qquad d\underline{\omega}_t = \begin{bmatrix} d\omega_{1t} \\ \vdots \\ d\omega_{nt} \\ d\omega_{Ht} \\ d\omega_{H't} \end{bmatrix}$$

which has instantaneous ((n+2)x(n+2)) covariance matrix Ω:

$$(8) \qquad \Omega = \begin{bmatrix} \Sigma & 0 & 0 \\ 0 & \sigma_P^2 & \sigma_H \\ 0 & \sigma_H & \sigma_{P'}^2 \end{bmatrix}$$

By specifying Ω as a block diagonal matrix, the model imposes the assumption that the stochastic component of house prices, both in the current market and in the household's next market, are uncorrelated with the returns to any of the financial assets. While the analytical results concerning the composition of the optimal portfolio require the block diagonality of the covariance matrix Ω, note that no restrictions are imposed on σ_H, the covariance of house prices in the current market with house prices in the household's next market. The block diagonality of the covariance matrix does not impose the (extremely implausible) assumption that house prices movements are uncorrelated across regions.

To characterize the household's maximization problem, let $V(H,W,P,P')$ denote the supremum of household expected utility, conditional on initial conditions (H,W,P,P').

(9) $$V(H_0, W_0, P_0, P_0') = \sup_{\underline{X}_s, C_s, \tau_1} E\left[\int_0^{\tau_1} e^{-\delta s} u\left(H_0, C_s\right) ds + e^{-\delta \tau_1} V(H_{\tau_1}, W_{\tau_1}, P_{\tau_1}, P_{\tau_1}')\right]$$

At any moment, the household decides whether to "stop", i.e., incur the transactions cost and sell the current house. Optimal stopping times are denoted $\tau_1, \tau_2, \tau_3, \ldots$. At any stopping time, the household chooses the size of the new house in order to maximize expected utility. Between stopping times, when the level of housing is fixed, the household chooses the path of nondurable consumption and the path of financial asset holdings. We are primarily interested in the household's determination of nondurable consumption and financial asset holdings during a short time interval (0,t) within which stopping does not occur. During such a time interval, wealth evolves according to:

(10) $$dW_t = \left[P_t H_0 \mu_H + \underline{X}_t \underline{\mu} - C_t\right] dt + \underline{X}_t d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht}$$

and the Bellman equation is:

(11) $$V(H_0, W_0, P_0, P_0') = \sup_{\underline{X}_s, C_s} E\left[\int_0^{t} e^{-\delta s} u\left(H_0, C_s\right) ds + e^{-\delta t} V(H_0, W_t, P_t, P_t')\right]$$

subject to the budget constraint (10) and the process for house prices (6). Subtracting $V(H_0, W_0, P_0, P_0')$ from both sides, dividing by t and taking the limit as $t \to 0$ gives:

(12) $$0 = \lim_{t \to 0} \sup_{\underline{X}_s, C_s} E\left[\frac{1}{t}\int_0^{t} e^{-\delta s} u\left(H_0, C_s\right) ds + \frac{1}{t}\left(e^{-\delta t} V(H_0, W_t, P_t, P_t') - V(H_0, W_0, P_0, P_0')\right)\right]$$

Evaluating the integral and using Ito's lemma, equation (12) can be rewritten as:

(13) $$\begin{aligned} 0 = \sup_{\underline{X}_0, C_0} \{ & u(H_0, C_0) - \delta V(H_0, W_0, P_0, P_0') + \frac{\partial V}{\partial W}(P_0 H_0 \mu_H + \underline{X}_0 \underline{\mu} - C_0) + \frac{\partial V}{\partial P} P_0 \mu_H \\ & + \frac{\partial V}{\partial P'} P_0' \mu_{H'} + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + P_0^2 H_0^2 \sigma_P^2\right) + \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{1}{2}\frac{\partial^2 V}{\partial P'^2} P_0'^2 \sigma_{P'}^2 \\ & + \frac{\partial^2 V}{\partial W \partial P} P_0^2 H_0 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P'} P_0 P_0' H_0 \sigma_H + \frac{\partial^2 V}{\partial P \partial P'} P_0 P_0' \sigma_H \ \} \end{aligned}$$

Because nondurable consumption is assumed to be costlessly adjustable, the household equates the marginal utility of nondurable consumption with the marginal value of wealth:

$$(14)\qquad \frac{\partial u}{\partial C} = \frac{\partial V}{\partial W}$$

Only two of the terms in equation (13) actually depend on financial asset holdings, $\underline{X}_0$. Thus the household chooses its portfolio of financial assets according to the rule:

$$(15)\qquad \sup_{\underline{X}_0}\left\{ \frac{\partial V}{\partial W}(P_0H_0\mu_H + \underline{X}_0\underline{\mu} - C_0) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0\Sigma\underline{X}_0^T + P_0^2H_0^2\sigma_P^2\right)\right\}$$

Restating financial asset holdings and the value of the house as shares of current wealth, define:

$$(16)\qquad \underline{x} = \frac{\underline{X}_0}{W_0}, \qquad h = \frac{P_0H_0}{W_0}$$

so that the optimization problem can be rewritten, after including the term $\frac{\partial V}{\partial W}C_0$ in the constant term, as:

$$(17)\qquad \sup_{\underline{x}}\left\{ \frac{\partial V}{\partial W}W_0(h\mu_H + \underline{x}\underline{\mu}) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}W_0^2\left(\underline{x}\Sigma\underline{x}^T + h^2\sigma_P^2\right)\right\}$$

Thus the household chooses asset shares, $\underline{x}$, in order to maximize:

$$(18)\qquad \text{objective function} = (h\mu_H + \underline{x}\underline{\mu}) - \frac{1}{2}A_0\left(\underline{x}\Sigma\underline{x}^T + h^2\sigma_P^2\right)$$

where $A_0 \equiv \dfrac{-\dfrac{\partial^2 V(H_0, W_0, P_0, P_0')}{\partial W_0^2}\,W_0}{\dfrac{\partial V(H_0, W_0, P_0, P_0')}{\partial W_0}} \geq 0$

subject to the constraint

$$(19)\qquad 1 = h + \underline{x}\underline{\ell}$$

and the nonnegativity constraints (equation (4b)) on the elements of $\underline{x}$.

Equation (18) states that the household's objective function is an increasing function of the expected return, $\underline{x}\underline{\mu}$, and a decreasing function of the variance, $\underline{x}\Sigma\underline{x}^T$, of the portfolio of financial assets. Thus we can interpret equation (18) as saying that the optimal choice of $\underline{x}$ will be on the mean-variance efficient frontier of financial assets. The implication that the optimal portfolio will be mean-variance efficient does not require a specific assumption, such as constant relative risk aversion, on the instantaneous utility function.

The derivation of equation (18) required the assumption that the covariance matrix is block diagonal as specified in equation (8). The dependence of the mean-variance efficiency result on the assumption of block-diagonality can be understood intuitively. Due to the transactions costs associated with selling the house, the optimization problem has the following recursive structure. The household first considers whether it is optimal to sell the house immediately; i.e., considers whether the t=0 is a stopping time. If the event that t=0 is not a stopping time, the household has essentially decided to hold P_tH_t in the form of housing for (at least) this instant and is therefore subject to an instantaneous expected return and standard deviation of return on the house as determined by the parameters μ_H and σ_P^2. If the covariance matrix is block diagonal, returns to financial assets are uncorrelated with current house prices and with future house prices. In this case, even though the risk averse household will dislike the risk created by variability in current (P) or future (P') house prices, the household is unable to hedge either of these types of risk with the portfolio of financial assets. Since financial assets cannot be used to hedge the risks associated the current or future housing, the model implies that the optimal vector of financial assets will achieve mean variance efficiency with respect to the portfolio of financial assets.

Although the model has the implication that the optimal vector of financial assets holdings will be mean-variance efficient, this result does not imply that the household's optimal portfolio is independent of the current holding of housing assets, or of the level of housing prices. The state variables that characterize the housing sector (H_t, P_t, and $P_t^{'}$) influence the optimal portfolio of financial assets in two ways: First, in determining the location of the constrained mean-variance efficient frontier available to the household, and second, in determining the household's degree of risk aversion (A_t) and thus its optimal location on the constrained frontier. The effect of the housing state variables on the location of the constrained efficient frontier is a result of the assumptions that households can borrow only in the form of a mortgage, and, further, the size of the mortgage is limited to 100% of the value of the house. That is, the household optimizes its portfolio of financial assets subject to the collateral constraint given in equation (4): a household whose house value exceeds net worth must hold a mortgage with minimum size equal to $[P_t H_t - W_t]$ and with maximum size equal to $[P_t H_t]$. Because the minimum and maximum constraints on the holdings of one of the financial assets (the mortgage) depend directly on the house value, $P_t H_t$, the constrained mean-variance efficient frontier available to the household also depends on the house value.

If, as in Flavin and Nakagawa (2008), we dropped the collateral and nonnegativity constraints and simply assumed interior solutions for every element of $\underline{x}$, we could differentiate equation (18) with respect to $\underline{x}$ and obtain an analytical solution for the portfolio shares:

$$\text{(20)} \qquad \underline{x} = \left[\frac{-\dfrac{\partial V}{\partial W}}{\dfrac{\partial^2 V}{\partial W^2} W_0} \right] \Sigma^{-1} \underline{\mu}$$

which implies that all households hold risky assets in the same proportions, the mutual fund separation theorem holds, and that the CAPM holds. However, under the current assumption that the household faces the constraints in equation (4), the possibility of corner solutions for some of the elements of $\underline{x}$ implies that equation (18) requires numerical optimization rather than analytical solution.

The two terms in parentheses on the right hand side of equation (18) represent the expected return and the variance of the asset portfolio inclusive of housing. Because the covariance matrix is block diagonal and h is a state variable, any vector $\underline{x}$ that achieves mean-variance efficiency with respect to the portfolio of financial assets also achieves mean-variance efficiency with respect to the whole portfolio inclusive of housing. Thus the model not only implies that the optimal portfolio will be mean-variance efficient; it further implies that the mean-variance efficiency property applies both to the portfolio of financial assets and to the portfolio inclusive of housing. Whether we choose to think about the efficient frontier in terms of the expected return and standard deviation of the portfolio inclusive of housing or in terms of the expected return and standard deviation of the portfolio of financial assets alone, the state variable h affects the efficient frontier via the corner constraints.

From among the set of optimal portfolios on the constrained efficient frontier, the optimal portfolio is determined by the household's tradeoff between risk and return as represented by the curvature of the value function, A_t.

$$(21) \qquad A_t \equiv -\frac{\dfrac{\partial^2 V(W_t, H_t, P_t, P_t^{'})}{\partial W_t^2}}{\dfrac{\partial V(W_t, H_t, P_t, P_t^{'})}{\partial W_t}} W_t > 0$$

In general, the curvature of the value function will depend on the values of all of the state variables. Thus the optimization problem of the household can be written as:

$$\sup_{\underline{x}}\left(\mu - \frac{A_t}{2}\sigma^2\right) \quad \text{subject to the constraints (4) and} \quad 1 = h + \underline{x}\ell \quad \text{where} \tag{22}$$

$$\mu \equiv h\mu_H + \underline{x}\underline{\mu} \equiv \text{expected return on portfolio, inclusive of housing} \tag{23}$$

$$\sigma^2 \equiv \underline{x}\Sigma\underline{x}^T + h^2\sigma_P^2 \equiv \text{variance of return on portfolio, inclusive of housing} \tag{24}$$

From equation (22), the slope of the household's indifference curve is:

$$\frac{\partial\mu}{\partial\sigma} = A_t\sigma \tag{25}$$

With the constrained efficient frontier and the indifference curve, we can identify the household's optimal portfolio as a function of its constraints, as measured by h, and its degree of risk aversion, as measured by the curvature of the value function, A_t.

Section 2: Optimal portfolios as a function of risk aversion and the housing constraint

For tractability, the housing model in Section I relied on the assumption that the covariance matrix is block diagonal. In a previous paper, Flavin and Yamashita (2002), we estimated the mean return, and the covariance matrix of returns, to housing, a mortgage, T-bills, T-bonds, and stocks, using household level data from the PSID from 1968-1992. Table 1 reports the expected returns, covariance matrix, and correlation matrix based on the PSID data.

Table 1: Expected Returns and Covariance Matrix – PSID Data

	T-Bills	Bonds	Stocks	Mortgage	House
Mean Return (arithmetic)	-.0038	.0060	.0824	.0000	.0659
Standard Deviation	.0435	.0840	.2415	.0336	.1424
Covariance Matrix					
T-Bills	.0018920				
T-Bonds	.0025050	.0070613			
Stocks	.0002008	.0040381	.0583292		
Mortgage	.0007087	.0023854	.0025400	.0011274	
House	-.000119	-.000067	-.000178	-.0000057	.020284
Correlation Matrix					
T- Bills	1.0000				
T-Bonds	.68533	1.0000			
	(.09103)				
Stocks	.01912	.19897	1.0000		
	(.12498)	(.12251)			
Mortgage	.84119	.680286	.467954	1.0000	
	(.11529)	(.15626)	(.18842)		
House	-.03339	-.004506	-.000771	-.001192	1.0000
	(.21309)	(.21320)	(.21319)	(.21320)	

Source: Flavin and Yamashita (2002). Standard errors are in parentheses.

According to the bottom row of the correlation matrix, the assumption that the covariance matrix is block diagonal in the sense that the return to housing is uncorrelated with the return to each of the financial assets is fully consistent with the data from the PSID. In each case, the correlation of the return to T-bills, T-bonds, stocks, and mortgages has a correlation with the return to housing which is essentially zero both in terms of numerical size and statistical significance.

While the historical data is valuable for testing the validity of the block-diagonality assumption, examination of the vector of mean returns over this sample period provides an illustration of the distinction between sample moments and population moments. Ex post, the average after-tax, real return on T-bills was slightly negative, the average after-tax rate on mortgages was zero (to four decimal places!), and the return to Treasury bonds was only 60 basis points. While these statistics accurately characterize the historical returns, ex post, it seems unlikely that actual households were making their portfolio decisions based on the ex ante belief that the average returns to these nominal assets would be so low.

Since they don't know the exact population moments of asset returns, households make their portfolio decision based on some subjective assessment of the process generating asset returns. To calculate the optimal portfolios, we attempt to write down the subjective assumptions on the risk and return on which we base on own household portfolio decisions. Further, we calculate the optimal portfolios for several different sets of assumptions on the moments of asset returns to check the robustness of the results.

The baseline set of assumptions is reported in Table 2a; the after-tax real return on T-bills is assumed to be small but positive (.01), the return on bonds and mortgages is equal at .03, and the return on stocks (.07) is slightly higher than the return on housing (.05). For the baseline case, the assumed covariance matrix of returns is a rough approximation to the covariance matrix estimated from the PSID, although the numerical values are limited to only one or two significant digits.

Under the baseline assumptions, the optimal portfolios (that is, the solution to the optimization problem in equation (22) for a range of values of the state variable, h, and of the relative risk aversion of the household) are reported in Table 2b. The table reports the optimal holdings of T-bills, T-bonds, and stocks as percentages of the portfolio of financial assets and the size of the mortgage is expressed as a percent of the house value (thus a mortgage value of -1 reflects a 100% mortgage). Thus any cell that reports a value of unity or zero for the share of financial assets, or a negative one for the mortgage, represents a portfolio in which at least one of the corner constraints is binding.

The nonnegativity constraint on T-bills is almost always binding. Only when the total net worth is twice the value of the house, and risk aversion is high does the optimal portfolio contain a strictly positive amount of T-bills. For households that are highly risk tolerant (with relative risk aversion of unity), the optimal strategy is to borrow the maximum against the house, and put all of the household's net worth into stocks, independent of the value of h. For a given value of h, higher values of relative risk aversion induce the household to decrease the share of the portfolio held in stocks, and at the same time reduce leverage by reducing the loan-to-value ratio on the house.

For a given value of risk aversion, as the value of the h declines the optimal portfolio is characterized by a lower loan-to-value ratio, and, in general, and increase in the share of the portfolio devoted to stocks. The dependence of the portfolio share devoted to stocks is not monotonic in h over the whole range, however. If the ratio of house value to net worth is less than one, the optimal share devoted to stocks declines with further increases in h for moderate and high levels of risk aversion.

Table 2a: Baseline assumptions on mean returns and covariance matrix of returns

	T-Bills	Bonds	Stocks	Mortgage	House
Mean Return (arithmetic)	0.01	0.03	0.07	0.03	0.05
Standard Deviation	0.04	0.10	0.20	0.04	0.15
Covariance Matrix					
T-Bills	0.0016				
T-Bonds	0.0025	0.010			
Stocks	0.0005	0.005	0.040		
Mortgage	0.0010	0.003	0.003	0.0016	
House	0	0	0	0	0.0225

Table 2b: Optimal Portfolio Weights for Different Constraints on *h*

Housing-to NW Ratio	Assets in Portfolio	Curvature of value function, A				
		A = 1	A = 2	A = 4	A = 8	A = 10
3.50	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.3750	0.6132	0.7242	0.7506
	Stocks	1	0.6250	0.3868	0.2758	0.2494
	Mortgage	-1	-1	-0.9871	-0.9512	-0.9440
3.00	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.3750	0.5679	0.6934	0.7242
	Stocks	1	0.6250	0.4321	0.3066	0.2758
	Mortgage	-1	-1	-0.9380	-0.8961	-0.8878
2.50	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.3342	0.5037	0.6468	0.6837
	Stocks	1	0.6658	0.4927	0.3532	0.3163
	Mortgage	-1	-0.9698	-0.8693	-0.8191	-0.8090
2.00	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.2372	0.4057	0.5679	0.6132
	Stocks	1	0.7628	0.5943	0.4321	0.3868
	Mortgage	-1	-0.8920	-0.7663	-0.7035	-0.6910
1.50	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.0977	0.2372	0.4057	0.4640
	Stocks	1	0.9023	0.7628	0.5943	0.5396
	Mortgage	-1	-0.7621	-0.5946	-0.5109	-0.4941
1.00	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0	0	0	0
	Stocks	1	1	1	1	1
	Mortgage	-1	-0.5618	-0.2809	-0.1404	-0.1124
0.75	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0	0	0.3750	0.4750
	Stocks	1	1	1	0.6250	0.5250
	Mortgage	-1	-0.4026	-0.0281	0	0
0.50	**Treasury Bills**	0	0	0	0.2157	0.3961
	Treasury Bonds	0	0	0.3750	0.4255	0.3086
	Stocks	1	1	0.6250	0.3588	0.2953
	Mortgage	-1	-0.0843	-0.0281	0	0

Note: Shares of T-bills, bonds, and stocks are stated as a percentage of the portfolio of financial assets, so that for each portfolio the shares of these three assets must sum to one. The mortgage is expressed as a percent of the house value, i.e., Mortgage = -1 indicates a 100% mortgage.

Table 3a: Relative to baseline, higher mean and s.d. of stocks; lower mean and s.d. of house

	T-Bills	Bonds	Stocks	Mortgage	House
Mean Return (arithmetic)	0.01	0.03	.07 to **0.09**	0.03	.05 to **0.03**
Standard Deviation	0.04	0.10	.20 to **0.25**	0.04	.15 to **0.10**
Covariance Matrix					
T-Bills	0.0016				
T-Bonds	0.0025	0.010			
Stocks	0.0005	0.005	**0.0625**		
Mortgage	0.0010	0.003	0.003	0.0016	
House	0	0	0	0	**0.010**

Table 3b: Optimal Portfolio Weights for Different Constraints on *h*

Housing-to-NW Ratio	Assets in Portfolio	Curvature of value function, A				
		A = 1	A = 2	A = 4	A = 8	A = 10
	Treasury Bills	0	0	0	0	0
	Treasury Bonds	0	0.4400	0.6545	0.7678	0.7946
3.50	Stocks	1	0.5600	0.3455	0.2322	0.2054
	Mortgage	-1	-1	-0.9725	-0.9396	-0.9330
	Treasury Bills	0	0	0	0	0
	Treasury Bonds	0	0.4400	0.6080	0.6934	0.7678
3.00	Stocks	1	0.5600	0.3920	0.2635	0.2322
	Mortgage	-1	-1	-0.9231	-0.8846	-0.8769
	Treasury Bills	0	0	0	0	0
	Treasury Bonds	0	0.3653	0.5418	0.6889	0.7265
2.50	Stocks	1	0.6353	0.4582	0.3111	0.2735
	Mortgage	-1	-0.9462	-0.8539	-0.8077	-0.7985
	Treasury Bills	0	0	0	0	0
	Treasury Bonds	0	0.2632	0.4400	0.6080	0.6545
2.00	Stocks	1	0.7368	0.5600	0.3920	0.3455
	Mortgage	-1	-0.8654	-0.7500	-0.6923	-0.6808
	Treasury Bills	0	0	0	0	0
	Treasury Bonds	0	0.1148	0.2632	0.4400	0.4694
1.50	Stocks	1	0.8852	0.7368	0.5600	0.5031
	Mortgage	-1	-0.7308	-0.5769	-0.500	-0.4941
	Treasury Bills	0	0	0	0	0
	Treasury Bonds	0	0	0	0	0
1.00	Stocks	1	1	1	1	1
	Mortgage	-1	-0.5164	-0.2582	-0.1291	-0.1033
	Treasury Bills	0	0	0	0	0
	Treasury Bonds	0	0	0	0.4400	0.5360
0.75	Stocks	1	1	1	0.5600	0.4640
	Mortgage	-1	-0.3471	-0.0029	0	0
	Treasury Bills	0	0	0	0.2157	0.3961
	Treasury Bonds	0	0	0.4400	0.4529	0.3322
0.50	Stocks	1	1	0.5600	0.3086	0.2520
	Mortgage	-1	-0.0086	0	0	0

Table 4a: Relative to baseline, lower mean and s.d. of stocks; higher mean and s.d. of house

	T-Bills	Bonds	Stocks	Mortgage	House
Mean Return (arithmetic)	0.01	0.03	.07 to **0.05**	0.03	.05 to **0.07**
Standard Deviation	0.04	0.10	.20 to **0.15**	0.04	.15 to **0.20**
Covariance Matrix					
T-Bills	0.0016				
T-Bonds	0.0025	0.010			
Stocks	0.0005	0.005	**0.0225**		
Mortgage	0.0010	0.003	0.003	0.0016	
House	0	0	0	0	**0.040**

Table 4b: Optimal Portfolio Weights for Different Constraints on *h*

Housing-to-NW Ratio	Assets in Portfolio	Curvature of value function, A				
		A = 1	A = 2	A = 4	A = 8	A = 10
3.50	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.3333	0.5556	0.6548	0.6766
	Stocks	1	0.6667	0.4444	0.3452	0.3243
	Mortgage	-1	-1	-1	-0.9724	-0.9653
3.00	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.3333	0.5227	0.6291	0.6548
	Stocks	1	0.6667	0.4773	0.3709	0.3452
	Mortgage	-1	-1	-0.9571	-0.9158	-0.9076
2.50	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.3162	0.4671	0.5900	0.6210
	Stocks	1	0.6838	0.5329	0.4100	0.3790
	Mortgage	-1	-0.9851	-0.8861	-0.8366	-0.8267
2.00	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.22270	0.3805	0.5227	0.5614
	Stocks	1	0.7730	0.6195	0.4773	0.4386
	Mortgage	-1	-0.9035	-0.7797	-0.7178	-0.7054
1.50	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0.0951	0.2270	0.3805	0.4291
	Stocks	1	0.9049	0.7730	0.6195	0.5709
	Mortgage	-1	-0.7621	-0.6023	-0.5198	-0.5033
1.00	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0	0	0	0
	Stocks	1	1	1	1	1
	Mortgage	-1	-0.5525	-0.2762	-0.1381	-0.1105
0.75	**Treasury Bills**	0	0	0	0	0
	Treasury Bonds	0	0	0	0.3333	0.4222
	Stocks	1	1	1	0.6667	0.5778
	Mortgage	-1	-0.3775	-0.0092	0	0
0.50	**Treasury Bills**	0	0	0	0.1882	0.3710
	Treasury Bonds	0	0	0.3333	0.3925	0.2785
	Stocks	1	1	0.6667	0.4193	0.3505
	Mortgage	-1	-0.0276	0	0	0

Table 3a states an alternative set of assumptions on the stochastic process of asset returns.

Here stocks are assumed to have a higher expected return (.09 instead of .07) and higher standard deviation of return (.25 instead of .20), while housing is assumed to have a lower expected return (.03 instead of .05) and lower standard deviation (.10 instead of .15). The resulting optimal portfolios are reported in Table 3b. Comparison of Tables 2b and 3b indicates that the quantitative effect on the optimal portfolio shares is modest.

A third set of assumption is considered in Table 4a. Here the expected return and standard deviation of stocks is lower than in the baseline case (expected return reduced from .07 to .05 and standard deviation of return reduced from .20 to .15), while the expected return and standard deviation of returns to housing are increased (mean return increased from .05 to .07 and standard deviation increased from .15 to .20). Again, the optimal portfolios generated under the new set of assumptions are not dramatically different from those generated from the baseline assumptions. The portfolios generated by any of the three sets of assumptions conform to the same set of qualitative characteristics: First, the nonnegativity constraint on T-bills is almost always binding, second, the share of the portfolio held in the form of stocks is decreasing in the value of h over most of its range.

Section 3: Cross-Year, Cross-Section and Cohort Analysis

Our model predicts that the share of the portfolio held in the form of risky assets generally increases as the ratio of house value to net worth declines, which is often associated with life-cycle wealth accumulation patterns. In this section we examine the empirical relationships among the house-value-to-net-worth ratio (h), the stock-to-financial-assets ratio (s), and the loan-to-value (LTV) ratio using the six waves of the Survey of Consumer Finances (SCF). Given the limitations of the available asset data, and because the model abstracts from many other factors that affect portfolio composition (variation in the riskiness of labor income, health risk, factors that can lead to limited participation in some markets, dramatic innovations to the financial market themselves in the form of the proliferation of available mutual funds, credit scoring, and flexible rate mortgages, to name a few), we do not attempt to test the model statistically. Instead, we use the SCF data to illustrate the relationship between the variables of interest in the cross section, and by constructing synthetic cohorts from the repeated cross section of the SCF. The sample period includes two recessions (1990-1991 and 2001) and two asset-price booms, the .com boom in the stock markets led by the technology stocks (1995-2001) and the housing boom (2001-2004). We construct household balance sheets using the program provided by the Board of Governors in its SCF web page (Federal Reserve Board (2008)).[2] We limit our sample to households with heads between age of 24 and 89 at the time of survey.[3]

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[2] The SCF imputes missing values of responses to the survey questions. We use all five imputations and take

Table 5 summarizes key sample statistics that describe the evolution of American households' wealth holdings between 1989 and 2004. The level of net worth of the median household increased from $92,000 in 1989 to $125,000 in 2004, with a decline after the 1991 recession and a peak in 2001 after a stock market boom. The median value of the principal residence, which stayed around $110,000 until 1995, increased rapidly after 1998 and reached $167,000 in 2004. The importance of housing is reflected in the change in the value of *h*, from 0.79 in 1989 to 0.84 in 2004. This ratio declined to below 0.65 in 2001 reflecting growing financial wealth of the household in the late 1990s but the stagnant housing markets at the time. Homeownership has been stable around 67 percent until 1995, but then started to climb in 1998 and reached a high of 72.5 percent in 2004. Gabriel and Rosenthal (2005) show that the demographic changes could explain a large part of this increase in homeownership rate.

The growth of stock ownership and the increase in the portfolio share of stocks represent the most remarkable changes in the household balance sheets in the late 1990s. In 1989, only 32.7

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arithmetic means of the five replicates. We then use the main replicate weight to arrive at estimates of sample statistics such as means and medians.

[3] We also limit our sample to observations that have non-negative net worth.

Table 5: Summary Statistics from the Survey of Consumer Finances, 1989-2004

(dollar amounts in constant 2004 dollars)

	1989		1992		1995		1998		2001		2004	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Age of Head	49.8	47	50.1	47	50.0	47	50.4	48	50.7	48	51.2	50
Transaction accounts	24,065	2,932	21,278	2,768	20,241	2,462	23,456	3,594	33,131	4,260	28,815	3,808
Stocks												
directly held	19,144	0	20,275	0	28,593	0	54,548	0	66,562	0	58,399	0
pensions & retirement a/c	6,054	0	11,112	0	16,253	0	30,099	0	42,293	0	39,083	0
other	3,823	0	2,123	0	4,090	0	8,139	0	15,681	0	10,189	0
Total	28,957	0	33,511	0	48,936	0	92,786	232	124,537	820	107,672	2,500
Financial assets	126,736	17,440	133,908	17,214	160,877	21,605	233,558	36,171	296,717	40,271	210,066	27,224
Primary residence [a/]	163,953	109,955	150,859	112,023	148,634	113,259	168,275	117,092	202,463	138,463	257,569	167,000
Mortgage outstanding [a/]	43,937	17,593	49,481	22,721	52,538	27,083	60,578	34,780	67,338	40,474	89,564	58,000
Net worth	316,244	91,805	303,205	86,495	327,041	96,171	428,631	121,521	542,116	138,357	501,945	124,600
$\frac{\text{stocks}}{\text{financial assets}}$	0.093	0	0.121	0	0.144	0	0.195	0.054	0.212	0.070	0.271	0.170
$\frac{\text{house value}}{\text{net worth}}$ [a/]	1.164	0.788	1.614	0.792	2.816	0.754	1.383	0.650	1.142	0.646	1.656	0.844
Homeownership rate (%)	67.1		67.1		66.6		69.2		70.5		72.5	
Ownership of stocks (%)	32.7		38.5		41.1		50.7		52.6		57.1	
No. of obs.	2,806		3,442		3,958		3,911		4,073		4,129	

Note: The sample is limited to the households with the head 24 years or older, and with non-negative net worth. Nominal values are adjusted for inflation using the CPI-U deflator of each survey year provided in the Federal Reserve's web page of the SCF. Summary statistics are calculated with the analysis weight provided with the SCF.

a/ Mean and median are calculated only for homeowners.

percent of the sample households owned stocks, but in 1998 the stock ownership rate exceeded 50 percent. As defined-benefit retirement plans are replaced with defined-contribution plans, more workers are enrolled in 401(k)-type retirement plans, which often include stock portfolios. In 2004, 57.1 percent of American households report that they own stocks directly or indirectly. Despite the wide-spread ownership of stocks, however, the median household holds relatively small amount in stocks ($2,500) or only 17 percent of total financial wealth in 2004.

In spite of the considerable increase in homeownership between 1998 and 2004, its life-cycle and cohort patterns have stayed relatively stable (figure 1). The homeownership rate increases rapidly for younger households, surpassing the 50 percent mark around age 30, and then stabilizes at about 80 percent after age 49. The homeownership rate shows a decline only at the very advanced age of 79 and older. In figure 1 (b), which plots cohort dynamics of homeownership,[4] we detect little variation across cohorts in becoming a homeowner early in life and then again a renter at a very late stage in life.

We plot the median of *h* against age of homeowners in figure 2. During the most of the sample period, *h* remains relatively stable until 2001. The value of *h* increased considerably in

[4] We follow the standard practice and group households into 21 three-year cohorts by the age of the head, from 25 to 85 in 1989. For example, the youngest cohort in 1989 includes households whose head was between 24 and 26 years old in 1989, and we follow this group as they age to 39 to 41 in 2004. The oldest group was between age 84 and 86 in 1989, reaches age 87 to 89 in 1992, and then drops out of the sample. As older cohorts exits our sample, newer cohorts enter. For example, the cohort between 24 and 26 years old in 2004 was nine to 11 years old in 1989. Each cohort is identified by the mid-point within each age group in 1989. For example, the “cohort 25” denotes the households whose heads were between 24 and 26 in 1989.

2004, principally for those younger than 37 years old, reflecting the financial stretch of younger households to buy into homeownership in the booming markets. The increase of *h* between 2001 and 2004 is particularly remarkable for cohorts 16, 19, and 22 for whom the ratio increased as much as 90 percent. For older cohorts, the increases in *h* between 2001 and 2004 are less pronounced and the ratio exhibits similar patterns throughout time and across different cohorts.

We interpret the variation in *h* to arise from life-cycle patterns of asset accumulation. At a young age, investment demand for housing is often constrained by consumption demand for housing services. Consequently, young homeowners are highly leveraged and invest in homes that are two to three as much as their net worth. As they accumulate financial assets over the course of the life cycle, *h* declines steadily to about 0.6 in their late 50s. This ratio stays relatively constant until a very late age. Only at a very advance age homeowners seem to decrease financial assets faster than they downsize their home, as evidenced by an increase in *h* after age 76. Alternative interpretations of the age profile of *h* are of course possible. To explain the observed pattern, however, we would have to resort to a complex combination of age, time and cohort effects. Because we cannot identify the pure lifecycle factors, or the age effect, from the cohort and time effects in the available data, we would have to make an assumption. In this paper, we attribute the observed pattern in *h* to the age effect as the age-effect assumption is a simple and intuitive one, and fits well to explain its relative invariance across years under varying housing market circumstances. Because we cannot test our identifying assumptions, our criteria for choosing one assumption over another are its simplicity and its reasonableness, and the age-effect assumption satisfies both. As we will see later, this assumption helps us explain the life-cycle pattern of stockholdings.

Figure 3 portrays changes of stock ownership rates over the life cycle. There is a mild hump shape in the age profile of stockownership rate peaking in the mid-50s and declining thereafter. Later-born cohorts are often more likely to own stocks than their earlier counterparts, although such cohort patterns are less obvious among the older cohorts.

Figures 4 and 5 plot the mean and median portfolio shares of stocks, *s*, respectively, by year and by cohort. These four figures reveal two facts: the share of financial assets invested in stocks rose throughout the sample period with a largest increase taking place between 2001 and 2004, and the increases concentrate among the younger cohorts. For the cohorts younger than 55 in 1989, each subsequent cohort almost always invests a higher fraction of assets in stocks than its older counterpart at the same age. For the older cohorts, *s* has grown at a much slower pace and cohort patterns are less pronounced.

Figure 4 also demonstrates that there is a gentle rise in the portfolio share of stocks from a young age to middle age peaking in the late 50s. This hump-shaped age profile is most pronounced in 2004 when house values increased relative to other financial assets for many households. The hump-shaped age profile is even more pronounced in figure 5 that plots the median *h*. In addition, the cohort profiles of the younger cohorts rose parallel to each other (figures 4(b) and 5(b)), indicating each cohort has raised the portfolio share of stocks at the same pace.[5] A combination of age and time effects could possibly explain this pattern. A time effect (e.g., the stock market boom) causes all individuals to raise their portfolio share of stocks every period, while an age effect leads them to increase stock investment as they get older. In terms of our model, a hump-shaped pattern of the cross-sectional profile is coming from age effects related to housing investment, and the stock market boom in the late 1990s caused the parallel rise in cohort profiles. While other interpretation is possible,[6] we are not able to distinguish the age effects from other factors without a theory. Our model, however, suggests that investment in housing exerts strong effects on the portfolio composition of financial assets and that the life-cycle pattern of *s* might be induced by the life-cycle variation in *h*. Our model thus offers implications for how the proportion of financial assets invested in stocks varies with *h*, hence with age. In other words, if we assume that the variation of *h* over life cycle is driven by the

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[5] Ameriks and Zeldes (2000) offer interpretations for three patterns of cross-section and cohort variations in portfolio shares. For example, the pattern of *s* (in Figure 4) corresponds to Exhibit 1 for the younger cohorts and Exhibit 3 for the older cohorts in their paper.

[6] For example, a combination of age and cohort effects without time effect may also explain this pattern, i.e., each cohort holds more equity at any given age than an earlier cohort, and all individuals increase their holdings of stocks as they age, regardless of their birth cohort.

pure age effect, then our model provides explanations for age profiles of stockholding observed in the data.[7]

We plot in figure 6 the mean and median of s against h to examine whether investment in housing is related to stockholding patterns.[8] Even disregarding a dent around $h = 1$, there is a negative relationship between investment in stocks and the ratio of house value to net worth: Homeowners with a higher value of h invest a smaller proportion of financial assets in stocks. This negative pattern is more strongly articulated in figure 6(b) that plots the median s.

The pattern of stock ownership reveals that this negative relationship between s and h arises from non-participation in the stock markets by homeowners with high values of h.[9] We plot the stock-ownership rate against h in figure 7, which shows a mirror image of the equity share in figure 6; homeowners with high values of h are less likely to hold stocks, and the probability of owning stocks increases as h goes down. The dip near $h = 1$ is explained by looking at types of

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[7] In contrast to younger cohorts, the age profiles of stockholding for older cohorts are more complex, as the slopes of the cohort profiles differ by cohort. The age at which this happens coincides with the age at which the value of h stabilizes at around $h = 0.6$. We suspect that the reason why at an older age the age profile becomes more complex is partly that the effects of h become weaker as the value of h gets smaller and heterogeneity among investors manifests more strongly at an older age.

[8] Households with the value of h between 0.05 and 1.05 are grouped into 0.1 intervals of h. For example, those with $0.05 \leq h < 0.15$ was grouped as $h = 0.1$, $0.15 \leq h < 0.25$ as $h = 0.2$, and so on. For larger values of h, the categories are coarser because the numbers of observations are small; we classify $1.05 \leq h < 1.2$ as $h = 1.125$, $1.2 \leq h < 1.45$ as $h = 1.325$, $1.45 \leq h < 2.05$ as $h = 1.75$, $2.05 \leq h < 2.95$ as $h = 2.5$, and $2.95 \leq h$ as $h = 3$. For very small values of h ($0 < h < 0.05$), the value of h is set to 0.001 to draw figures. This classification keeps the number of observations in each category roughly equal.

[9] Attanasio, Banks, and Tanner (2002) and Vissing-Jorgenson (2002) analyze the importance of non-participation in stock market for asset pricing. Vissing-Jorgenson estimates that a small fixed cost of participation and transaction is sufficient to explain non-participation of the majority of non-stockholders.

households included in these groups. Many of the households that have the value of *h* near 1 are those whose assets primarily consist of equity in owner-occupied housing with very little financial assets and a small amount of mortgages. They also tend to be older.[10] If we limit our sample to those who have at least $3,000 in financial assets, then the dip near *h* = 1 becomes less prominent.

To verify the importance of non-participation by some homeowners, we also plot the mean and median of *s* against *h* only for stockholders in figure 8. The *s*-*h* profiles for stockholders are flat, indicating that the downward sloping curves in figure 6 are induced by homeowners who do not invest in stocks. Effects of non-participation by risk-averse investors also manifest in the age profiles of *s* for stockholders. In figures 9 and 10, we plot the mean and median of *s* against age only for those with positive amounts of stockholding. In contrast to the age profiles of *s* for the entire sample (figures 4 and 5), the portfolio share of stocks does not vary with age. Therefore, the age profile of *s* must be driven by a changing mix of stockholders and non-stockholders among homeowners: Only risk-tolerant households enter the stock markets at a younger age, while those who are more risk averse delay entry into the stock markets because they bear high

[10] Households with the value of *h* near 1 have substantially smaller amount of financial assets compared to those with lower or higher values of *h*. For example, in 2004 the median value of financial assets for households with *h* = 0.9 and 1 is $16,288 and $8,000, respectively, compared to $40,204 and $22,000 for *h* = 0.8 and 1.125, respectively. Similarly, the median age for those with *h* = 0.9 and 1 is 58 and 52 in 2004, respectively, compared to 55 and 48 for *h* = 0.8 and 1.125, respectively.

risk from overinvestment in their homes. Consequently, the average share of financial assets invested in stocks appears low for the young with many non-participants (figures 4 and 5). If we limit our attention to stockholders only, on the other hand, the average young investor is more risk tolerant than the average old investor, which in turn produces flat age profiles (figures 9 and 10).

To investigate further our claim that investment in housing accounts for a large part of the variation in the portfolio composition, we plot the age profile of stockholding of non-homeowners in figure 11. If the stockholding pattern of non-homeowners exhibits similar age profiles to those of homeowners, then homeownership may not be an explanation for the pattern of stockholding. Conversely, if non-homeowners behave systematically differently from homeowners in terms of their investment decisions, it is likely that investment in housing is related to financial portfolios. In figure 11, the mean of *s* for non-homeowners exhibits no clear age pattern.[11]

Other factors, such as the homeowner's leveraged position, could induce the pattern of *s* against *h*, as homeowners with a high value of *h* tend to have a high LTV ratio. If the LTV ratio explains better the age pattern of stockholding, then *h* in our model may simply be a proxy for the LTV ratio and our model is misspecified. On the other hand, if there is no relationship

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[11] The median of *s* for non-homeowners is zero for all ages throughout all years. Thus we only plot the mean.

between the LTV ratio and *s*, then our model of households optimizing the composition of portfolios taking *h* as a state variable would be valid. In figure 12, we plot the mean and median of *s* against the LTV ratios.[12] Although there appears to be a negative association between *s* and the LTV ratio, the relationship is very weak at best and may not exist in some years. Thus the ratio of house value to net worth, *h*, seems to be a better candidate to explain the variation in the proportion of financial assets invested in stocks over the life cycle.

[12] The values of LTV ratio for homeowners are grouped into twelve categories: those with no mortgages (LTV = 0), 0 < LTV < 0.05, nine 0.1 intervals for those with LTV between 0.05 and 0.95, and those with LTV of 0.95 or bigger.

References

Ameriks, John and Stephen P. Zeldes, 2000, How Do Household Portfolio Shares Vary with Age? Unpublished manuscript, Columbia University.

Attanasio, Orazio P., James Banks, and Sarah Tanner, 2002, Asset Holding and Consumption Volatility, *Journal of Political Economy* 110, 771-792.

Cauley, Stephen Day, Andrey D. Pavlov, and Eduardo S. Schwartz, 2007, Homeownership as a Constraint on Asset Allocation, *Journal of Real Estate Finance and Economics* 34, 283-311.

Cocco, Joao, 2005, Portfolio Choice in the Presence of Housing, *Journal of Financial Studies* 18, 535-567.

Faig, Miguel, and Pauline Shum, 2002, Portfolio Choice in the Presence of Personal Illiquid Projects, *Journal of Finance* 57, 303-328.

Federal Reserve Board, 2008, SAS Program Used to Create Summary Variables: Detailed Definitions, (Washington, DC).

Flavin, Marjorie, and Shinobu Nakagawa, 2008, A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence, *American Economic Review* 98, 474-495.

Flavin, Marjorie, and Takashi Yamashita, 2002, Owner-Occupied Housing and the Composition of the Household Portfolio, *American Economic Review* 92, 345-362.

Gabriel, Stuart A., and Stuart S. Rosenthal, 2005, Homeownership in the 1980s and 1990s: Aggregate Trends and Racial Gaps, *Journal of Urban Economics* 57, 101-127.

Gomes, Francisco, and Alexander Michaelides, 2005, Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence, *Journal of Finance* 60, 869-904.

Grossman, Sanford J., and Guy Laroque, 1990, Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods, *Econometrica* 58, 25-51.

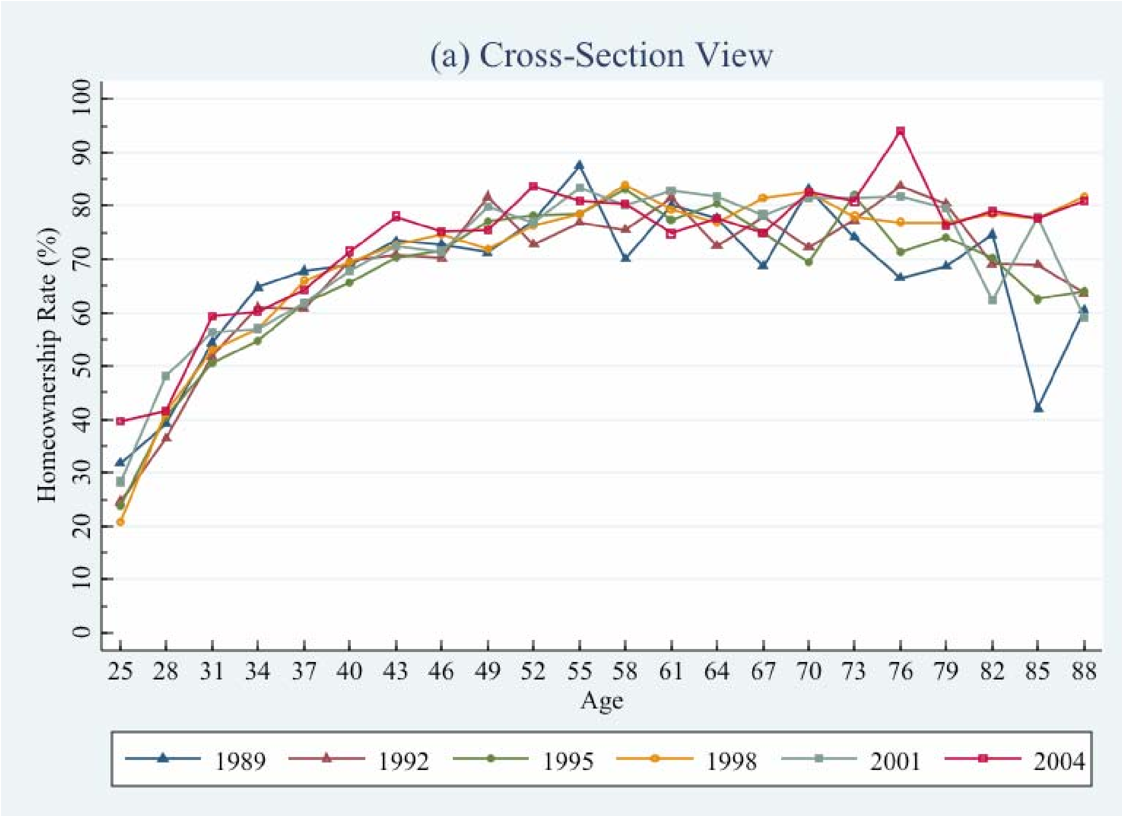
Hu, Xiaoqing, 2005, Portfolio Choices for Homeowners, *Journal of Urban Economics* 58, 114-136.

Hurst, Erik, and Annamaria Lusardi, 2004, Liquidity Constraints, Household Wealth, and Entrepreneurship, *Journal of Political Economy*, 112, 319-347.

Vissing-Jorgensen, Annette, 2002, Limited Asset Market Participation and the Elasticity of Intertemporal Substitution, *Journal of Political Economy* 110, 825-853.

Yao, Rui, and Harold H. Zhang, 2005, Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints, *Journal of Financial Studies* 18, 197-239.

Figure 1 Homeownership Rate, 1989-2004



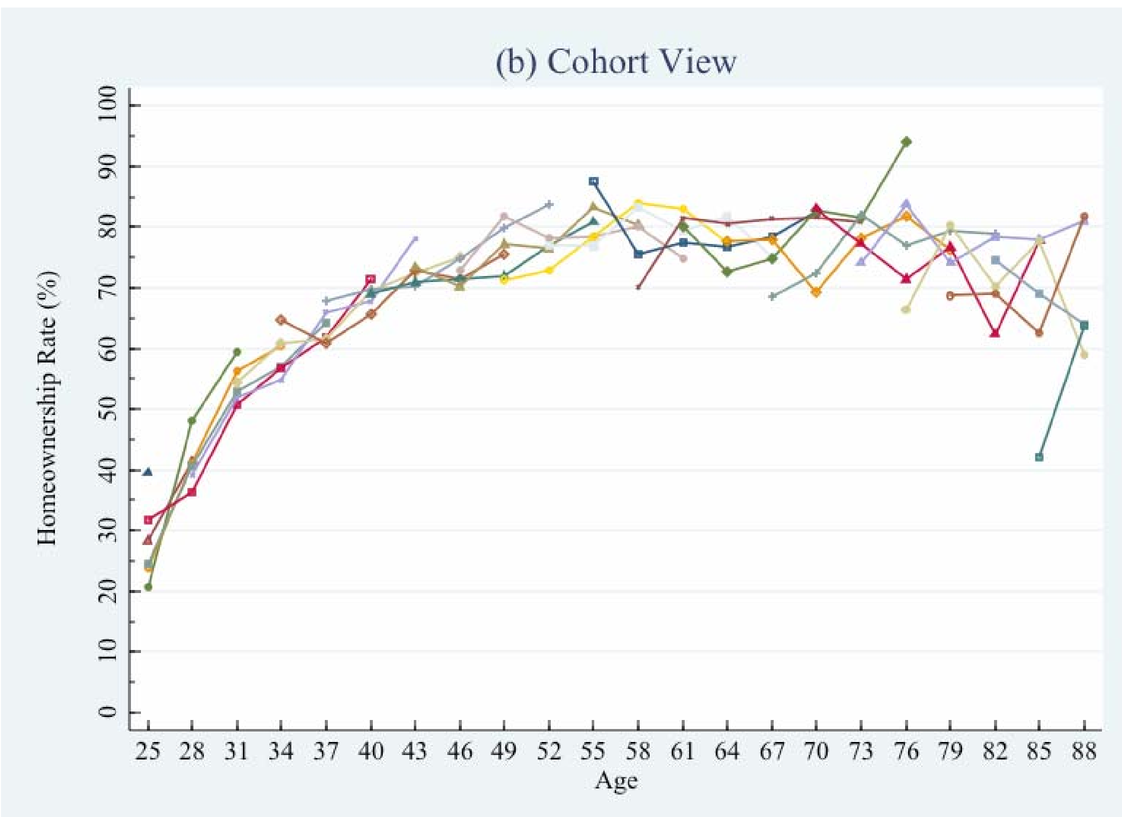


Figure 2 Median House-to-Net-Worth Ratio, 1989-2004

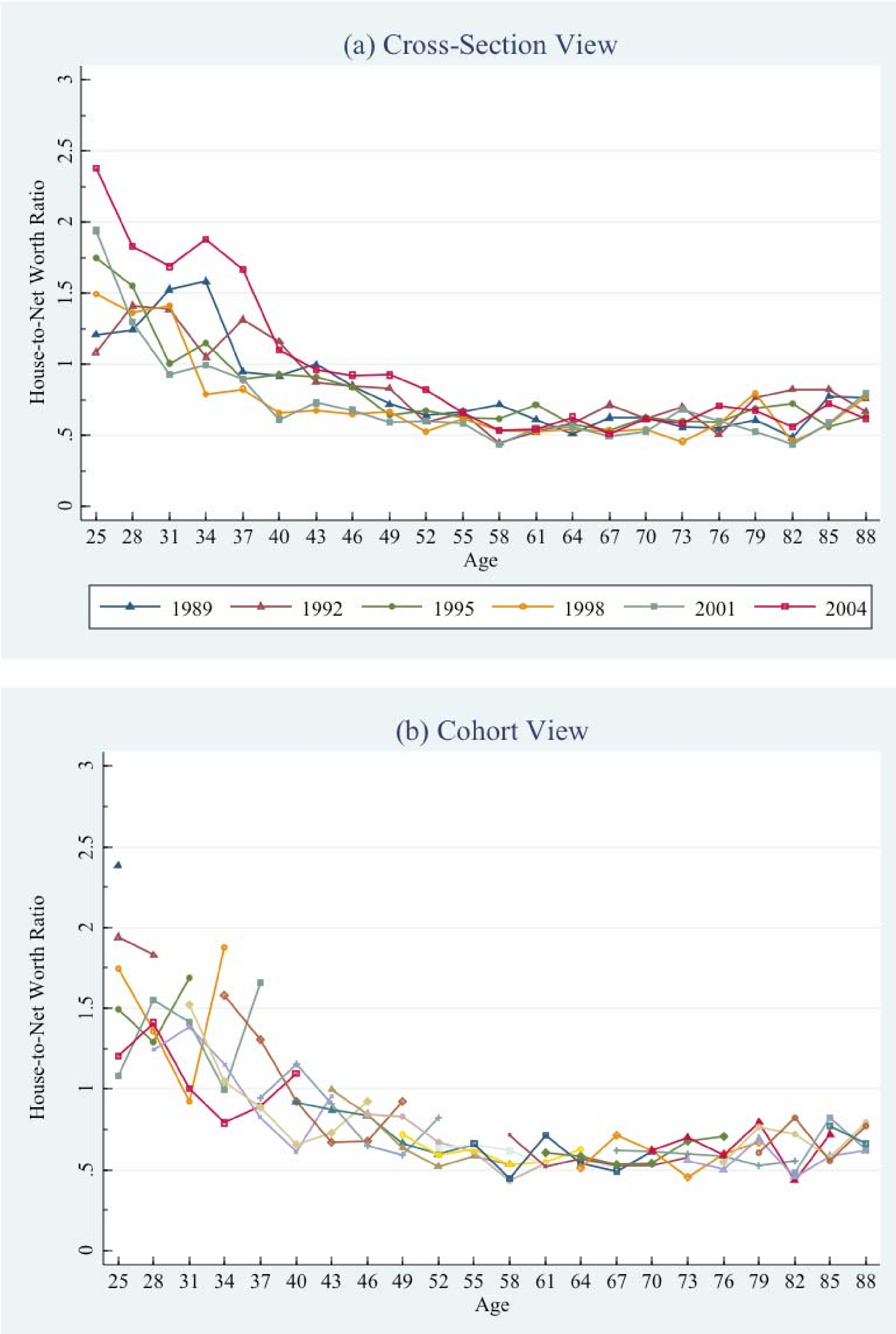
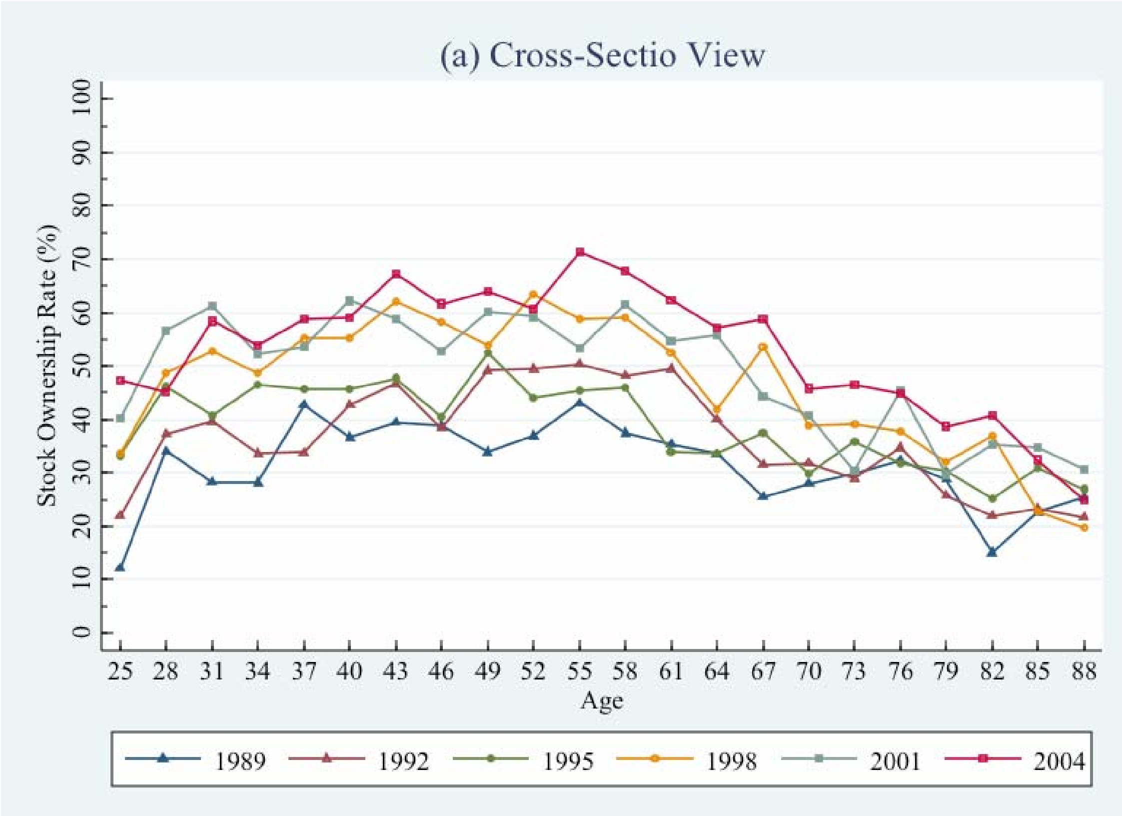


Figure 3 Stockownership Rate, 1989-2004



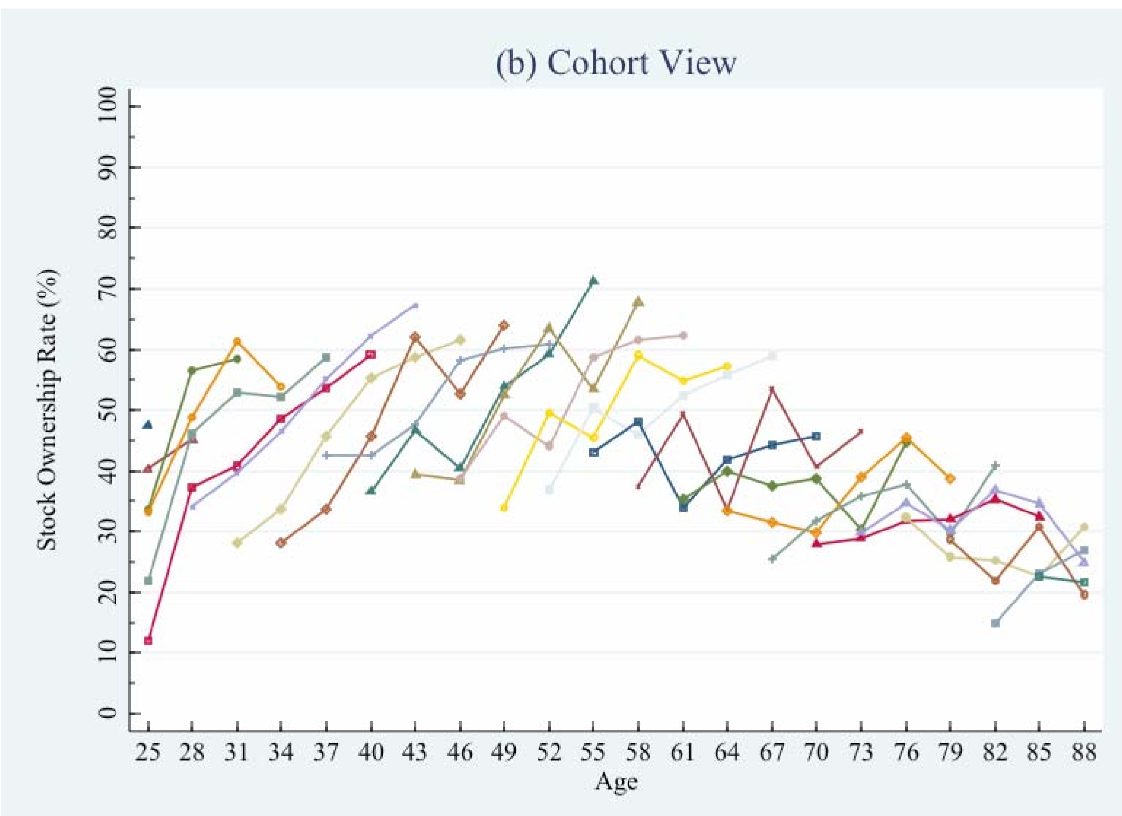
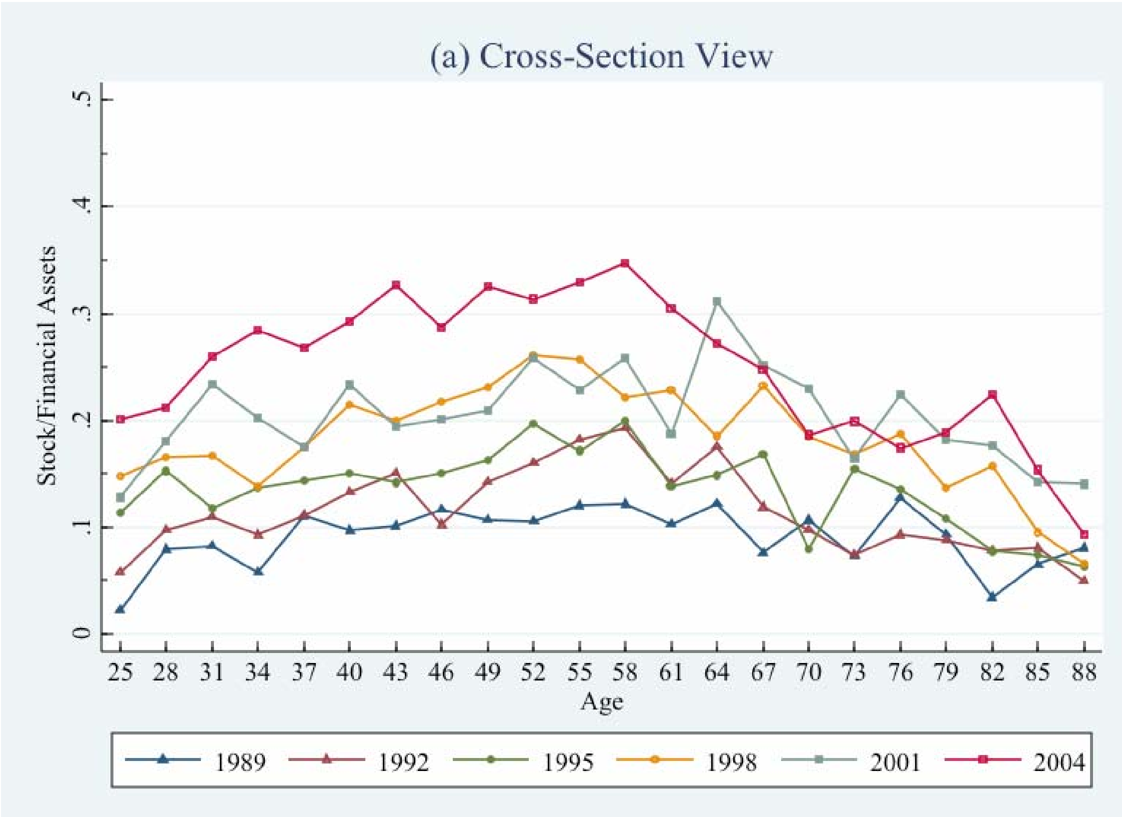


Figure 4 Mean Portfolio Share of Stock Investment, 1989-2004



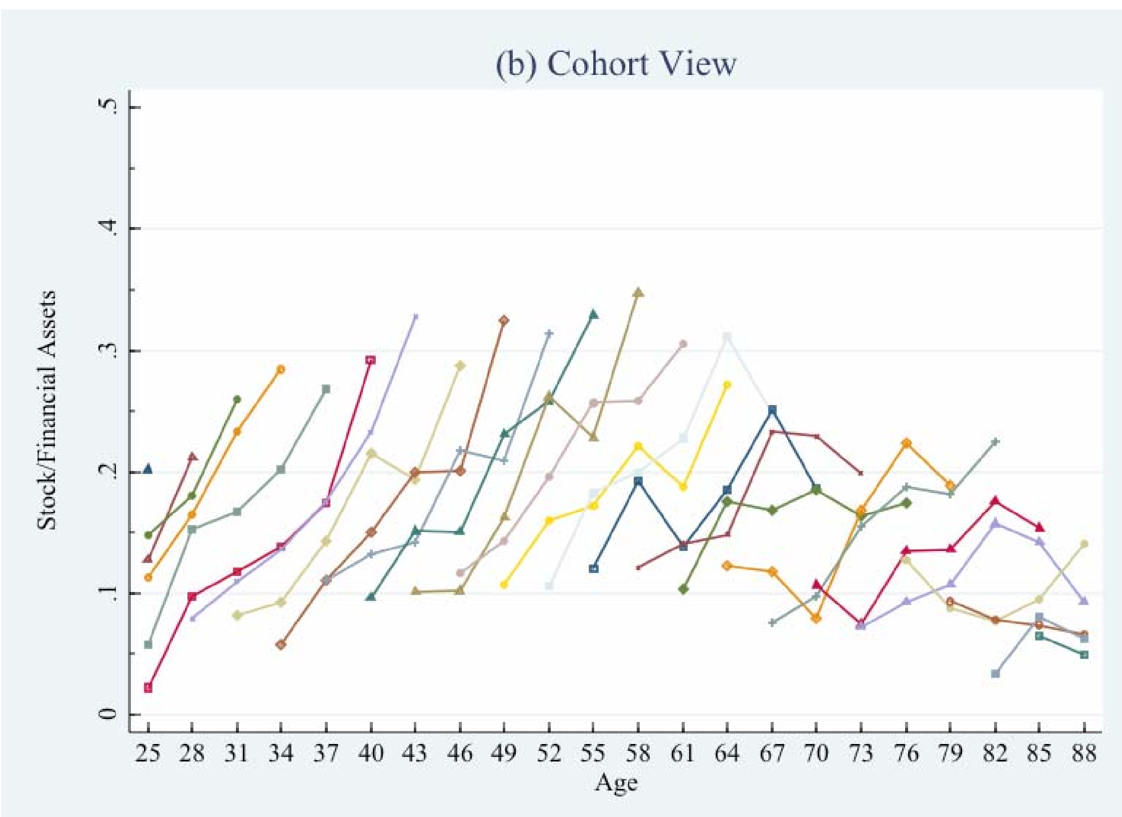
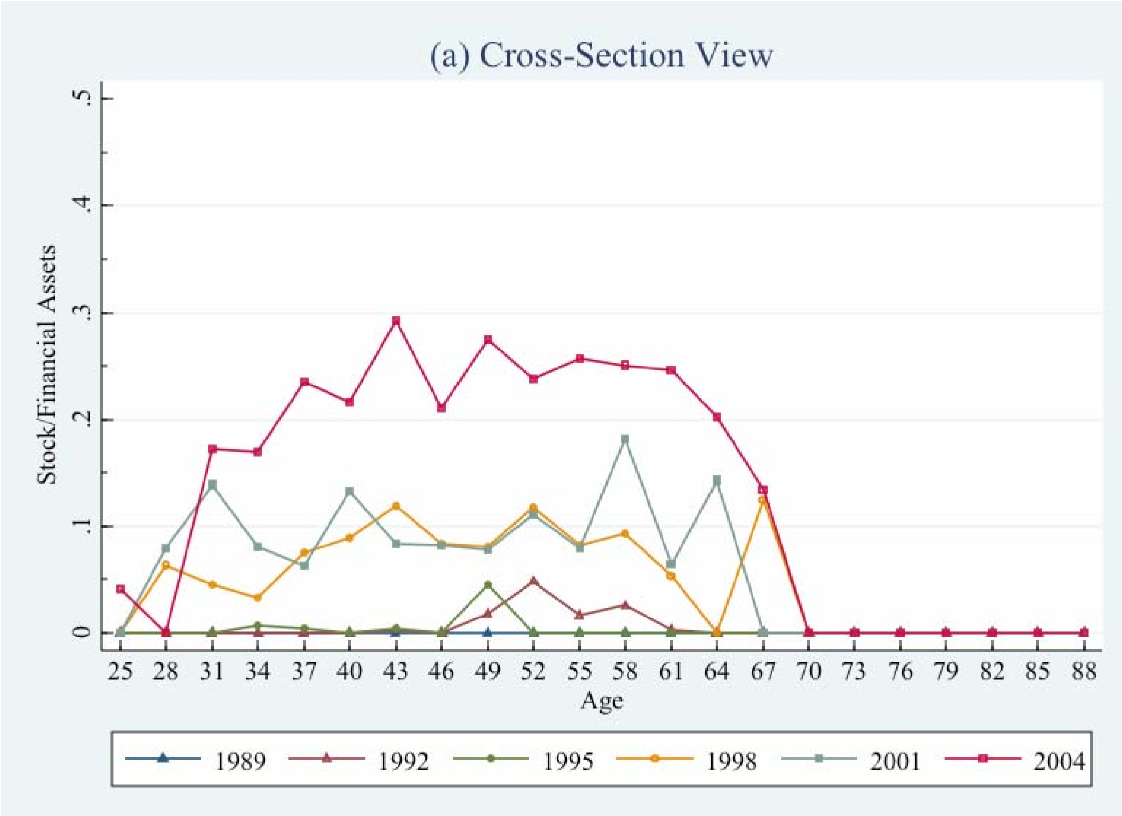


Figure 5 Median Portfolio Share of Stock Investment, 1989-2004



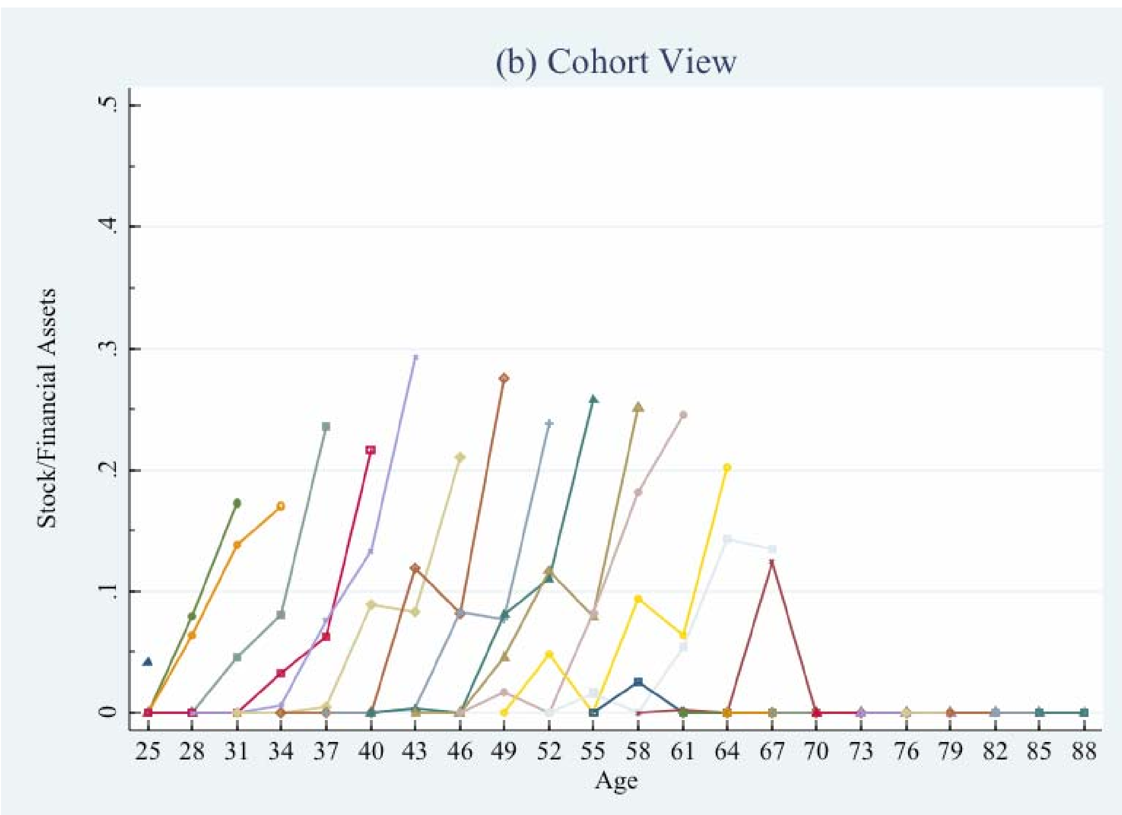


Figure 6 Portfolio Share of Stock Investment versus House-to-Net-Worth Ratio, 1989-2004

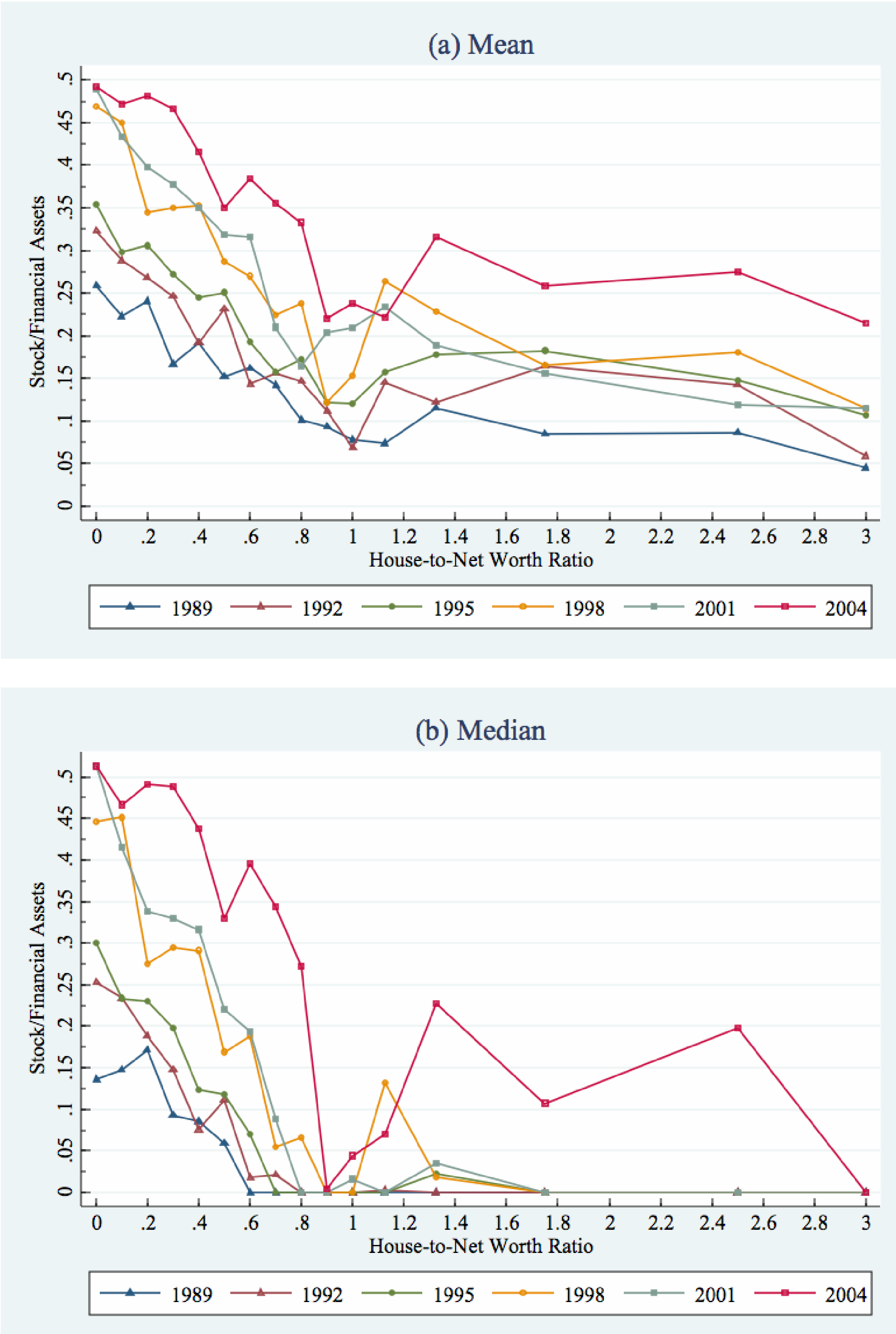


Figure 7 Stockownership Rate versus House-to-Net-Worth Ratio, 1989-2004

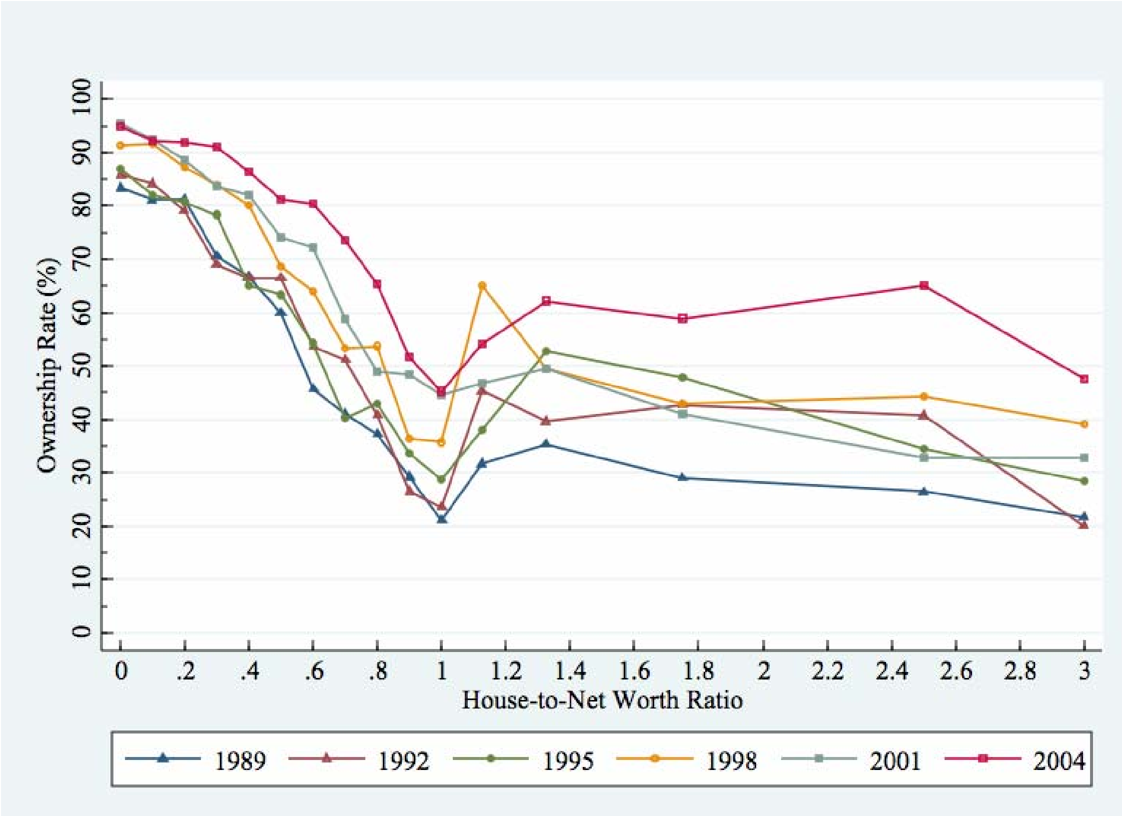


Figure 8 Portfolio Share of Stocks vs. House-to-Net-Worth Ratio (Stockholders Only), 1989-2004

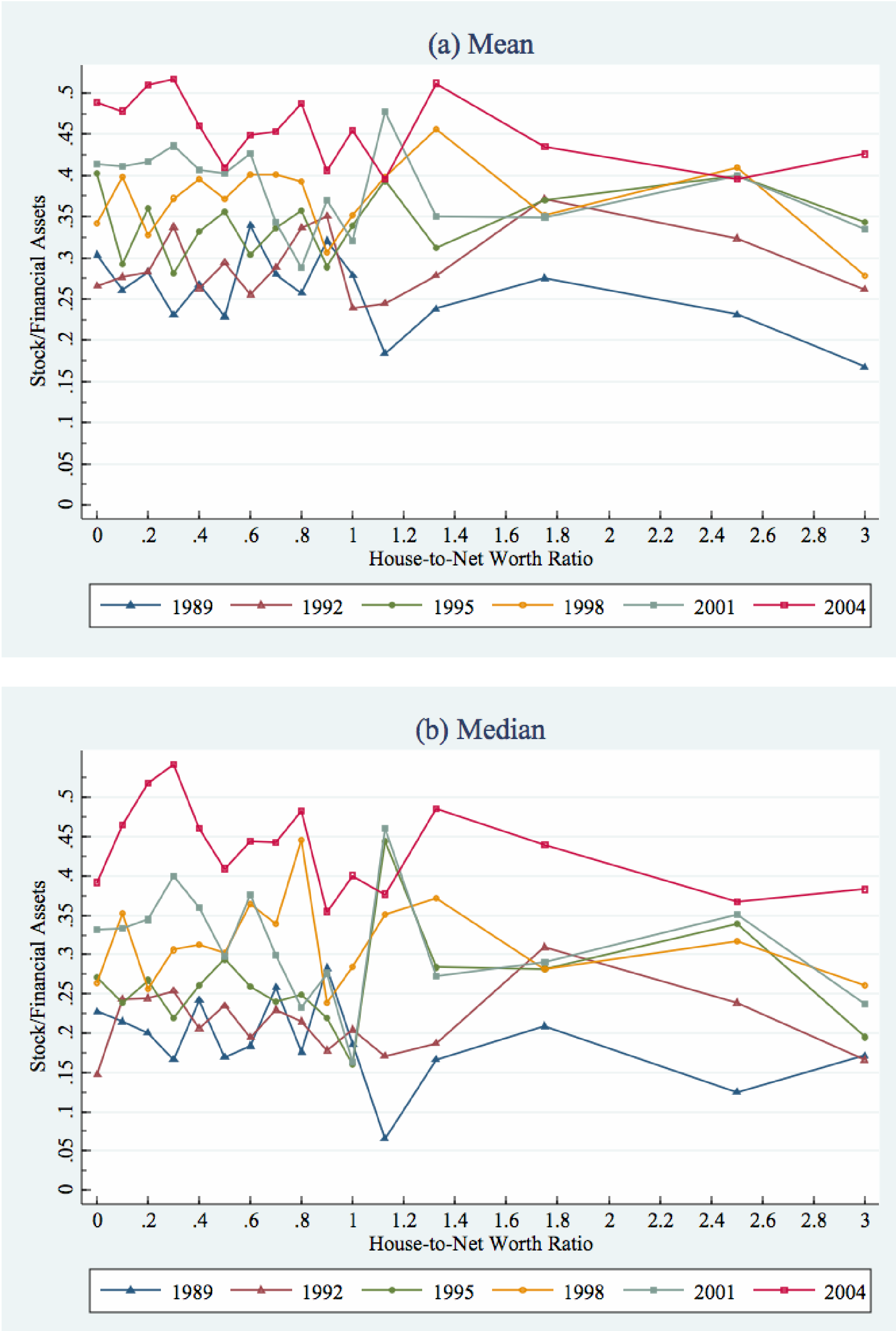
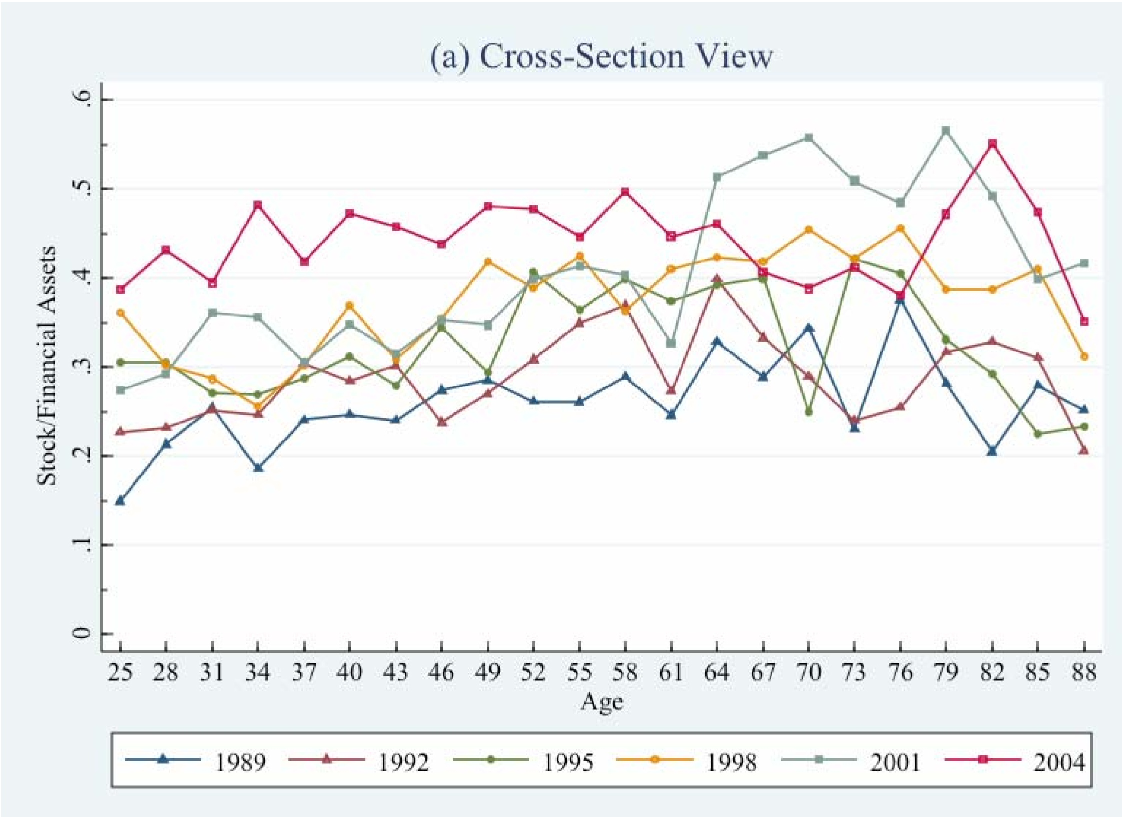


Figure 9 Mean Portfolio Share of Stock Investment (Stockholders Only), 1989-2004



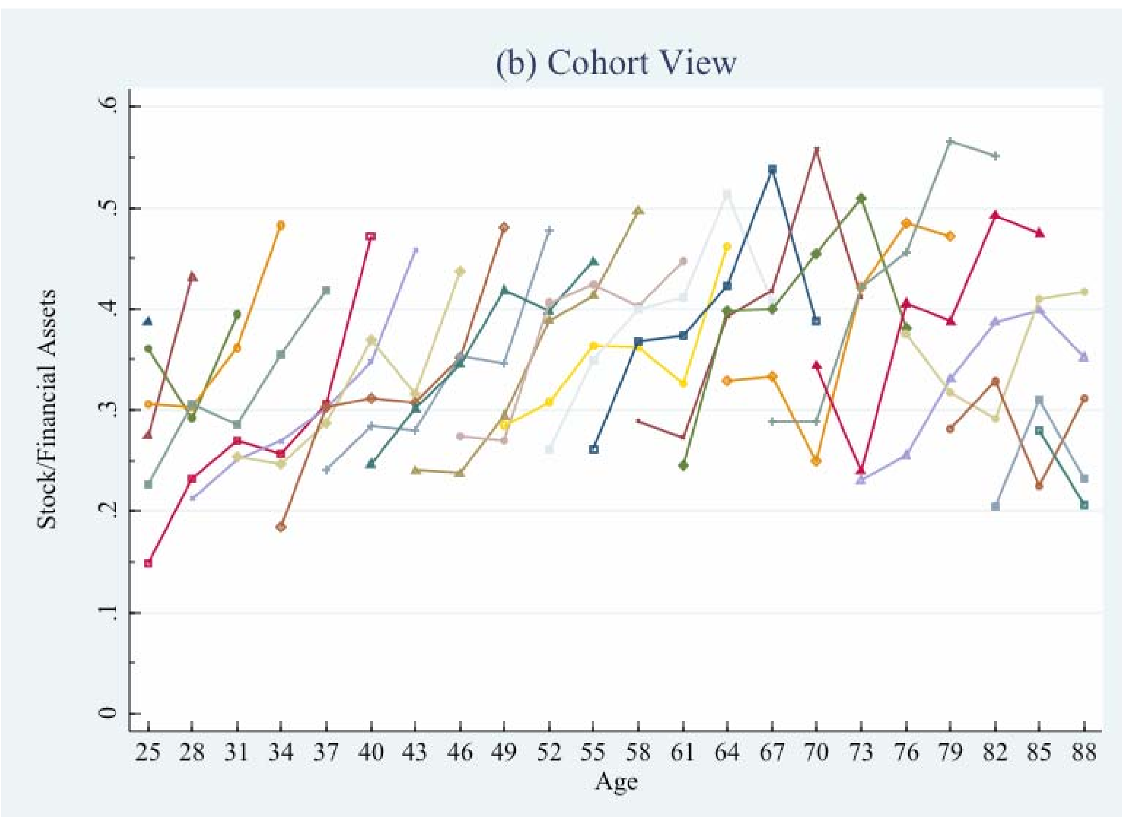
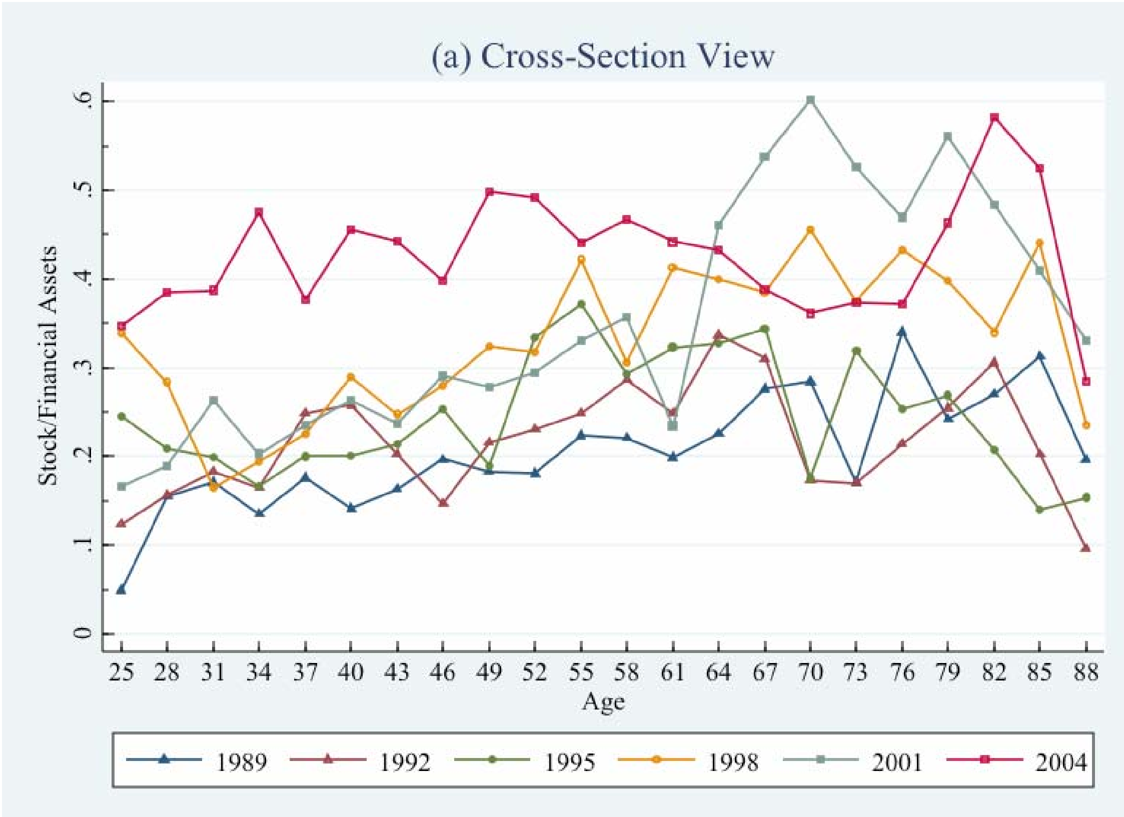


Figure 10 Median Portfolio Share of Stock Investment (Stockholders Only), 1989-2004



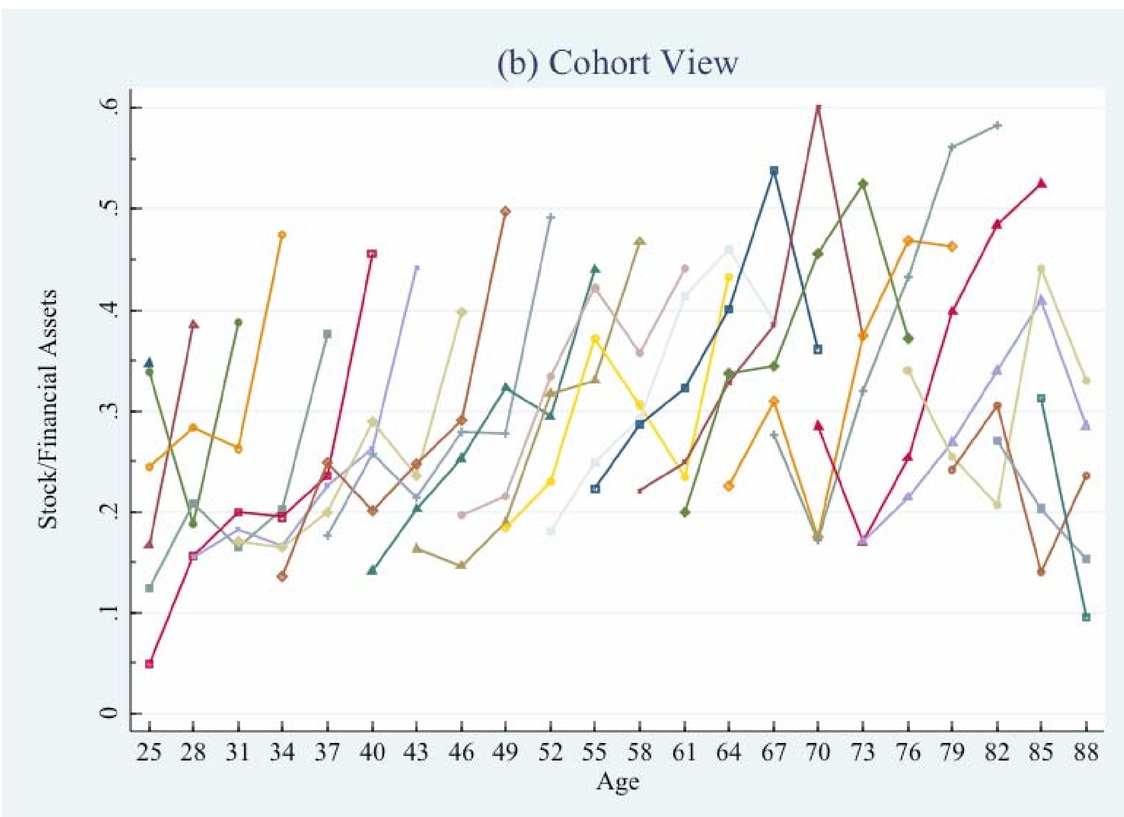
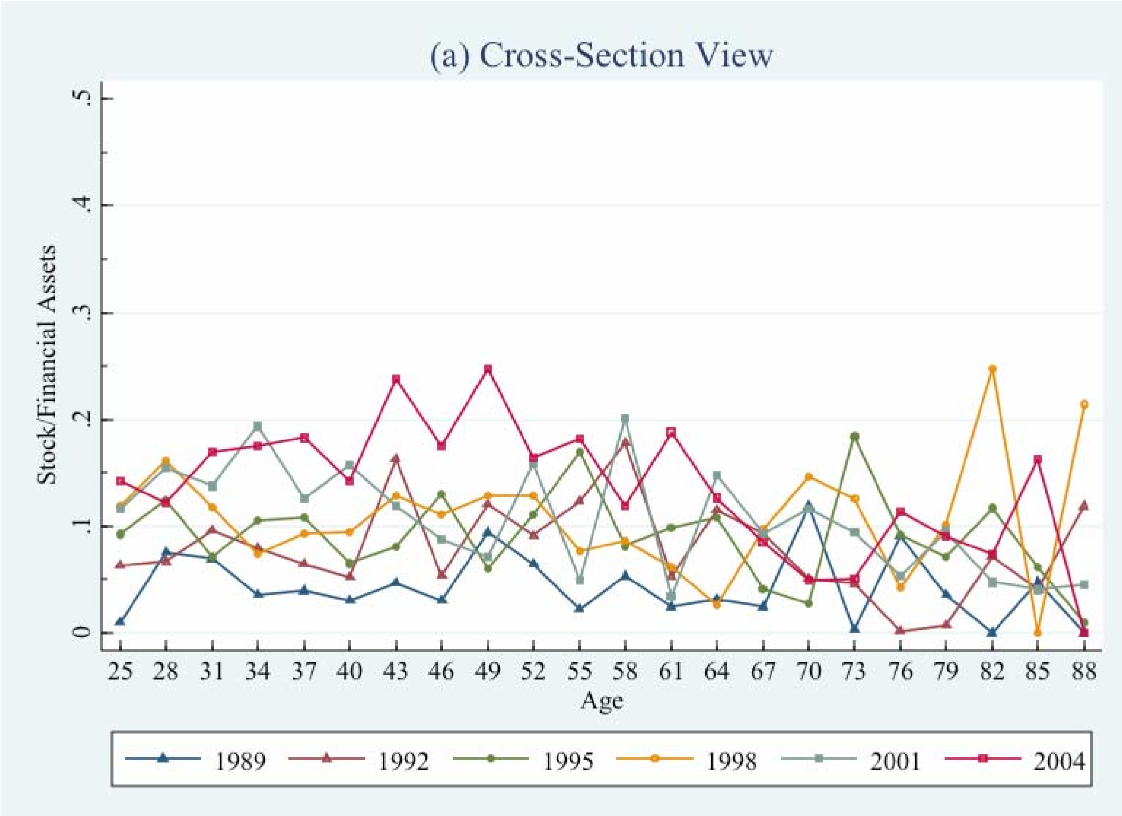


Figure 11 Mean Portfolio Share of Stock Investment (Non-Homeowners Only), 1989-2004



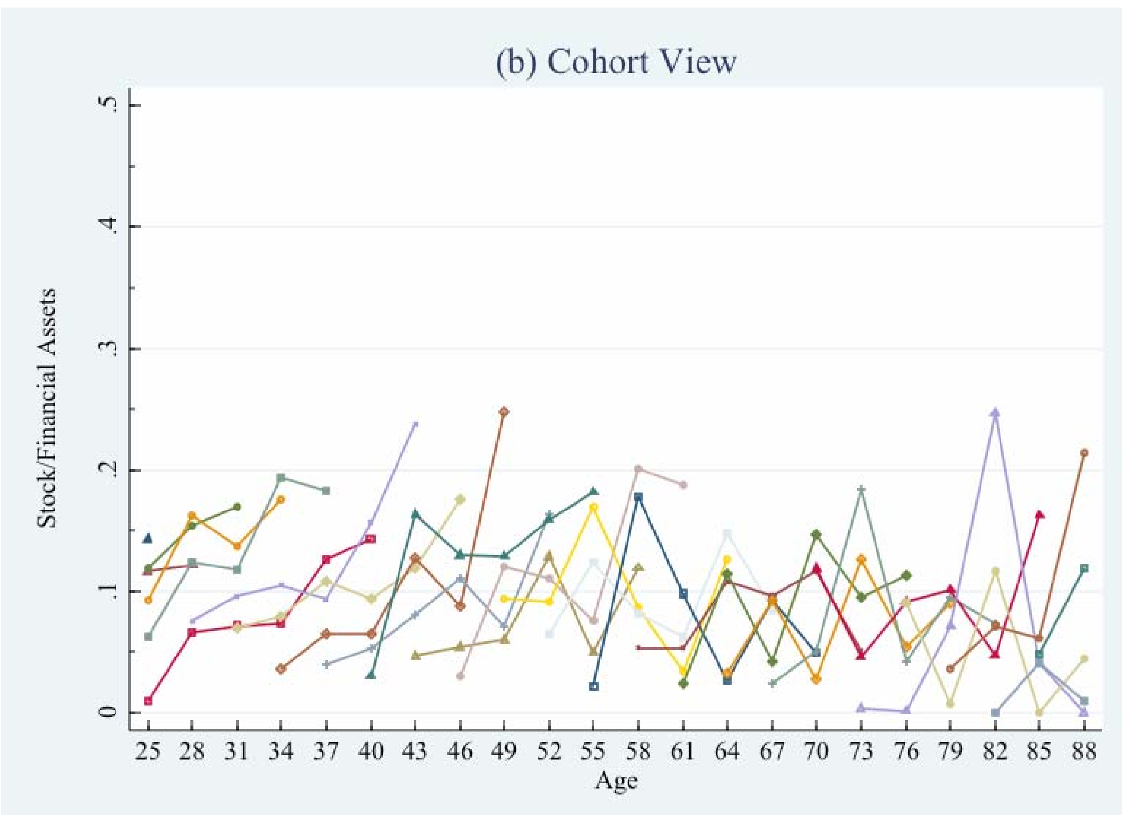


Figure 12 Portfolio Share of Stocks vs. LTV Ratio, 1989-2004